PROCEEDINGS

OF THE

AMERICAN SOCIETY OF CIVIL ENGINEERS

Vol. 63 DECEMBER, 1937 No. 10

TECHNICAL PAPERS

AND

DISCUSSIONS

Published monthly, except July and August, at Prince and Lemon Streets, Lancaster, Pa., by the American Society of Civil Engineers. Editorial and General Offices at 33 West Thirty-ninth Street, New York, N. Y. Reprints from this publication may be made on condition that the full title of Paper, name of Author, page reference, and date of publication by the Society, are given.

Entered as Second-Class Matter, September 23, 1937, at the Post Office at Lancaster, Pa., under the Act of March 3, 1879. Acceptance for mailing at special rate of postage provided for in Section 1103, Act of October 3, 1917, authorized on July 5, 1918.

Subscription (if entered before January 1) $8.00 per annum. Price $1.00 per copy.

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AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
PAPERS

ECONOMIC PIPE SIZES FOR WATER DISTRIBUTION SYSTEMS

BY THOMAS R. CAMP,[1] M. AM. SOC. C. E.

SYNOPSIS

The method for determining the economic size of a single pipe line is well established and has been used for some time. In the past some efforts have been directed toward the extension of this method to a water distribution system, but the procedure to be used was not sufficiently developed for practical application. Had the method been available, it would have been of little use in design because of the difficulty of determining the distribution of flow in a pipe network. Practical methods for analyzing the distribution of flow in pipe networks have been published by the writer elsewhere[2] as well as by Hardy Cross,[3] M. Am. Soc. C. E., and James J. Doland,[4] M. Am. Soc. C. E. In order to apply one of these methods to the actual design of a distribution system, it becomes desirable to give further thought to the economic sizes of pipes. It is the purpose of this paper to develop methods for determining the proper sizes of pipes in distribution systems for best economy.

INTRODUCTION

The general method for determining the economic size of pipe is to express the total cost of the system as a function of the pipe sizes and find the pipe sizes that will make this cost a minimum. The total cost consists of the fixed cost, capital plus depreciation, and the operating cost which is the cost of pumping. Two general types of systems are encountered in practice, namely, those with pumped supplies, and gravity systems in which there is no cost of pumping. Mr. George W. Tuttle[5] has shown that in some gravity systems

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NOTE.—Discussion on this paper will be closed in **April, 1938,** *Proceedings.*

[1] Associate Prof. of San. Eng., Mass. Inst. Tech., Cambridge, Mass.

[2] "Hydraulic Analysis of Water Distribution Systems by Means of an Electric Network Analyzer," by T. R. Camp, M. Am. Soc. C. E., and H. L. Hazen, *Journal*, New England Water Works Assoc., December, 1934, p. 383.

[3] "Analysis of Flow in Networks of Conduits or Conductors," by Hardy Cross, *Bulletin 286*, Eng. Experiment Station, Univ. of Illinois, Urbana, Ill.

[4] "Simplified Analysis of Flow in Water Distribution Systems," by J. J. Doland, *Engineering News-Record*, October 1, 1936, p. 475.

[5] "The Economic Velocity of Transmission of Water Through Pipes," by George W. Tuttle, *Engineering Record*, September 7, 1895, p. 258.

the available head is so low that it is more economical to pump. In other words, a gravity system may not be the most economical type for a given case. Nevertheless, once a gravity system is decided upon, the cost of pumping ceases to be a part of the total cost.

Most modern American distribution systems consist largely of cast-iron pipe. The cost of mains in terms of the pipe diameter is presented hereafter only for such pipe. If some of the mains are constructed of other materials, or are built as tunnels, the economic sizes may be derived by methods similar to those presented, provided the costs of such mains can be stated in terms of their sizes. In the following development it is assumed that the cost of pumping, which necessarily includes capital cost of station, equipment, and operator's wages as well as power cost, is directly proportional to the head and to the discharge. It is also assumed that the cost of elevated storage will not be influenced by changes in pipe sizes and that this cost, therefore, need not be considered.

In the design of a distribution system, the location of the mains is usually determined by the location of the consumers. Preparatory to the determination of pipe sizes, the system is divided into districts, and the domestic and fire drafts are estimated for each district. The system is then "skeletonized" to show only the larger mains which play important parts in the delivery of water to each critical point in the system. The domestic and fire loads for each district are assumed to be concentrated at some point or points on the skeleton system. In simplifying the system, the errors introduced by neglect of small pipes must be examined and corrections made if they are found to be too large. Moreover, the location of a fire load in one district may require a modification of the skeleton system used for the fire load in another district.

By the trial-and-error method of, alternately, selecting pipe sizes for the skeleton system and computing corresponding pressure drops, a set of pipe sizes may be found which produces the desired residual pressures at each critical point in the system. The set of pipe sizes thus selected will be only one set of an infinite number each of which may produce the desired residual pressures. In order to obtain the most economical system, it will be necessary to examine the relation of the sizes for best economy as the trial-and-error process is in progress.

Notation.—The symbols used in this paper are defined where they first appear and are assembled for reference in the Appendix.

COST IN TERMS OF PIPE DIAMETER

The first cost of laying cast-iron pipe, in cents per linear foot, based upon cost data cited by the late Dabney H. Maury,[6, 7] M. Am. Soc. C. E., may be expressed as follows:

$$C_c = B\,D = [(0.06 + 0.02\,d)\,C_w + 0.19\,C_l + 0.007\,C_y]\,D \ldots\ldots (1)$$

in which,

$$B = (0.06 + 0.02\,d)\,C_w + 0.19\,C_l + 0.007\,C_y \ldots\ldots\ldots\ldots (2)$$

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[6] *Engineering News-Record*, May 11, 1922, p. 779.

[7] "Water Supply Engineering," by H. E. Babbitt and J. J. Doland, Members, Am. Soc. C. E., McGraw-Hill, 1931, p. 424.

d = the depth of the trench, in feet; C_w = the wage rate for common labor, in cents per hour; C_l = the cost of lead in cents per pound; C_y = the cost of yarn, in cents per pound; and, D = the diameter of the pipe, in inches.

The costs given by Mr. Maury apply to trenches in ordinary soil without pumping, sheeting, rock, or other unpredictable costs. One joint was assumed for every 11 ft of pipe. As Mr. Maury's data made no allowance for contractor's profit, engineering, insurance, and interest during construction, the cost given by Equation (1) has been made approximately 20% greater than Mr. Maury's cost, in order to cover these items. The unpredictable costs may be omitted safely from the equation inasmuch as they will not be affected much by the pipe size. The cost of hydrant installations is also independent of the size of the main and may be omitted from the equation.

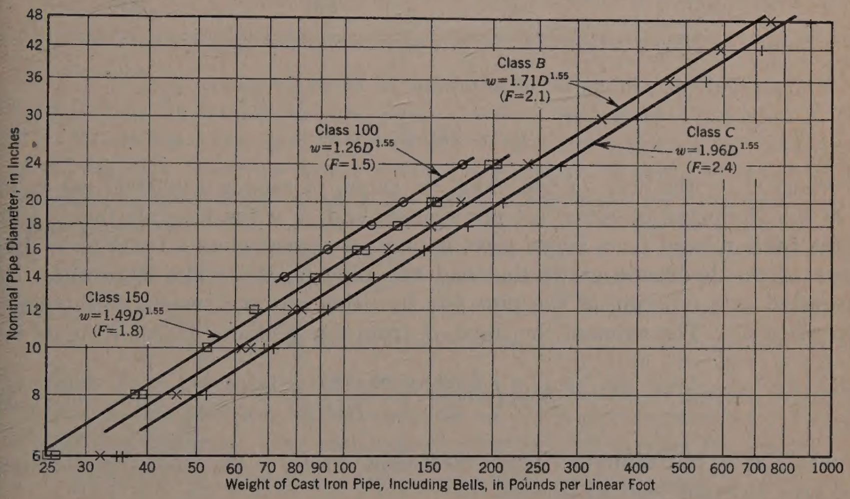

FIG. 1

A study of the weights of standard bell-and-spigot cast-iron pipe (Fig. 1) indicates that the weight, in pounds per linear foot, varies as the 1.55 power of the diameter with errors not exceeding 10% for any single pipe size. A study of the weights of fittings, which are frequently paid for by the pound, indicates that these weights may be assumed to vary as $D^{1.55}$. A study of valve costs shows that they also may be assumed to vary as $D^{1.55}$ for practical purposes. A study of the cost of linings, as given by Mr. E. T. Killam,[8] indicates that either cement or bitumastic enamel lining will cost about 7% of the cost of unlined cast-iron pipe. Therefore, the first cost of materials, including cast-iron pipe, valves, fittings, and pipe lining, in cents per linear foot of pipe, may be expressed by the following empirical equation:

$$C_m = F\,C_f\,D^{1.55} \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

in which C_f = the cost of cast-iron pipe, in cents per pound, with freight allowed; and, F = a coefficient giving the cost of different types of pipe at the trench, including appurtenances and 20% allowed for contractor's profit, engineering, etc. Typical values of F are given in Table 1.

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[8] "Economic Significance of Pipe Linings," by E. T. Killam, *Water Works and Sewerage*, March, 1933, p. 73.

TABLE 1.—VALUES OF F, EQUATION (3)

Class	UNLINED PIPE		LINED PIPE (CEMENT OR BITUMASTIC ENAMEL)	
	Pipe only	Pipe with one valve, one tee, and one cross for each 500 ft	Pipe only	Pipe with one valve, one tee, and one cross for each 500 ft
100	1.50	2.05	1.65	2.20
150	1.80	2.35	1.95	2.50
B	2.10	2.65	2.25	2.80
C	2.40	2.95	2.55	3.10

The annual cost, in cents, of pumping, or of water power, is:

$$C_a = 236\, C_p\, Q\, h \qquad (4)$$

in which C_p = the cost, or the value, in cents, of raising 1 000 000 gal 1 ft; Q = the discharge, in cubic feet per second; and, h = the head, in feet. If h is the friction head for a single pipe, it may be expressed as a function of the slope of the hydraulic grade line and the pipe length. The slope may be expressed as a function of the pipe size by using any convenient formula for pipe friction. The value of the slope, S, from the Williams-Hazen formula is:

$$S = \left(\frac{1\,594}{\mathsf{C}}\right)^{1.85} \frac{q^{1.85}}{D^{4.87}} \qquad (5)$$

in which C = the Williams-Hazen coefficient; and q = the discharge through the pipe, in cubic feet per second.

GRAVITY SYSTEMS

A gravity system is defined as one in which the total head available for loss through pipes is fixed by the topography. Flow is by gravity from a distributing reservoir, the elevation of which has been fixed without regard to the economic size of pipes in the distribution system. In such a system the cost of pumping, if pumping is required, is fixed arbitrarily by the elevation of the distributing reservoir and, hence, is independent of the size of pipes in the distribution system.

The economic pipe sizes for a gravity system will be one set that utilizes all the available head in friction when delivering the peak flow to each critical point in the system. If the system consists of a single pipe line, as is the case of a gravity water supply line, the determination of the economic size is direct and simple. When there are several pipes in series, however, and the costs per foot differ, there are an infinite number of ways in which the total head may be distributed among the pipes. Only one of these ways will produce a minimum cost for the entire series.

Let the cost of a series of gravity pipe lines be:

$$C = c_1 + c_2 + c_3 + \cdots c_n = f_1(h_1) + f_2(h_2) + f_3(h_3) + \cdots f_n(h_n) \ldots (6)$$

in which $c_1, c_2, c_3 \cdots c_n$ = the cost of Sections 1, 2, 3, $\cdots$ n; and, $f_1(h_1)$, $f_2(h_2)$, $f_3(h_3) \cdots f_n(h_n)$ = these costs expressed as functions of the head loss in each section. The cost of any two connecting pipes in the series may be made a minimum for any given total head for these two pipes by equating to zero the first derivative of this cost with respect to the head loss in either section, thus:

$$\frac{d(c_1 + c_2)}{dh_1} = f_1'(h_1) - f_2'(h_2) = 0 \ldots\ldots (7)$$

since $dh_1 = - dh_2$. Hence, for minimum cost of the first two sections:

$$\frac{dc_1}{dh_1} = \frac{dc_2}{dh_2} \ldots\ldots (8)$$

Since this is the condition of minimum cost for any given total head in the first two sections, it is also the condition of minimum cost for the total head consistent with minimum cost for the entire series. A similar relation exists for each two connecting pipes in the series. Therefore, for minimum cost of the entire series:

$$\frac{dc_1}{dh_1} = \frac{dc_2}{dh_2} = \frac{dc_3}{dh_3} = \cdots \frac{dc_n}{dh_n} \ldots\ldots (9)$$

and,

$$h_1 + h_2 + h_3 + \cdots h_n = H \ldots\ldots (10)$$

in which H = the total head available for loss in the series. A graphical solution of this problem for an aqueduct is given in an example by Messrs. Babbitt and Doland.[9] For cast-iron pipes in a gravity series, the cost is,

$$C = [B_1 D_1 + F C_f D_1^{1.55}] l_1 + [B_2 D_2 + F C_f D_2^{1.55}] l_2 + \cdots \ldots (11$$

in which l_1, l_2, etc., equal the lengths of Sections 1, 2, etc. From the Williams-Hazen formula,

$$D = 16.5 \frac{q^{0.38} l^{0.205}}{C^{0.38} h^{0.205}} \ldots\ldots (12)$$

Substituting this value of D for each pipe size in Equation (11) and differentiating as in Equation (9), the following result is obtained:

$$3.38 \frac{B_1 q_1^{0.38}}{C_1^{0.38} S_1^{1.205}} + 24.5 F \frac{C_f q_1^{0.59}}{C_1^{0.59} S_1^{1.318}} = 3.38 \frac{B_2 q_2^{0.38}}{C_2^{0.38} S_2^{1.205}}$$

$$+ 24.5 F \frac{C_f q_2^{0.59}}{C_2^{0.59} S_2^{1.318}} \ldots\ldots (13)$$

Equation (13) gives the relation between the slopes of the hydraulic grade lines of any two connecting pipes in a gravity series for best economy. It may be solved by trial and error, but the process is difficult and time-consuming.

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[9] "Water Supply Engineering," by H. E. Babbitt and J. J. Doland, Members, Am. Soc. C. E., McGraw-Hill, 1931, p. 237.

By assuming that the exponent of the slope in every term of Equation (13) is 1.25 an approximate solution may be had, as follows:

$$\frac{S_1}{S_2} = \left[\frac{3.38 \dfrac{B_1\, q_1^{0.38}}{\mathsf{C}_1^{0.38}} + 24.5\, F\, \dfrac{C_f\, q_1^{0.59}}{\mathsf{C}_1^{0.59}}}{3.38 \dfrac{B_2\, q_2^{0.38}}{\mathsf{C}_2^{0.38}} + 24.5\, F\, \dfrac{C_f\, q_2^{0.59}}{\mathsf{C}_2^{0.59}}} \right]^{0.8} \ldots\ldots\ldots\ldots\ldots (14)$$

Results obtained by means of Equation (14) will usually vary by not more than 2% from those obtained by Equation (13). In applying it to the design of a distribution system, every pipe series should be considered that delivers water to a critical point for the condition of peak flow to that point. For another loading condition (that is, with the fire load at another critical point) other pipe series will come into play. The main feeders, however, will be parts of series for several loading conditions. The economic sizes of these mains, as determined for the several loading conditions, will not be identical, and it will be necessary to select some standard pipe size which comes nearest to satisfying all the conditions.

Classification of Pumped Supplies

A number of cases arise with pumped supplies which affect the manner of determining the economic pipe sizes. In all these cases, it will be assumed that the minimum head is fixed at the delivery end of the pipe or pipe series at the required residual value for peak flow. The pumping head over and above this static head is chargeable to pipe friction.

If the discharge is constant, this additional pumping head has only one value consistent with best economy for the system; and there is only one set of pipe sizes that will give the least total cost. A distribution reservoir may be inserted in this system without affecting the economic pipe sizes, provided its elevation does not disturb the hydraulic grade lines.

If the discharge is variable, the pumping head may be either fixed or variable, depending upon whether a distribution reservoir is provided near the pump station. Each case will entail a different set of economic pipe sizes, and both sets will differ from the economic sizes for constant discharge.

Case I.—Pumped Supply with Constant Discharge.—If the system consists of a single pipe line discharging only at the lower end, the total annual cost of pipe and pumping against friction per linear foot of pipe is:

$$C_{at} = B\, r\, D + F\, C_f\, r\, D^{1.55} + 236\, C_p \left(\frac{1\,594}{\mathsf{C}} \right)^{1.85} \frac{q^{2.85}}{D^{4.87}} \ldots\ldots\ldots (15)$$

in which r = the annual rate of interest plus depreciation of the pipe. Differentiating this cost with respect to the diameter and equating the result to zero, the following relation is obtained:

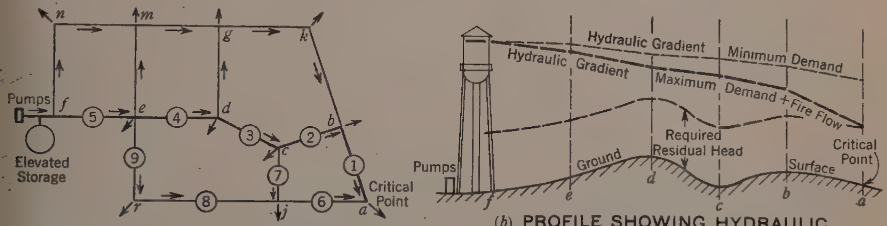
$$B\, r\, D^{5.87} + 1.55\, F\, C_f\, r\, D^{6.42} = 1\,150\, C_p \left(\frac{1\,594}{\mathsf{C}} \right)^{1.85} q^{2.85} \ldots\ldots (16)$$

Equation (16) may be solved by trial and error for the economic diameter of pipe. However, if the exponent of D in both terms is assumed to be 6.15, following the procedure of Babbitt and Doland,[10] the following approximate solution for the economic diameter is obtained:

$$D_{ec} = \frac{27.4}{C^{0.30}} \left[\frac{C_p}{r\,(B + 1.55\,F\,C_f)} \right]^{0.163} q^{0.46} \ldots\ldots\ldots\ldots\ldots (17)$$

The coefficient in Equation (17) has been adjusted to 27.4 to compensate for the error introduced by changing the exponents. This formula gives values of the economic pipe diameter within 3% of those given by Equation (16).

If the system consists of a network of pipes with a number of take-offs, the pressure at only one take-off point may be made to correspond with the desired residual pressure. At each of the other take-off points there is an excess of pressure, and there is a power loss, therefore, corresponding to the draft from the system and this excess pressure. Some of the power loss at take-offs is due to the topography of the city and is not influenced by pipe sizes; but some of it is properly chargeable to pipes inasmuch as larger pipes down stream from a take-off point would reduce the excess pressure at the point.



(a) PLAN OF SYSTEM

(b) PROFILE SHOWING HYDRAULIC GRADIENT ALONG ONE ROUTE

FIG. 2.—ILLUSTRATING CASE I, PUMPED SUPPLY WITH CONSTANT DISCHARGE THROUGHOUT

In the simple network represented by Fig. 2(*a*) all the excess head at Point *b* is chargeable to Pipe 1. The part of the excess head at Points *c*, *e*, and *f*, corresponding to the lost head in Elements 2, 4, and 5, is chargeable, respectively, to those pipes. The excess head at Point *d* is due to Pipes 1 and 2; and the difference between the total head charged to these pipes and the excess at Point *d* is chargeable to Pipe 3. Since this value is negative in the example, the take-off power charge for Pipe 3 is a credit. To formulate a rule:

> (1) The head corresponding to the take-off power loss chargeable to any pipe is the head lost by friction in the pipe, or the difference between the excess head at the upper end of the pipe and the sum of the heads chargeable to the pipes down stream, whichever is the smaller.

The take-off power loss chargeable to Element 1, Fig. 2, is due to the draft from the system at Points *b* and *k*, and a part of the draft from Points *c*, *d*, *e*, *m*, and *n*. Since the loss due to the take-off at Point *c* is chargeable to Elements 6 and 7 as well as to Pipes 1 and 2, it is convenient to apportion the loss by distributing the draft at Point *c*, between the two routes. A similar distribution of the draft from other points should be made between the routes down stream from these points. To formulate a rule:

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[10] "Water Supply Engineering," by H. E. Babbitt and J. J. Doland, Members, Am. Soc. C. E., McGraw-Hill, 1931, p. 425.

> (2) The discharge corresponding to the take-off power loss chargeable to any pipe is the sum of the drafts from the system at all points up stream from the pipe, a suitable apportionment of drafts being made between parallel routes.

For any pipe of a network through which water is pumped at a constant rate, the total annual cost, per linear foot of pipe, of pipe and power loss due to pipe friction and take-offs is given by:

$$C_{at} = B\, r\, D + F\, C_f\, r\, D^{1.55} + 236\, C_p \left(\frac{1\,594}{\mathrm{C}}\right)^{1.85} (q + m\, Q_t)\, \frac{q^{1.85}}{D^{4.87}} \quad (18)$$

in which q = the discharge through the pipe, in cubic feet per second; Q_t = the total take-off discharge chargeable to the pipe, in cubic feet per second, estimated as described herein; and, m = the ratio of the take-off head to the head lost by friction in the pipe, the take-off head being estimated as described herein.

Following the procedure used in the derivation of Equation (17), the economic diameter of pipe for any element in a network through which water is pumped at a constant rate, is given by the following equation:

$$D_{ec} = \frac{27.4}{\mathrm{C}^{0.30}} \left[\frac{C_p\,(q + m\, Q_t)}{r\,(B + 1.55\, F\, C_f)}\right]^{0.163} q^{0.30} \quad (19)$$

Case II.—Pumped Supply with Varying Demand and Varying Pumping Head; No Storage in System.—If throughout the design period, the discharge and head loss for any element vary such that q_a, q_b, q_c, etc. = the discharge for times, t_a, t_b, t_c, etc., and h_a, h_b, h_c, etc. = the corresponding friction heads, the average annual cost of the power loss is:

$$\begin{aligned} C_a &= 236\, C_p \left[q_a\, h_a \frac{t_a}{t} + q_b\, h_b \frac{t_b}{t} + q_c\, h_c \frac{t_c}{t} + \cdots \right] \\ &= 236\, C_p\, q\, h \left[\frac{q_a\, h_a}{q\, h} \frac{t_a}{t} + \frac{q_b\, h_b}{q\, h} \frac{t_b}{t} + \frac{q_c\, h_c}{q\, h} \frac{t_c}{t} + \cdots \right] = 236\, C_p\, A\, q\, h \quad (20) \end{aligned}$$

in which q and h = the average discharge through the element and the corresponding head, respectively, over the design period; t = the design period; and,

$$A = \left[\frac{q_a\, h_a}{q\, h} \frac{t_a}{t} + \frac{q_b\, h_b}{q\, h} \frac{t_b}{t} + \cdots \right] \quad (21)$$

Using the Williams-Hazen formula,

$$A = \left(\frac{q_a}{q}\right)^{2.85} \frac{t_a}{t} + \left(\frac{q_b}{q}\right)^{2.85} \frac{t_b}{t} + \left(\frac{q_c}{q}\right)^{2.85} \frac{t_c}{t} + \cdots \quad (22)$$

For a particular case of a single pipe line with a 20-yr design period and with a constant pumping rate throughout any one day, the writer estimated a value of 2.1 for A. For the same pipe line designed to care for hourly fluctuations in demand, the value of A was estimated at 2.9.

The economic diameter of any pipe in a system through which water is pumped at a varying rate and against a varying head (Fig. 3) is given by the following equation:

$$D_{ec} = \frac{27.4}{C^{0.30}} \left[\frac{C_p\, A\, (q + m\, Q_t)}{r\, (B + 1.55\, F\, C_f)} \right]^{0.163} q^{0.30} \ldots\ldots\ldots\ldots (23)$$

in which q = the average discharge through the pipe throughout the design period; and m and Q_t = values corresponding to q estimated as described for Equation (18).

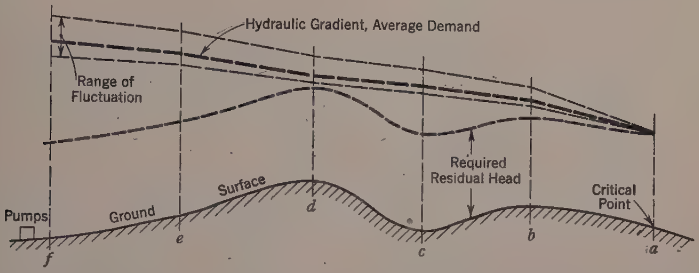
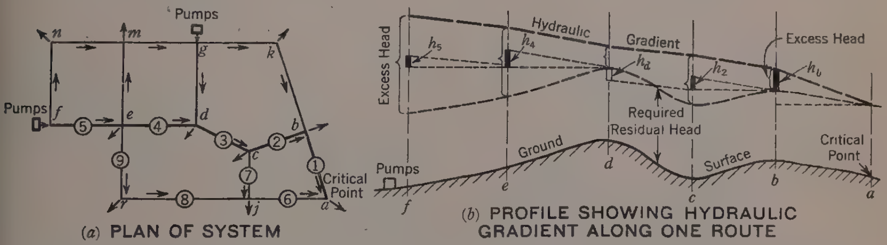


FIG. 3.—PROFILE SHOWING HYDRAULIC GRADIENT ALONG ONE ROUTE, FOR CASE III, PUMPED SUPPLY WITH VARYING DEMAND AND VARYING PUMPING HEAD; PLAN OF SYSTEM SAME AS FIG. 2; NO STORAGE

The foregoing analysis is based upon the assumption that the flow is distributed throughout the system in the same ratios at all times. Fire flows are excluded since they account for a negligible part of the yearly pumping cost.

It is assumed in the foregoing derivations that for each discharge there is a corresponding friction loss for each pipe. This is not true throughout the design period for pipes in which the friction coefficients change with age. Since the economic diameter of a pipe varies inversely as $C^{0.30}$, a reduction in the value of C during the design period as great as from 130 to 80, for example, will produce a variation in the economic diameter as computed from the foregoing equations of only 15 per cent. If the average value of C expected throughout the design period is used in the equations, the error in the computed economic pipe size will be small.



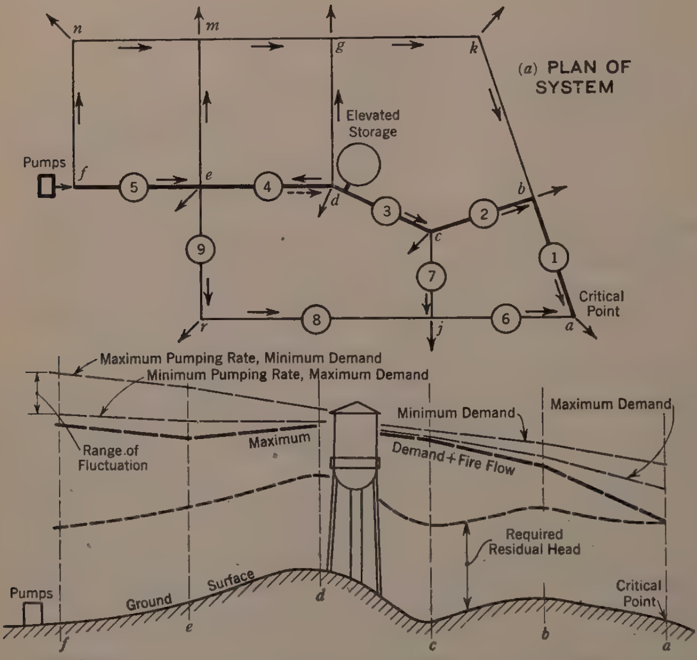
(a) PLAN OF SYSTEM

(b) PROFILE SHOWING HYDRAULIC GRADIENT ALONG ONE ROUTE

FIG. 4.—ILLUSTRATING CASE III, SUPPLY WITH VARYING DEMAND AND CONSTANT PUMPING HEAD. ELEVATED STORAGE AT PUMPS

Case III.—Pumped Supply with Varying Demand and Constant Pumping Head; Elevated Storage at Pumps.—In this case (Fig. 4) the pumping head will be fixed by the peak flow through the system, including fire flow, and will be maintained constant by a distribution reservoir or elevated tank placed near the pumps at the elevation for best economy. The annual cost of pumping is not affected by the rate of pumping.

The power loss chargeable to each pipe for friction and take-offs is determined from the maximum friction head and from the average discharge over the design period. The elevation of the tank is established by the maximum friction head through the system.



(b) PROFILE SHOWING HYDRAULIC GRADIENT ALONG ONE ROUTE.

FIG. 5.—ILLUSTRATING CASE IV, PUMPED SUPPLY WITH VARYING DEMAND AND VARYING PUMPING HEAD. ELEVATED STORAGE REMOTE FROM PUMPS

The economic diameter of any pipe in the system for a varying discharge and constant pumping head is given by the following equation:

$$D_{ec} = \frac{27.4}{C^{0.30}} \left[\frac{C_p \left(q + m\, Q_t \right)}{r \left(B + 1.55\, F\, C_f \right)} \right]^{0.163} q_{\max}{}^{0.30} \qquad (24)$$

in which q = the average discharge in the pipe throughout the design period; and, $q_{\max}$ = the peak discharge, including fire flow, and other quantities are the same as for Equation (23).

Case IV.—Pumped Supply with Varying Demand and Varying Pumping Head; Elevated Storage Remote from Pumps.—In this case (Fig. 5) the elevation of the distributing reservoir will be determined by the maximum friction loss through the pipes delivering water from the reservoir to the critical point. This will occur with the maximum draft from the system, including a fire load at the critical point. Presumably, the pumps would be operating at their maximum capacity during a fire, as this practice would make for the greatest economy in the design of the system.

The variation in discharge for normal demand is not similar for all pipes in a system of this type. In some of the pipes, as in Pipe 4 of Fig. 5, the direction of flow reverses. For others, such as Pipes 1, 2, and 3, the flow is always in the same direction for normal demand. Although the pumping head varies, the manner of variation is controlled by both the rate of pumping and the demand, which do not vary similarly and are not equal. The increased cost of pumping due to variable head is chargeable to friction loss in the pipes between the pumps and the elevated storage, and is substantially independent of the pipes down stream from the elevated storage.

In so far as the manner of determining economic sizes is concerned, three types of pipes may be distinguished in a system of this class, as follows:

(1) Pipes down stream from the elevated storage on direct routes to the critical point, such as Pipe 2, Fig. 5. These pipes may be considered as falling in Case III, and their sizes may be determined by means of Equation (24).

(2) Pipes on direct routes between pumps and elevated storage, such as Pipe 5, Fig. 5, may be considered as falling in Case II and their sizes determined by Equation (23). The values of A and q can be approximated only very roughly since they are influenced by both demand and pumping rates.

(3) Pipes that are similar in nature to both Types 1 and 2, such as Pipe 4, Fig. 5, which is down stream from the storage for the fire load at Point a and is en route from pumps to storage for minimum demand. Both Equations (23) and (24) may be used for estimating the economic size of pipes of this type. Equation (23) should be given greater weight for Pipe 4 since the influence of this pipe on the fluctuations of the pumping head is greater than its influence upon the established elevation of the storage. About equal weight should probably be given to both equations for finding the size of Pipe 8.

In the application of Equation (23) to the determination of the size of pipes in which there is a reversal of flow, such as in Pipe 4, the effect of the reversal is to increase the range of fluctuation of the pumping head. Since negative friction losses correspond with savings in pumping head, however, they should be credited to the pipe by using negative values of the corresponding terms in Equation (22) for the solution of A, which value will thus be small for pipes in which there is a reversal of flow for normal demands.

ECONOMIC VELOCITY

The discussion of economic pipe sizes in terms of economic velocity has been purposely avoided in this paper. Notwithstanding the fact that many engineers have become accustomed to thinking of the economics of pipes in terms of velocities, the practice is valid only in very special cases.

If the system consists of a single pipe through which water is pumped at a constant rate at all times (Equation (17)), the velocity does bear a definite relation to the diameter for best economy for a given set of costs. Even for this special case, which is unusual in practice, the relation varies with the unit costs.

If the pumping rate is constant and the system consists of a network of pipes, even for a single set of unit costs, there is no single relation between the economic pipe size and the velocity. The size is determined by both the discharge through the pipe and the take-offs from the system.

If the layout is a gravity system, there is no relation between the velocity and the pipe size for best economy which is independent of the head available. There is no such thing, therefore, as an economic velocity for a given pipe size in a gravity system.

If the system consists of pipes through which water is pumped at a variable rate, the question arises immediately as to which discharge rate should correspond with the economic velocity. If it is assumed that the economic velocity should be determined from the average discharge through the pipe, it is found that it does not bear the same relation to the diameter as in the case of constant discharge. Moreover, since the economic diameter is affected by the manner of variation of discharge, the corresponding velocity is also affected in a similar manner. In this case, again, the take-offs from the system affect the relation of diameter to velocity for best economy.

APPENDIX

NOTATION

The symbols introduced in this paper are defined as follows:

A = a substitution constant (see Equation (21));

B = a substitution constant (see Equation (2));

C = cost of a series of gravity pipe lines; C_a = annual cost, in cents, of pumping, or of water power; C_c = first cost of laying cast-iron pipe, in cents per linear foot; C_f = cost of cast-iron pipe, in cents per pound, with freight allowed; C_l = cost of lead, in cents per pound; C_m = cost of materials, including cast-iron pipe, valves, fittings, and pipe lining, in cents per linear foot; C_p = cost or value, in cents, of raising 1 000 000 gal 1 ft; C_w = the wage rate for common labor, in cents per hour; C_y = cost of yarn, in cents per pound; C_{at} = total annual cost of pipe and pumping, in cents per linear foot of pipe;

$\mathbf{C}$ = Williams-Hazen coefficient;

c = cost of a section of pipe = $f(h)$;

D = diameter of pipe, in inches;

d = depth of pipe trench, in feet;

F = a coefficient giving the cost of different types of pipe at the trench, including contractors' profit, engineering, etc.;

f = function of: $f(h)$ = cost, c, expressed as a function of the head loss of the section;

H = total head available for loss in a series of pipe lines;

h = head loss through a single pipe or element;
l = length of sections of pipe;
m = ratio of take-off head to head lost by friction in the pipe;
Q = discharge, in cubic feet per second; Q_t = take-off discharge;
q = discharge through a single pipe or element, in cubic feet per second;
r = annual rate of interest plus depreciation of the pipe;
S = slope;
t = the design period; and,
w = weight, in pounds per linear foot.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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PAPERS

EARTHQUAKE STRESSES IN AN ARCH DAM

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Assoc. Members, Am. Soc. C. E.

Synopsis

The forces of inertia manifested in a structure during an earthquake are ordinarily taken into account in the design. In this paper formulas are derived for determining the stresses due to the horizontal effect of an earthquake on an arch dam for two different conditions: First, the acceleration of the earthquake is assumed to occur along the canyon, that is, horizontally up stream; and, second, the acceleration is assumed to occur across the canyon, that is, horizontally at right angles to the first condition. The vertical effect of the earthquake is omitted from consideration because of its minor importance. Arches with hinged ends as well as with fixed ends are considered, and illustrative numerical examples are given (see Table 1). The derivation of the formulas is based on Castigliano's Theorem of Least Work.

Introduction

Stresses in thin circular arches due to water load have been investigated at length, but not much has been written of the stresses induced by an earthquake in the arch due to its own inertia. In this paper an attempt is made to investigate these stresses and to find their relative importance in the design of an arch dam. Algebraic symbols are defined where first introduced and are assembled, for convenience of reference, in the Appendix. The formulas derived are for moments, thrusts, and shears induced in an arch by an earthquake for two conditions: (1) Acceleration is assumed to occur along the canyon; and (2) it is assumed to occur across the canyon.

Assumption (1) Applied to Arches with Fixed Ends.—Consider an element within an arch ring, 1 ft deep and having an elementary weight, dW, equal to (see Fig. 1):

$$dW = w_c \, t \, r \, d\theta \quad \text{(1)}$$

in which w_c = unit weight of concrete, in pounds per cubic foot; t = uniform

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Note.—Discussion on this paper will be closed in April, 1938, *Proceedings*.

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radial thickness of the arch, in feet; and r = the radius of the center line of the arch, in feet. With the earthquake acting horizontally and the plane of the arch ring inclined at an angle, ψ, with the vertical, as in the case of a multiple arch, the elementary force, dP, exerted upon each arch element in the plane of the arch ring will be,

$$\frac{dP}{a} = \frac{a_q}{g}\, dW \sin \psi \qquad (2)$$

in which a is the acceleration of the earthquake, and g is the acceleration of gravity. The force, dP, acts through the center of gravity of the arch element, which lies slightly outside the neutral axis at a distance, r', from the arch center. In this case, it can readily be shown that $r' = r\left(1 + \frac{k^2}{r^2}\right)$,

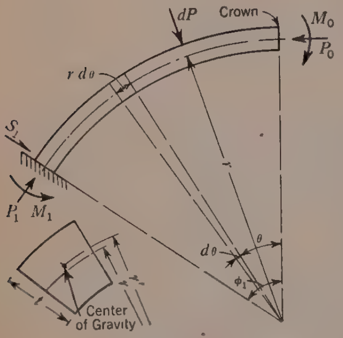

FIG. 1

in which k^2 is substituted for $\frac{t^2}{12}$. Since, for the symmetrically loaded arch, there is no rotation of the crown section under load and no translation of the crown section in the direction of the crown thrust, the partial derivatives of Castigliano's equation for internal elastic work with respect to the crown moment, M_0, and the crown thrust, P_0, are equal to zero.[3] The derivation of the formulas for moment, thrust, and shear for this case is similar to that for the dead load of the arch,[3] and is omitted herein.

With the introduction of r', $\frac{a}{g}$, $w_c' = w_c \sin \psi$, and $C = \frac{a}{g}\, w_c'\, t\, r$, the thrust, moment, and shear at the crown are, respectively,

$$P_0 = C\, Z' \qquad (3a)$$

$$M_0 = C\, r\, Z_2' \qquad (3b)$$

and,

$$S_0 = 0 \qquad (3c)$$

and at the abutments,

$$P_1 = C\, Z_1' \qquad (4a)$$

$$M_1 = C\, r\, Z_3' \qquad (4b)$$

and,

$$S_1 = C\, Z_4' \qquad (4c)$$

in which,

$$Z' = \left[2\left(1 + \frac{r'}{r}\right)\frac{\sin^2 \phi_1}{\phi_1} - \frac{\sin 2\,\phi_1}{2}\left(\frac{5}{2} + \frac{r'}{r}\right) + \frac{\phi_1 \cos 2\,\phi_1}{2} - \phi_1 \frac{r'}{r}\left(\frac{2.88\, r'}{r} - 1\right)\frac{k^2}{r^2}\left(\frac{\sin 2\,\phi_1}{4} - \frac{\phi_1 \cos 2\,\phi_1}{2}\right)\right] \div \left[\left(1 + \frac{k^2}{r^2}\right)\left(\phi_1 + \frac{\sin 2\,\phi_1}{2}\right) - \frac{2 \sin^2 \phi_1}{2} + \frac{2.88\, k^2\, r'}{r^2\, r}\left(\phi_1 - \frac{\sin 2\,\phi_1}{2}\right)\right] \qquad (5)$$

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[3] "Stresses in Inclined Arches of Multiple Arch Dams," by George E. Goodall and Ivan M. Nelidov, Assoc. Members, Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 98 (1933), p. 1208.

and,

$$Z_1' = Z' \cos \phi_1 + \phi_1 \sin \phi_1 \ldots\ldots \tag{6a}$$

$$Z_2' = \left(Z' + \frac{r'}{r}\right)\left(1 - \frac{\sin \phi_1}{\phi_1}\right) - \frac{\sin \phi_1}{\phi_1} + \cos \phi_1 \ldots\ldots \tag{6b}$$

$$Z_3' = \left(Z' + \frac{r'}{r} + 1\right)\left(\cos \phi_1 - \frac{\sin \phi_1}{\phi_1}\right) + \phi_1 \sin \phi_1 \ldots\ldots \tag{6c}$$

and,

$$Z_4' = \phi_1 \cos \phi_1 - Z' \sin \phi_1 \ldots\ldots \tag{6d}$$

Assumption (1) Applied to Arches with Hinged Ends.—The derivation of the formulas for moment, thrust, and shear for this case is similar to that for dead load, as previously mentioned, except, again, for r', $\frac{a}{g}$, $w_c' = w_c \sin \psi$, and C. The moment, thrust, and shear at the crown are, respectively,

$$P_0 = C\, Z'' \ldots\ldots \tag{7a}$$

$$M_0 = C\, r\, Z_2'' \ldots\ldots \tag{7b}$$

and,

$$S_0 = 0 \ldots\ldots \tag{7c}$$

and at the abutments,

$$P_1 = C\, Z_1'' \ldots\ldots \tag{8a}$$

$$M_1 = 0 \ldots\ldots \tag{8b}$$

and,

$$S_1 = C\, Z_4'' \ldots\ldots \tag{8c}$$

in which:

$$\begin{aligned} Z'' = \Bigg[& \frac{3}{4} \sin 2\,\phi_1 + \frac{3\, r' \sin 2\,\phi_1}{2\, r} - \phi_1 \frac{3}{2} \cos 2\,\phi_1 + \phi_1 \sin 2\,\phi_1 \\ & + \frac{r'}{r}(2 + \cos 2\,\phi_1) + \left(\frac{2.88\, r'}{r} - 1\right)\frac{k^2}{r^2}\left(\frac{\sin 2\,\phi}{4} - \phi_1 \frac{\cos 2\,\phi_1}{2}\right)\Bigg] \\ & \div \left[\phi_1 (2 + \cos 2\,\phi_1) - \frac{3}{2} \sin 2\,\phi_1 + \frac{k^2}{r^2}\left(\phi_1 + \frac{\sin \phi_1}{2}\right)\right. \\ & \left. + \frac{2.88\, k^2\, r'}{r^2\, r}\left(\phi_1 - \frac{\sin 2\,\phi_1}{2}\right)\right] \ldots\ldots \end{aligned} \tag{9}$$

and,

$$Z_1'' = Z'' \cos \phi_1 + \phi_1 \sin \phi_1 \ldots\ldots \tag{10a}$$

$$Z_2'' = \left(Z'' + \frac{r'}{r}\right)(1 - \cos \phi_1) - \phi_1 \sin \phi_1 \ldots\ldots \tag{10b}$$

and,

$$Z_4'' = \phi_1 \cos \phi_1 - Z'' \sin \phi_1 \ldots\ldots \tag{10c}$$

In the foregoing equations, if $\frac{r'}{r} = 1$, the value of Z' and Z'' will be identical to those derived[3] elsewhere for the dead load of the arch. For a ratio, $\frac{t}{r} = 0.3$, the value of $\frac{r'}{r} = 1.0075$ and decreases with the decrease of $\frac{t}{r}$, which indicates the error involved by such a substitution.

Assumption (2) Applied to Arches with Fixed Ends.—It is assumed for this case that the acceleration of the earthquake is from right to left, acting parallel to a line drawn through the abutments. The forces producing deformation in the arch ring, induced by the inertia of each elementary part of the arch, act toward the right, opposite in direction to that of the acceleration.

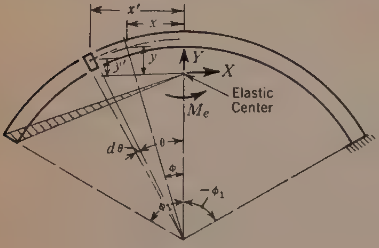

FIG. 2

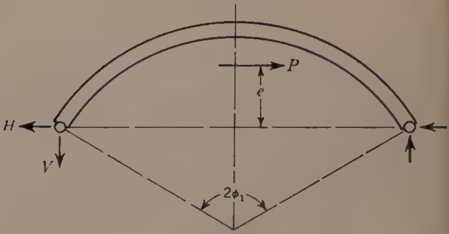

FIG. 3

Since the loading on the arch ring is not symmetrical about the crown and, therefore, the rotation and translation of the crown section are unknown, it is necessary to consider the entire arch ring (see Fig. 2). Assume the arch ring to be fixed, at its right abutment, to the canyon wall, and the left abutment to be held by an imaginary rigid bracket connected to the elastic center of the arch and kept stationary by the forces, X and Y, and the moment, M_e. For a circular arch ring of uniform thickness the elastic center lies at the center of gravity of the neutral axis.

The total internal elastic work in the entire arch ring is:

$$W = \int_{-\phi_1}^{\phi_1} \frac{M^2\,ds}{2\,E\,I} + \int_{-\phi_1}^{\phi_1} \frac{P^2\,ds}{2\,E\,A} + 2.88 \int_{-\phi_1}^{\phi_1} \frac{S^2\,ds}{2\,E\,A} \quad \ldots\ldots\ldots (11)$$

and since there is no rotation or translation of the rigid bracket at the elastic center the partial derivative of the work with respect to X, Y, and M_e is equal to zero, or:

$$\frac{\partial W}{\partial X} = \int_{-\phi_1}^{\phi_1} \frac{M}{E\,I} \frac{\partial M}{\partial X} ds + \int_{-\phi_1}^{\phi_1} \frac{P}{E\,A} \frac{\partial P}{\partial X} ds + \frac{2.88}{E\,A} \int_{-\phi_1}^{\phi_1} S \frac{\partial S}{\partial X} ds = 0 \ldots\ldots (12a)$$

$$\frac{\partial W}{\partial Y} = \int_{-\phi_1}^{\phi_1} \frac{M}{E\,I} \frac{\partial M}{\partial Y} ds + \int_{-\phi_1}^{\phi_1} \frac{P}{E\,A} \frac{\partial P}{\partial Y} ds + \frac{2.88}{E\,A} \int_{-\phi_1}^{\phi_1} S \frac{\partial S}{\partial Y} ds = 0 \ldots\ldots (12b)$$

and,

$$\frac{\partial W}{\partial M_e} = \int_{-\phi_1}^{\phi_1} \frac{M}{E\,I} \frac{\partial M}{\partial M_e} ds + \int_{-\phi_1}^{\phi_1} \frac{P}{E\,A} \frac{\partial P}{\partial M_e} ds + \frac{2.88}{E\,A} \int_{-\phi_1}^{\phi_1} S \frac{\partial S}{\partial M_e} ds = 0 \,.\,. (12c)$$

At any radial section in the arch ring (x, y), the moment, thrust, and shear are,

$$M = M' + X_y + Y_x + M_e \ldots\ldots\ldots\ldots\ldots\ldots\ldots (13a)$$

$$P = P' + X \cos\phi + Y \sin\phi \ldots\ldots\ldots\ldots\ldots\ldots\ldots (13b)$$

and,

$$S = S' + X \sin\phi - Y \cos\phi \ldots\ldots\ldots\ldots\ldots\ldots\ldots (13c)$$

in which M', P', and S' are the moment, thrust, and shear at the section (x, y), produced by the external loads on the arch ring to the left of the section. From Equations (13),

$$\frac{\partial M}{\partial X} = y \qquad (14a)$$

$$\frac{\partial M}{\partial Y} = x \qquad (14b)$$

$$\frac{\partial M}{\partial M_e} = 1 \qquad (14c)$$

$$\frac{\partial P}{\partial X} = \cos \phi \qquad (15a)$$

$$\frac{\partial P}{\partial Y} = \sin \phi \qquad (15b)$$

$$\frac{\partial P}{\partial M_e} = 0 \qquad (15c)$$

and,

$$\frac{\partial S}{\partial X} = \sin \phi \qquad (16a)$$

$$\frac{\partial S}{\partial Y} = -\cos \phi \qquad (16b)$$

$$\frac{\partial S}{\partial M_e} = 0 \qquad (16c)$$

Substituting Equations (13) to (16) in Equations (12), and solving for X, Y, and M_e, the results are as follows:

$$-X = \frac{\int_{-\phi_1}^{\phi_1} \frac{M'}{I} y \, ds + \int_{-\phi_1}^{\phi_1} \frac{P' \cos \phi}{A} ds + 2.88 \int_{-\phi_1}^{\phi_1} \frac{S' \sin \phi}{A} ds}{\int_{-\phi_1}^{\phi_1} \frac{y^2 \, ds}{I} + \int_{-\phi_1}^{\phi_1} \frac{\cos^2 \phi \, ds}{A} + 2.88 \int_{-\phi_1}^{\phi_1} \frac{\sin^2 \phi \, ds}{A}} \qquad (17a)$$

$$-Y = \frac{\int_{-\phi_1}^{\phi_1} \frac{M'}{I} x \, ds + \int_{-\phi_1}^{\phi_1} \frac{P' \sin \phi \, ds}{A} - 2.88 \int_{-\phi_1}^{\phi_1} \frac{S' \cos \phi \, ds}{A}}{\int_{-\phi_1}^{\phi_1} \frac{x^2}{I} ds + \int_{-\phi_1}^{\phi_1} \frac{\sin^2 \phi \, ds}{A} + 2.88 \int_{-\phi_1}^{\phi_1} \frac{\cos^2 \phi \, ds}{A}} \qquad (17b)$$

and,

$$M_e = \frac{\int_{-\phi_1}^{\phi_1} \frac{M' \, ds}{I}}{\int_{-\phi_1}^{\phi_1} \frac{ds}{I}} \qquad (17c)$$

It is now necessary to find the values of M', P', and S', so that they may be substituted in Equations (17). The force acting on each element of the arch through its center of gravity, induced by the earthquake, is,

$$dP = \frac{a}{g}\,dW = \frac{a}{g}\,w_c\,t\,r\,d\theta = C\,d\theta \tag{18}$$

and the mathematical summation through the angle, $\theta = \phi$ to $\theta = \phi_1$ of these forces, times their respective lever arms, and the mathematical summation of the proper components, will produce M', P', and S', the moment, thrust, and shear, respectively, of the "external" loads at the point, x, y. Since:

$$x = r \sin \phi \tag{19a}$$

$$x' = r' \sin \theta \tag{19b}$$

$$y = r\left(\cos \phi - \frac{\sin \phi_1}{\phi_1}\right) \tag{19c}$$

$$y' = r' \cos \theta - r\,\frac{\sin \phi_1}{\phi_1} \tag{19d}$$

then,

$$dM' = C\,r\left(\frac{r'}{r}\cos \theta - \frac{\sin \phi_1}{\phi_1}\right) d\theta \tag{20a}$$

and,

$$M' = C\,r\left[(\phi_1 - \phi)\cos \phi + \frac{r'}{r}\sin \phi_1 - \sin \phi\right] \tag{20b}$$

Likewise,

$$P' = C \cos \phi\,(\phi_1 - \phi) \tag{21a}$$

and,

$$S' = C \sin \phi\,(\phi_1 - \phi) \tag{21b}$$

Substituting Equations (21) into Equations (17), replacing ds by $r\,d\phi$, and integrating:

$$X = -\,C\,\phi_1 \tag{22a}$$

$$\begin{aligned} Y = -\,C\,\phi_1 &\left[\frac{1}{2} - \frac{\sin 2\,\phi_1}{4\,\phi_1} - \sin^2 \phi_1\right]\left[1 + \frac{k^2}{r^2}\left(1 - 2.88\,\frac{r'}{r}\right)\right. \\ &\left. + \frac{r'}{r}\left(1 - \frac{\sin 2\,\phi_1}{2\,\phi_1}\right)\right] \div \left[\left(\phi_1 - \frac{\sin 2\,\phi_1}{2}\right)\left(1 + \frac{k^2}{r^2}\right)\right. \\ &\left. + 2.88\,\frac{k^2}{r^2}\,\frac{r'}{r}\left(\phi_1 + \frac{\sin 2\,\phi_1}{2}\right)\right] = -\,C\,C_Y \end{aligned} \tag{22b}$$

and,

$$M_e = -\,C\,r\left(1 - \frac{r'}{r}\right)\sin \phi_1 \tag{22c}$$

Transfer the forces, X and Y, and the moment, M_e, to the abutment and the following moments, thrusts, and shears in the arch are obtained:

At the crown ($\phi = 0$),

$$M_0 = 0 \tag{23a}$$

$$P_0 = 0 \tag{23b}$$

and,

$$S_0 = -\,C\,C_Y \tag{23c}$$

At the left abutment ($\phi = \phi_1$),

$$M_1 = C\,r\left[\left(\frac{r'}{r} - C_Y\right)\sin\phi_1 - \phi_1\cos\phi_1\right] \dots\dots\dots\dots (24a)$$

$$P_1 = C\,(-\,\phi_1\cos\phi_1 - C_Y\sin\phi_1) \dots\dots\dots\dots (24b)$$

and,

$$S_1 = C\,(-\,\phi_1\cos\phi_1 + C_Y\cos\phi_1) \dots\dots\dots\dots (24c)$$

and, at the right abutment ($\phi = -\,\phi_1$),

$$M_2 = -\,M_1 \dots\dots\dots\dots (25a)$$

$$P_2 = -\,P_1 \dots\dots\dots\dots (25b)$$

and,

$$S_2 = S_1 \dots\dots\dots\dots (25c)$$

Assumption (2) Applied to Arches with Hinged Ends.—In the case of an arch with hinged ends (and also with fixed ends), it may be readily proved that $P = 2\,H$ (see Fig. 3) in which P is the total force induced by the earthquake acting through the center of gravity of the arch. The problem of finding the stresses in a hinged arch is then rendered statically determinate. From Fig. 2,

$$P = 2\,\frac{a}{g}\,w_c\,t\,r\,\phi_1 = 2\,C\,\phi_1 \dots\dots\dots\dots (26a)$$

$$H = C\,\phi_1 \dots\dots\dots\dots (26b)$$

$$e = r'\,\frac{\sin\phi_1}{\phi_1} - r\cos\phi_1 \dots\dots\dots\dots (26c)$$

and,

$$V = \frac{P\,e}{L} = C\,\phi_1\,\frac{\left(\dfrac{r'}{r}\,\dfrac{\sin\phi_1}{\phi_1} - \cos\phi_1\right)}{\sin\phi_1} \dots\dots\dots\dots (26d)$$

in which L = one-half the span length. The moment, thrust, and shear for any value of ϕ within the arch is,

$$M = M' - H\,r\,(\cos\phi - \cos\phi_1) + V\,r\,(\sin\phi_1 - \sin\phi) \dots\dots (27a)$$

$$P = P' - H\cos\phi - V\sin\phi \dots\dots\dots\dots (27b)$$

and,

$$S = S' - H\sin\phi + V\cos\phi \dots\dots\dots\dots (27c)$$

At the crown ($\phi = 0$):

$$M_0 = 0 \dots\dots\dots\dots (28a)$$

$$P_0 = 0 \dots\dots\dots\dots (28b)$$

and,

$$S_0 = C\left(\frac{r'}{r} - \phi_1\cos\phi_1\right) \dots\dots\dots\dots (28c)$$

At the left abutment ($\phi = \phi_1$):

$$M_1 = 0 \quad \text{(29a)}$$

$$P_1 = -C \frac{r'}{r} \sin \phi_1 \quad \text{(29b)}$$

and,

$$S_1 = C \phi_1 \left(\frac{r' \cos \phi_1}{r \phi_1} - \frac{1}{\sin \phi_1} \right) \quad \text{(29c)}$$

and, at the right abutment ($\phi = -\phi_1$):

$$M_2 = 0 \quad \text{(30a)}$$

$$P_2 = -P_1 \quad \text{(30b)}$$

and,

$$S_2 = S_1 \quad \text{(30c)}$$

Numerical Example.—Assume an arch with a thickness, $t = 19$ ft; mean radius, $r = 171$ ft; central angle, $2 \phi_1 = 103°$; and depth of water, $h = 70$ ft. Let: $\frac{a}{g} = 0.1$; $w_c = 150$ lb per cu ft; $r' = r$; and $\psi = 0$.

The stresses due to uniform water load are computed from known formulas[4] to serve as a basis for comparison to the earthquake stresses, as shown in Table 1. The stresses designated as s_e, s_i, and s_s are combined bending and direct stresses at the extrados and the intrados of the arch and shear stresses, respectively.

TABLE 1.—COMPARISON OF EARTHQUAKE STRESSES, IN POUNDS PER SQUARE INCH

(The minus sign indicates tension)

Load	CROWN			LEFT ABUTMENT			RIGHT ABUTMENT		
	Extrados, s_e	Intrados, s_i	Shear, s_s	Extrados, s_e	Intrados, s_i	Shear, s_s	Extrados, s_e	Intrados, s_i	Shear, s_s
(*a*) ARCH WITH FIXED ENDS									
Uniform water load	341	127	0	33	447	−14	33	447	14
Earth acceleration along the canyon	26	2	0	18	26	− 2	18	26	− 2
Earth acceleration across to canyon	0	0	3	−104	80	−11	104	−80	−11
(*b*) ARCH WITH HINGED ENDS									
Uniform water load	353	217	0	286	286	−27	286	286	27
Earth acceleration along the canyon	27	3	0	22	22	2	22	22	2
Earth acceleration across to canyon	0	0	5	−14	−14	− 9	+14	14	− 9

CONCLUSIONS

The example chosen represents an arch ring at about the mid-height of an average arch dam and is assumed to be a typical one. It will be noted that an earthquake which accelerates the dam up stream produces stresses that seldom exceed about 10% of the water-load stresses; but for an earthquake acting across the canyon the stresses from this cause may be quite large. Of course, these stresses do not include the possible restraining effect of adjacent arch rings, but this effect would probably tend to reduce their magnitude, as the trial-load method of analysis indicates for water-load stresses.

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[4] "The Circular Arch Under Normal Loads," by the late William Cain, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., LXXXV (1922), p. 233.

Furthermore, it will be noted by observation that the less the central angle is, the less vulnerable will be the arch for the cross-wise acceleration. Therefore, it is fortunate perhaps that the thick lower parts of some arch dams subtend a much smaller central angle than the thin upper parts.

APPENDIX

NOTATION

The following symbols, defined where first introduced in the paper, are rearranged herein for convenience of reference. An effort has been made to conform essentially with "Symbols for Mechanics, Structural Engineering, and Testing Material"[5] compiled by a committee of the American Standards Association, with Society representation, and approved by the Association in 1932:

a = linear acceleration of an earthquake;

C = a coefficient (see Equations (3)); C_Y = coefficient, C, when forces are translated to the elastic center of the arch (see Equation (23*c*));

e = eccentricity; lever arm of Load P, measured vertically from the springing;

g = acceleration due to gravity;

I = moment of inertia;

k = a constant ratio $= \frac{t^2}{12}$;

L = one-half the span length of an arch;

M = moment; M_1 = bending moment at the springing due to the elemental external load, dP; M_c = bending moment at the crown, due to the elemental external load, dP; M_e = bending moment translated, and assumed acting at the elastic center of an arch; and M' = moment at the section, x, y, produced by the external loads on the arch ring to the left of the section;

P = a concentrated force; total force induced by an earthquake, acting through the center of gravity of the arch; P_1 = the normal thrust at the springing due to an elemental external load, dP, on the arch; P_c = the normal thrust at the crown due to an elemental external load, dP, on the arch; and P' = the thrust at the section, x, y, produced by the external loads on the arch ring to the left of the section;

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[5] A S A—*Z*10*a*—1932.

p = pressure per unit area; pressure due to the head, y; p_0 = pressure due to the head, h_0;

r = radius of the center line of the arch; r_e = radius of the extrados of an arch; r_i = radius of the intrados of an arch; and r' = radial distance to the center of gravity of an arch element;

S = shearing component; S_1 = total shear at the springing; S' = shear at the section, x, y, produced by the external loads on the arch ring to the left of the section;

s = stress; s_e = combined direct stress and bending at the extrados; s_i = combined direct stress and bending at the intrados; and s_s = shear stress;

t = uniform, radial thickness of an arch;

W = total, internal, elastic work;

w = unit weight of water; w_c = unit weight of concrete; and w_c' = component unit weight of concrete = $w_c \sin \psi$;

X = horizontal load assumed acting at the elastic center of an arch;

Y = vertical load assumed acting at the elastic center of an arch;

Z = a coefficient; also Z' and Z'' (see Equations (3) to (10), inclusive);

θ = polar co-ordinate; angular distance; angle between the tangents to the arch ring, at the crown and any point; and $d\theta$ = angle between the radial sides of an elemental segment of an arch of depth, t;

ϕ = the central angle of an arch; ϕ_1 = one-half the central angle of an arch;

ψ = inclination of the plane of the arch ring.

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PAPERS

GRAPHICAL REPRESENTATION OF THE MECHANICAL ANALYSES OF SOILS

By Frank B. Campbell,[1] Assoc. M. Am. Soc. C. E.

Synopsis

In view of the lack of uniformity in proposals for the graphical representation of grain-size distribution, this paper is presented in an effort to correlate various existing methods and concentrate them into one method which the writer believes would make a good standard for graphical representation. A semi-logarithmic co-ordinate system is used with the grain sizes plotted as abscissas on a logarithmic scale and cumulative percentages by weight plotted as ordinates. With curves so plotted, the grain-size distribution curves have been conventionalized by introducing straight lines. Such lines can be designated briefly by their slopes and intercepts so that data from various soils may be tabulated or plotted on drawings for purposes of comparison. The value of mechanical analysis is discussed briefly. Further recommendation is made for nomenclature of soil fractions. In doing so there is an attempt to conform closely to the widely accepted classification used by the Bureau of Chemistry and Soils, United States Department of Agriculture, but to make the separations at recurring cycles of log 2 and log 6. Other classifications are also discussed.

Introduction

On considering the various types of soils which may be encountered in Nature, the writer has attempted to develop some simplified designation for both mean diameter and gradation of particle size. It is often desirable to adopt a set of symbols on a plan or profile which indicates the type of grain-size distribution. In the past, such drawings have usually indicated the soils as clay, sand, or gravel with many variations. Such descriptions depend upon the conception of an individual as to the meaning of such terms. These conceptions vary widely. Sometimes, a set of symbols or numbers is used, in which case, a student of the drawing must learn a different system of designations each time he encounters a separate report. In certain studies, the

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Note.—Discussion on this paper will be closed in **April, 1938,** *Proceedings.*

[1] Associate Soil Conservationist, S C S, Spartanburg, S. C.

engineer wishes to make a tabular comparison of a group of analyses of various samples. The method used in the past has been to list the quantity of each soil fraction, or to separate them in columns. Although the information thus recorded is definite, it is difficult for one to obtain a unified "picture" of the comparative grain-size distributions.

MECHANICAL ANALYSES GRAPHS

The importance of the mechanical analysis graph, or grain-size distribution curve, to technical literature lies in the fact that it is a method of international expression, or a universal language. The ordinate is the percentage by weight of the total sample, smaller than a corresponding particle diameter. Percentages involve no physical unit and are often called a ratio or a pure number. The millimeter is used as a unit of particle size. Technically trained men are familiar with the millimeter as a unit of length, and it proves to be a convenient scale for particle diameters of soils. The use of the English inch involves so many ciphers before the first significant figure as to be cumbersome. The purpose of the semi-logarithmic scale is obvious when the nature of grain-size distribution of soils is considered.

In discussing methods of graphical representation of the mechanical analysis of soils, other methods in occasional use should be mentioned. The older method of plotting the weight of soil separates or grain-size fractions, in percentages of the total weight, against particle diameter, has certain advantages of analysis as shown by Mr. Arthur J. Weinig,[2] in 1933. The tri-linear method of plotting mechanical analyses of soil is convenient for an approximate classification of soils according to texture. The method is explained by Messrs. R. O. E. Davis and H. H. Bennett.[3]

Recently some engineers have used repeating cycles of log 1, log 3, and log 5 for the abscissa lines. There is an increasing use in technical literature of log 2 and log 6. It should be noted that since $\log_{10} 2 = 0.301$ and $\log_{10} 6 = 0.778$, the scalar difference between successive abscissa lines is 0.477 and 0.523. These two scalar values offer a spacing of approximately equal distance as may be seen in Fig. 1.

The writer has plotted mechanical analyses selected from several hundred samples from widely different localities and types. Six types are shown in Fig. 1, as follows: Sample (*a*) is a black clay taken from the up-stream face of a dam which failed partly by sliding into the reservoir; Sample (*b*) is a red sandy clay that exhibits a broad variation of grain sizes; Sample (*c*) is an aeolian soil or "blow sand," and is a typical uniform grain-size material; Sample (*d*) represents a residual soil derived from sedimentary rock and has an excellent distribution of soil particles; Sample (*e*) is similar to Sample (*d*), except that it is coarser and indicates that the disintegration process has not been carried so far; and Sample (*f*) is a coarse gravel which contains some sand. It was selected from a dam that is quite stable, although it is very porous and continues to leak.

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[2] "A Functional Size-Analysis of Ore Grinds," by Arthur J. Weinig, *Quarterly*, Colorado School of Mines, Vol. XXVII, No. 3, July, 1933.

[3] "Grouping of Soils on Basis of Mechanical Analysis," by R. O. E. Davis and H. H. Bennett, *Department Circular No. 419*, U. S. Dept. of Agriculture.

It should be mentioned that the shape of the mechanical analysis curve is often an indication of type; that is, whether it is residual, alluvial, or aeolian; and, if it is residual, the curve indicates its degree of maturity. The term, "residual," is used in the sense of a soil that has been derived by the disinte-

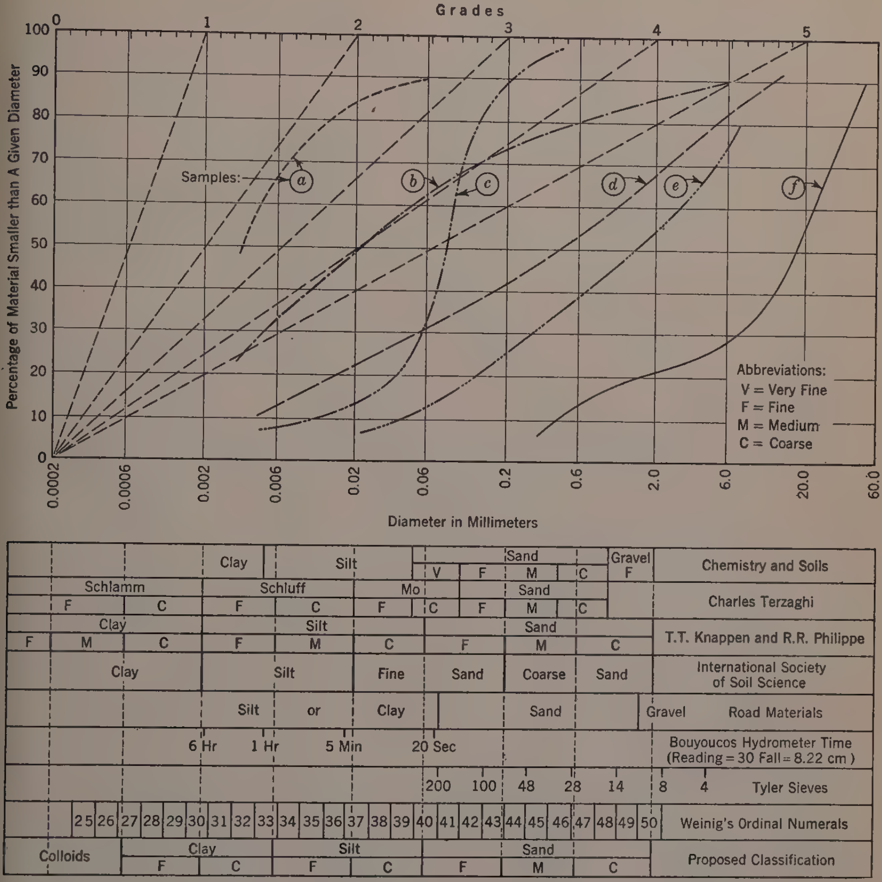

FIG. 1.—COMPARISON OF MECHANICAL ANALYSES AND NOMENCLATURE OF SOILS

TABLE 1.—DIAMETERS AND GRADE CHARACTERISTICS OF SAMPLES IN FIGURE 1

Sample	Mean diameter, in millimeters	Grade line deviation*	Grade, 20% to 80%	Diameter–grade designation	Sample	Mean diameter, in millimeters	Grade line deviation*	Grade, 20% to 80%	Diameter–grade designation
a	0.0038	+0.0002	1.9†	D 0.0038 – G 1.9	*d*	0.47	– 0.19	4.2	D 0.47 – G 4.2
b	0.022	+0.021	3.9	D 0.022 – G 3.9	*e*	1.45	– 0.62	3.3	D 1.45 – G 3.3
c	0.081	–0.022	1.0	D 0.081 – G 1.0	*f*	17.6	–10.0	2.4	D 17.6 – G 2.4

* Convexity is positive. † Estimated by extrapolation.

gration of igneous and certain metamorphosed rock. The shape of the curve may or may not be indicative of the degree of maturity of a soil derived from sedimentary rock. Certain alluvial and aeolian soils are difficult to classify as mature or immature from the standpoint of the disintegration process. In general, the principle follows that immature soils have a size distribution curve that is concave upward, and mature soils show a curve that is convex upward.

By the same reasoning, a soil that is represented by a straight line is often intermediate between maturity and immaturity. Such a soil usually contains particles ranging in size from clay to gravel. It should not be confused with soil of uniform grain size which is a vertical line on the graph. Such a soil is usually an alluvium or an aeolian type. Many observers have noted that water and wind are excellent sorting agents. In view of the foregoing facts, the writer believes that mechanical analysis graphs have a definite value in determining the history of a soil. It is often helpful to know the history of the soil development so that certain other chemical and physical characteristics may be anticipated and tests made for such qualities.

The need of a simplified classification according to grain size has often been recognized. The writer has attempted to develop one, first, by expressing the mean grain diameter, and second, by expressing the gradation of particle size according to the slope of the line on the mechanical analysis graph. The mean grain size is defined as that diameter, in millimeters, of which 50% of the sample by weight has larger diameters and 50% has smaller diameters. The gradation is assumed to be the slope of the straight line passing through the 20% smaller, and the 80% smaller, points of any one curve. The grade lines originating at the lower left-hand corner are shown in Fig. 1. A grade of 0 means that all particles have the same diameter. Grade 1 slopes from 0 to 100%, spanning one logarithmic cycle horizontally; Grade 2 spans two logarithmic cycles, etc. A decimal scale is shown along the top so that the grade may be read to tenths. The method is easily applied by two triangles for transferring the straight line through the 20% and 80% points to a parallel position through the origin at the lower left-hand corner of the graph. The grade can then be read from the horizontal scale at the top.

Table 1 shows the mean diameters and grades of the samples in Fig. 1. The letter designations of the curves on the graph are listed in the first column. The mean diameter is shown in the second column. The "Grade Line Deviation" is the difference between the curve and the grade line measured along the 50% ordinate. The writer has included this column to show the diameter difference, in millimeters, between the actual curve and the grade line. A minus deviation indicates a convex curve. However, the magnitude of the deviation is scarcely an indication of degree of convexity or concavity in a Cartesian sense, for the reason that the scale is logarithmic. The fourth column is the grade, or slope of the grade line, determined by the method described in this paper. The last column shows an abbreviated designation which includes both mean diameter, in millimeters, and the grade. Such a set of symbols may be utilized by plotting it on a soil map or profile.

Various nomenclatures of soil-size groups, or rather soil separates, are shown below the mechanical analysis graph. The most widely used classification is that of the Bureau of Chemistry and Soils, U. S. Department of Agriculture. Charles Terzaghi,[4] M. Am. Soc. C. E., has given a classification of soil separates in use in Germany. Recently, T. T. Knappen, M. Am. Soc. C. E., and R. R. Philippe,[5] Assoc. M. Am. Soc. C. E., have published a classification which they credit to the Massachusetts Institute of Technology. According to this classification, the upper limit of clay corresponds to that of the German "Schlamm." The classification advanced by the International Society of Soil Science is used by some soil scientists both in America and abroad.[6] The Society's Committee on Road Materials has defined the lower limit of sand as a 200-mesh Tyler sieve opening. Some engineers have adopted this conception. The Bouyoucos hydrometer times, corresponding to certain sizes, are shown. It serves to emphasize the lengthening interval of time involved in any sedimentation method when it is desired to determine the content of particles much smaller than 0.006 mm. The corresponding location of Tyler sieve openings illustrates the limits of screen analysis in common use. Mr. Weinig's[2] ordinal numerals are size groups developed for graphical analysis of soil separates. The coincidence of these size groups with the Tyler sieve openings may be seen. The uniform spacing of the Massachusetts Institute of Technology classification, Tyler sieve openings, and Weinig ordinal numerals on a logarithmic scale, are worthy of note.

The writer proposes a classification based upon the log 2 and log 6 division which is not widely different in nomenclature from the much used classification developed by the Bureau of Chemistry and Soils. Although similar to the Terzaghi and Massachusetts Institute of Technology classifications in the use of log 2 and log 6 divisions, the upper limit of clay is moved to 0.006 mm. Since the particles smaller than 0.0006 (the lower clay limit of the writer's classification) are beyond the limits of practical sedimentation analyses and ordinary microscopic technique, such fines should properly be relegated to the fields of colloidal chemistry and ultra-microscopy. For this reason, particles less than 0.0006 mm in diameter are termed colloids. By placing the upper limit of clay at 0.006 mm, fair agreement is had with the 0.005-mm limit of the Bureau of Chemistry and Soils, and the time required for sedimentation analysis is reduced to allow efficient laboratory working schedules. The proposed silt limits of 0.006 mm to 0.06 mm embrace practically all the silt class as defined by the Bureau of Chemistry and Soils, overlapping a little into very fine sand. The proposed fine, medium, and coarse sands correspond roughly to the Chemistry and Soils groups although the ranges are extended somewhat. The proposed coarse sand class overlaps the present fine gravel class, which has an upper limit of 2.0 mm. The new classification presented herein has the distinct advantage of facilitating the transfer of tabular data on soil separates to a mechanical analysis graph sheet for reproducing its curve.

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[4] "Die Prüfung von Baumaterielien fürgewältze Erddämme," by Charles Terzaghi, Premier Congrès des Grandes Barrages, Stockholm (1933), p. 73.

[5] "Practical Soil Mechanics at Muskingum," by Theodore T. Knappen and R. R. Philippe, *Engineering News-Record*, March 26, 1936, p. 455.

[6] See "Soils," by G. W. Robinson, published by Murby, London, 1936.

Acknowledgments

The writer expresses appreciation to E. W. Lane, M. Am. Soc. C. E., and other former associates in the Bureau of Reclamation, U. S. Department of the Interior, for curve data used in this discussion and to his present associates of the Soil Conservation Service, U. S. Department of Agriculture, for helpful criticism.

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PAPERS

GRIT CHAMBER MODEL TESTS FOR DETROIT, MICHIGAN, SEWAGE TREATMENT PROJECT

By George E. Hubbell,[1] Assoc. M. Am. Soc. C. E.

Synopsis

In designing the grit chambers for the Sewage Treatment Plant, at Detroit, Mich., in 1936, several problems were solved by means of a small-scale model of the tank and large-scale tests of two existing tanks. The small model was used to observe flow distribution and grease removal, and the large-scale tests revealed the percentage removal of sand of various sieve sizes for any given length of tank. This paper presents actual data on the settling of sand in flowing water, under field conditions, in a form which greatly simplifies the computations involved in grit-chamber design. This is not a theoretical approach to the problem of sedimentation in flowing water, but rather a presentation of data in a practical and usable form heretofore not available to the sanitary engineer.

Introduction

Cost estimates of the grit chambers at Detroit indicated that a saving of $167 000 could be made by adopting a design consisting of grit chambers, 150 ft long by 15 ft deep, instead of the customary tanks, 75 ft long by 7.5 ft deep. The unusual depth of the proposed chambers raised a question as to their effectiveness for grit removal; and also as each unit was designed to handle a flow of 250 cu ft per sec, the matter of entrance configuration in changing from a velocity of 4 ft per sec to 1 ft per sec required careful consideration. Since the effluent from the grit chambers was to be transported to the settling tanks through submerged conduits, the problem of coercing grease and floating material into the conduits arose. Recourse was made to the performance of a small-scale model for flow distribution and grease removal, and field tests were made on two existing grit chambers for the data on sedimentation of sand in flowing water.

Note.—Discussion on this paper will be closed in **April, 1938,** *Proceedings.*

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LABORATORY TESTS OF A SMALL-SCALE MODEL

Description of the Model.—A one-fifteenth scale model was constructed of wood and equipped with a glass side wall for observation of flow distribution. The principal dimensions of the prototype are given in Fig. 1. The model was

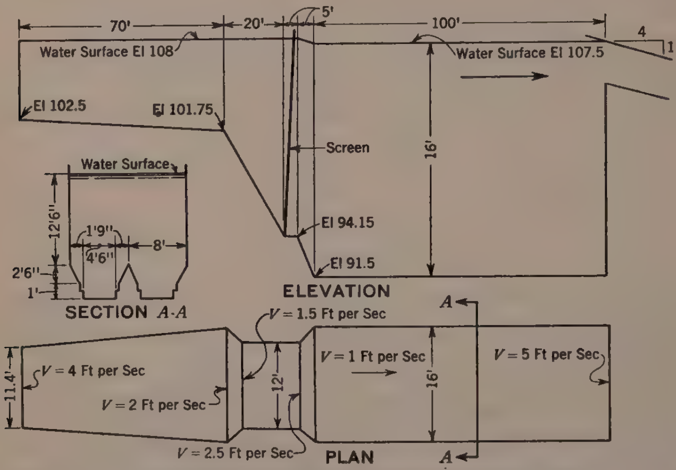

FIG. 1.—PRINCIPAL DIMENSIONS OF GRIT CHAMBER PROTOTYPE MODELED AT 1 : 15 SCALE; Q = 250 CUBIC FEET PER SECOND; AND V = 1 FOOT PER SECOND

operated in accordance with the Froude model law,[2] the actual flow in the model being 130 gal per min with an average velocity in the model of the grit chamber proper of 0.26 ft per sec. The supply was from a constant head tank entering through a calibrated valve. In the prototype the screen or rack consisted of bars, 3 in. by 0.5 in., with 0.75-in. openings. This screen was modeled of brass strips, 0.2 in. by 0.031 in., with a clear opening of 0.05 in.

Flow distribution was observed by introducing coloring matter, sawdust, and fine sand into the model. Observations indicated the following:

(1) There were no dead areas in the grit chamber;

(2) Flow followed the divergence angle of 2 on 5 in passing from the screen section, 12 ft wide, to the grit chamber section, 16 ft wide, without creating back eddies;

(3) Flow followed the bottom configuration in the approach to the rack, the slope being 2 ft on 5.26 ft;

(4) Flow followed the bottom configuration from the rack to the grit chamber, the slope being 2 ft on 3.77 ft;

(5) Coloring matter moved with an approximately vertical front in a remarkably uniform manner through the grit chamber (the first color flowed through in 74% of the theoretical time of 25.6 sec, the main body in about 90%, and the last color in 130%, of the time); and,

(6) No unusual horizontal or vertical velocity was observed. Coloring matter placed on one side of the channel remained on that side throughout the length of flow.

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[2] "Hydraulic Laboratory Practice," Edited by the late John R. Freeman, Past-President and Hon. M., Am. Soc. C. E., A. S. M. E., New York, N. Y., 1929.

The grit chamber actually adopted (see Figs. 2 and 3) conforms closely to the dimensions of the prototype shown in Fig. 1, except that the length was increased to 150 ft.

Screen.—The excellent distribution obtained is attributed to the inclined bar screen or rack. The tests were conducted with a rack having a slope of 1 horizontal to 3.44 vertical. Variation in the angle of slope of the screen had little effect on flow distribution.

Oil and Scum Removal.—Floating matter was removed from the surface of the model by means of the sloping top of the outlet conduit. A slope of 1 vertical to 4 horizontal is required to prevent undue accumulation of oil and grease. In the model, the draw-off velocity was 1.29 ft per sec, corresponding to a velocity of 5 ft per sec in the prototype. Since the effluent from the grit chambers is carried in submerged conduits to the sedimentation tanks, and since no mechanical means for grease removal will be installed in the grit chamber, it is important that all such matter be forced into the outlet conduit.

Settling of Sand in the Model.—When the investigation was first undertaken, it was felt that perhaps data could be obtained from the model relative to the manner in which sand would settle in the prototype. Direct application of the Froude law of model similitude would indicate that volumes to be used in a one-fifteenth scale model should be 3 375 times as small as the corresponding volumes in Nature. This would indicate that a 0.2-mm sand grain in the prototype would be the finest of dust in the model. In arriving at a proper grain diameter to use in the model, the following assumptions were made:

(1) Particles with diameters ranging from 0.2 mm to 0.1 mm settle in still water at constant temperatures in accordance with the expression,

$$V_s = k\, D^n \qquad (1)$$

in which V_s = velocity, in millimeters per second; D = diameter of particle, in millimeters; and, k and n = constants to be determined experimentally.

(2) The horizontal velocity, V_h, of the water in the model varies in accordance with the Froude law, $\frac{1}{\sqrt{l}}$, of the velocity in the prototype, in which l = a scale ratio for length $= \frac{L_n}{L_m}$; L_n = any linear dimension in the prototype (in Nature); and L_m = any linear dimension in the model.

(3) That it is desired to settle a particle having a settling velocity, V_s, a vertical depth, d, while the same particle moves forward with a horizontal velocity V_h a distance, L, in which d = the depth of a tank, in millimeters; and, L = the length of a tank, in millimeters.

Let the settling time, $t_s = \frac{d}{V_s}$; and, the time for corresponding horizontal movement, $t_h = \frac{L}{V_h}$. Then, for $t_s = t_h$:

$$V_s = \frac{V_h\, d}{L} \qquad (2)$$

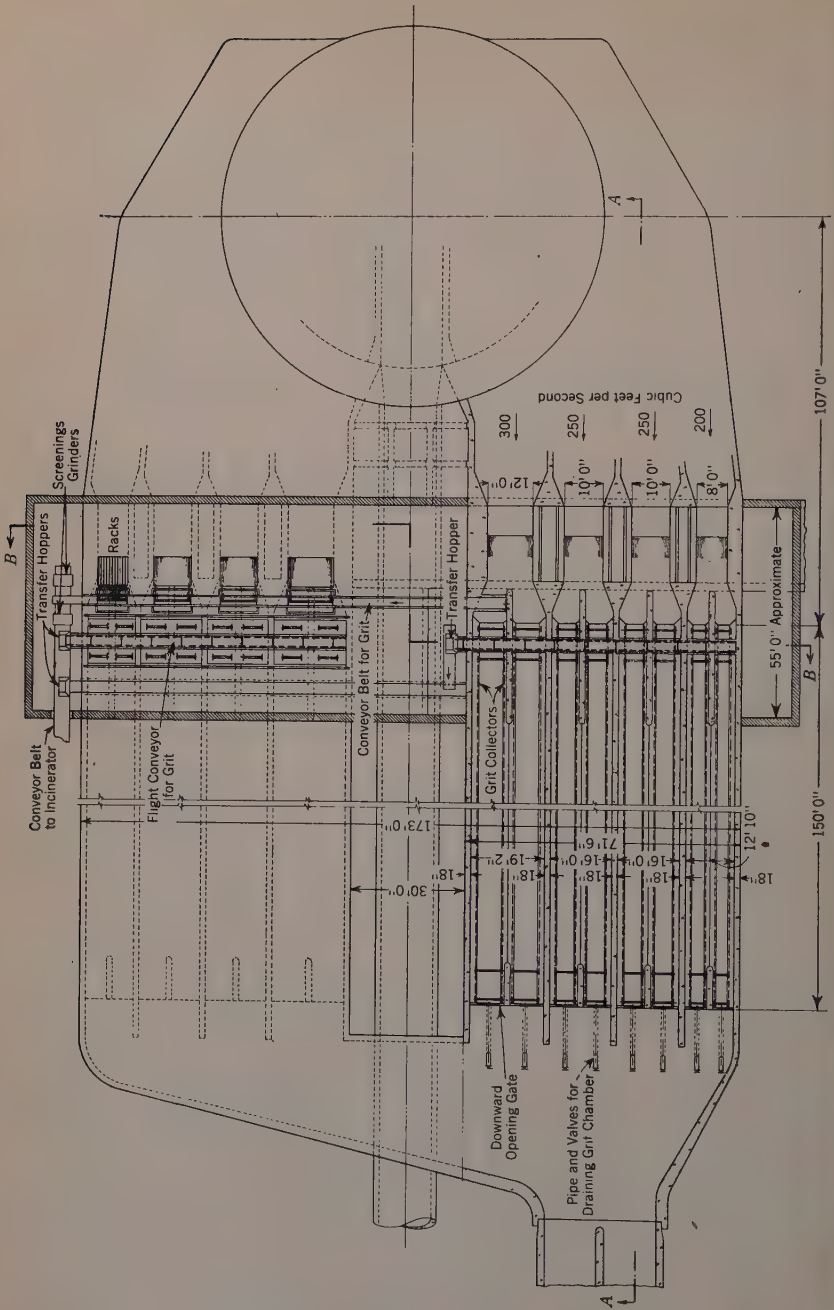

FIG. 2.—PLAN OF RACKS AND GRIT COLLECTORS, DETROIT SEWAGE TREATMENT PROJECT

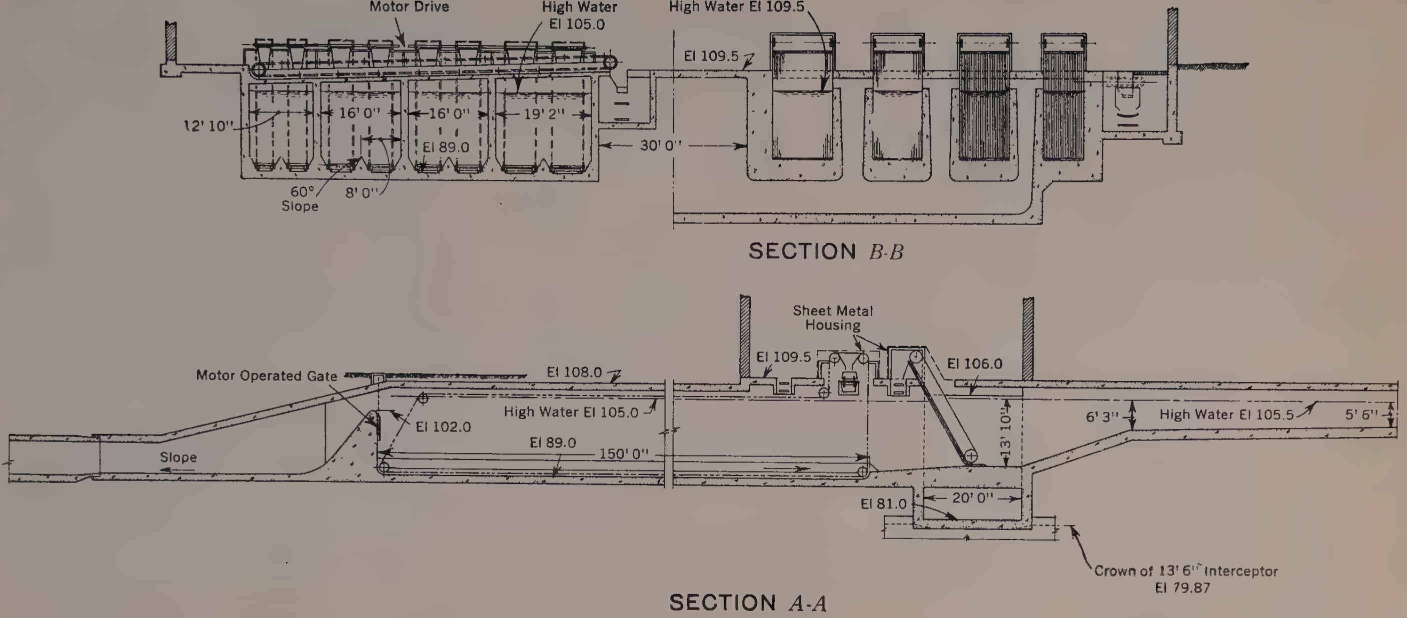

Fig. 3.—Sections of Racks and Grit Collectors

To obtain the ratio of grain diameter in the prototype to grain diameter in the model, equate the ratios of Equations (1) and (2): $\left(\frac{D_p}{D_m}\right)^n = \frac{l\sqrt{l}}{l}$ and,

$$D_m = \sqrt[n]{\frac{D_p{}^n}{l}} = \frac{D_p}{l^{1/2n}} \quad \text{.......................} \quad (3)$$

Although the foregoing development appears to be rational, the application has various limitations. For example, it is assumed that a logarithmic expression can be written for the manner in which sand settles in still water (Assumption (1)). If the particle size remained in the region governed by Stokes' law, or Newton's law, this might be done; but sand 0.2 mm in diameter lies in the critical region where the law of subsidence changes.

Stokes' law (see Assumption (2)) is true for particles settling in quiescent water. It does not hold for settlement in moving water in which, obviously, certain eddy currents occur due to turbulent flow conditions. Much information is available on the transportation of sediment by flowing water[3] but little is available on the sedimentation of material in flowing water at the velocity encountered in grit-chamber practice. In the prototype the average horizontal velocity is 1 ft per sec whereas in the model it is 0.26 ft per sec. Thus, it would appear that model results indicate greater removal than would actually occur in the prototype.

To operate the model in order to obtain data on the removal of sand, 0.2 mm in diameter, in the prototype (Assumption (3)) requires that sand having a diameter (by Equation (3)) of 0.10 mm be obtained for use in the model. Tyler sieves of 150 and 140 mesh have nominal openings of 0.104 and 0.107 mm, respectively. In the prototype, this range of size would correspond to 0.212 mm and 0.218 mm. Quartz sand, 0.1 mm in diameter, is not spherical, but rather is composed of angular particles of various shapes. In arriving at the required model size, it is assumed that the particles are spherical and of uniform size. Both these assumptions are difficult to achieve.

Field Tests of Large-Scale Models

Because of the inherent difficulties involved in the conduction and interpretation of small-scale tests of sand deposition, recourse was had to two existing grit chambers where tests could be conducted at a velocity in the grit chamber of approximately 1 ft per sec. Although the foregoing comment shows that it is possible to test small-scale models, it is quite evident that the use of a large-scale model in which the velocity ratio is 1.0, materially reduces the number of required assumptions for a transfer of results from model to prototype. Two tests were conducted, one at the sewage treatment works in Grand Rapids, Mich., and one at the East Side Sewage Treatment Plant, in Dearborn, Mich. Both tests were run to obtain data on the manner in which sand of various particle sizes settled in water flowing with an average velocity of 1.0 ft per sec. Actually, due to conditions surrounding the tests, the average velocity was

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[3] "Transportation of Detritus by Flowing Water," by F. T. Mavis, M. Am. Soc. C. E., Chitty Ho, Assoc. M. Am. Soc. C. E., and Yun Cheng Tu, Univ. of Iowa *Bulletin No. 5;* and "Transportation of Débris by Running Water," by G. K. Gilbert, *Professional Paper No. 86, U. S. Geological Survey.*

0.9 ft per sec. Two tests were necessary in order to substantiate the following theoretical development as to interpretation of test results.

Referring to the development of Equation (2), $\frac{L}{V_h} = \frac{d}{V_s}$; or, in terms of ratios: When the V_h-ratio = 1 and the V_s-ratio = 1, the L-ratio is $l = \frac{1.0}{1.0} \times h$-ratio. That is, the length ratio must be equal to the height ratio. Thus, in the model, if the height ratio is 2.0, and 60% of the sand of a given size is removed in 100 ft of tank, then in the prototype the same percentage removal of sand can be expected to occur in 200 ft. Since, in the settling of sand, there is no coagulation action, this conclusion appears rational and is substantially in accord with observations.

Grand Rapids Test.—Fig. 4 shows the general plan of the grit chamber tested. The channel selected was thoroughly cleaned for the test. Under normal flow conditions, the average depth in the channel is 4.7 ft. In order to secure a greater depth, the treatment plant was shut down and the sewage allowed to store in the influent sewer. When it had reached the lip of the overflow, all the sewage was passed through the selected channel, giving a depth at the midpoint of the grit chamber of 6.13 ft at the beginning of the test to 6.00 ft at the end of the test. When flow conditions had become constant, as evidenced by a uniform water level and Venturi meter readings, sand was fed into the grit chamber with shovels by two workmen standing on the platform at Elevation 18.0, dropping the sand into the water at Elevation 15.2. The sand was placed in the narrow end of the curved entrance channel (4 ft wide by 4.5 ft deep). The velocity in this approach section averaged 2.4 ft per sec. The sand was measured in a container having a volume of 0.77 cu ft which held 74.5 lb of sand, measured moist. No determination of moisture content was made. The same container was used to measure the sand removed by the chamber. The test was made on Friday and the grit chamber allowed to drain and dry until the following Monday. It has been assumed that bulking was the same during both sets of measurements. Fig. 5 shows the weight of

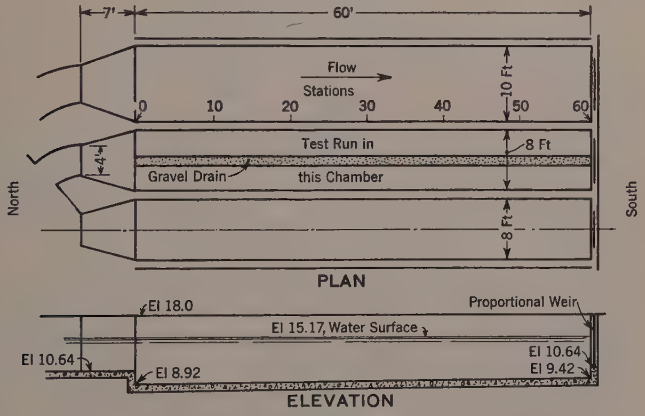
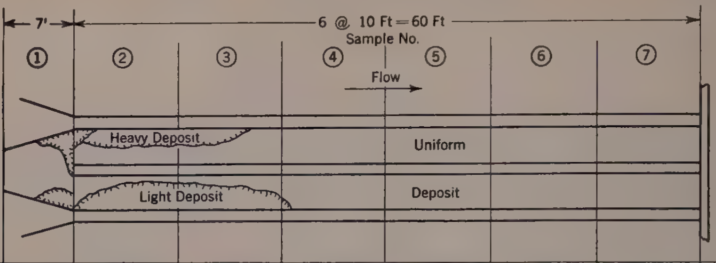


FIG. 4.—PLAN OF GRIT CHAMBER, GRAND RAPIDS, MICHIGAN

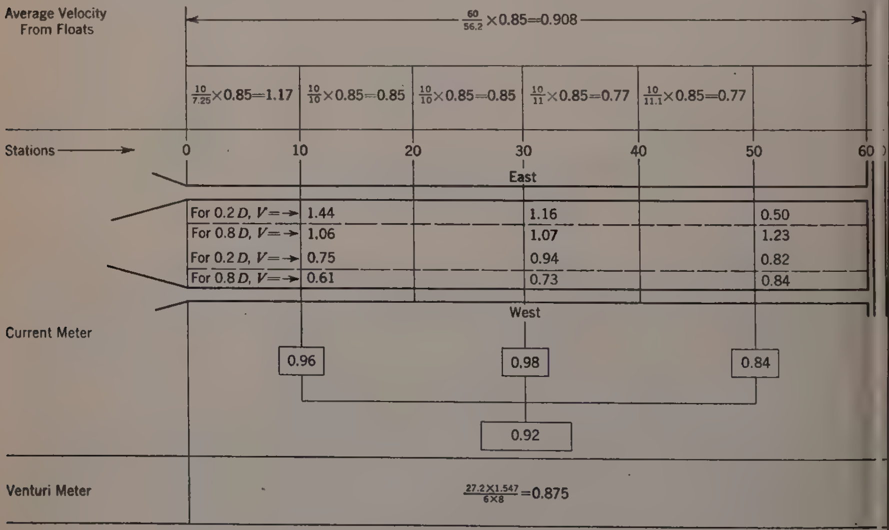
sand deposited in each 10-ft section of the chamber. The sand was carefully scraped into piles, a sample was taken in accordance with the A.S.T.M. standard method of sampling, and the volume determined. It was estimated



								Total
Pails	3.0	11.250	6.393	3.536	1.750	1.080	0.571	27.580
Pounds	224	838	476	263	131	78	43	2 053.0
On Drain Channel		0.5 Pails @ 74.5 Lb						37.0
Pounds	Sand Removed From Channel							2 090.0
Pounds	Sand Placed in Channel, (30×74.5 Lb)							2 235.0
Pounds	Sand Passing Out of Channel							145.0

FIG. 5.—WEIGHT AND LOCATION OF SAND DEPOSITED IN GRIT CHANNEL

that 0.5 pail of sand was deposited on top of the central gravel drain that runs the entire length of the chamber. This half pail of sand has been prorated to the various sections. Fig. 6 shows the velocity variation in the channel, the average of all readings being 0.901 ft per sec. There was an unequal distribution over the cross-section, the velocity on the east side being consistently greater than that on the west side.


Average Velocity From Floats
$\frac{60}{56.2}\times0.85=0.908$
$\frac{10}{7.25}\times0.85=1.17$
$\frac{10}{10}\times0.85=0.85$
$\frac{10}{10}\times0.85=0.85$
$\frac{10}{11}\times0.85=0.77$
$\frac{10}{11.1}\times0.85=0.77$
Stations
0
10
20
30
40
50
60
East
For 0.2 D, V=
For 0.8 D, V=
For 0.2 D, V=
For 0.8 D, V=
1.44
1.06
0.75
0.61
1.16
1.07
0.94
0.73
0.50
1.23
0.82
0.84
West
Current Meter
0.96
0.98
0.84
0.92
Venturi Meter
$\frac{27.2\times1.547}{6\times8}=0.875$

Average of all Measurements $\frac{0.908+0.920+0.875}{3}=0.901$

FIG. 6.—VELOCITY COMPUTATIONS, IN FEET PER SECOND

Samples of the sand placed in the tank, and of the sand removed in the various parts of it, were thoroughly dried and sieved for a period of 10 min in a

Tyler type of shaker, using standard Tyler sieves. No attempt was made to calibrate these sieves and the mesh openings are as indicated by the manufacturer. The sand retained on each sieve (see Table 1) was weighed on a balance accurate to 0.10 gram, and the percentage composition of each sample was determined. Knowing the percentage composition of each sample and the actual weight of sand from which the sample was taken, the quantity of sand of each sieve size removed in each 10-ft section of the grit chamber was computed. (The average water depth was 6.0 ft; the temperature of the water was 65° F; the nominal velocity of the water was 0.90 ft per sec; and the specific gravity of the sand was 2.65.) The relation between the percentage removal of sand, of a given sieve size, and the length of tank required to obtain this removal, is best shown by means of the semi-logarithmic plat. Fig. 7 treats the data for the plant at Grand Rapids in this manner. It appears that these curves can be extrapolated as straight lines, all values beyond the 60-ft length being so obtained.

TABLE 1.—SIEVE ANALYSIS OF SAND REMOVED FROM, AND ADDED TO, THE GRIT CHAMBER

SIEVE SIZES		MATERIAL, IN GRAMS,* RETAINED ON A SIEVE, IN ANALYSIS OF SAMPLES NOS.:							
Meshes per inch	Opening, in millimeters	1	2	3	4	5	6	7	8†
Station		....	0–10	10–20	20–30	30–40	40–50	50–60	....
8	2.362	74.7 (13.5)	35.6 (6.8)	7.2 (1.4)	10.6 (1.8)	61.8 (9.3)	80.5 (12.0)	17.2 (2.5)	45.4 (2.6)
9	1.981	62.0 (11.2)	13.8 (2.6)	1.0 (0.2)	0.3 (0.5)	0.9 (0.1)	1.2 (0.2)	0.7 (0.1)	44.6 (2.5)
20	0.833	248.2 (45.1)	151.6 (29.0)	50.0 (9.8)	18.9 (3.2)	11.9 (1.8)	8.2 (1.2)	7.1 (1.1)	362.1 (20.8)
28	0.589	73.8 (13.4)	120.95 (23.1)	117.1 (22.9)	84.8 (14.2)	42.4 (6.4)	20.5 (3.1)	13.3 (2.1)	319.1 (18.3)
35	0.417	50.5 (9.2)	108.05 (20.7)	165.4 (32.3)	200.7 (33.7)	163.5 (24.6)	111.4 (16.6)	81.2 (12.5)	400.8 (22.8)
48	0.295	29.4 (5.3)	66.7 (12.7)	121.5 (23.8)	189.6 (31.9)	242.6 (36.5)	259.8 (38.8)	278.3 (43.0)	344.1 (19.8)
65	0.208	8.3 (1.5)	19.75 (3.8)	37.7 (7.4)	67.1 (11.3)	111.1 (16.7)	148.0 (22.1)	192.9 (29.7)	164.9 (9.4)
100	0.147	2.7 (0.5)	5.5 (1.1)	10.2 (2.0)	16.8 (2.8)	27.4 (4.1)	34.8 (5.2)	49.8 (7.7)	57.9 (3.3)
150	0.104	0.5 (0.1)	0.6 (0.1)	0.7 (0.1)	1.5 (0.3)	2.4 (0.3)	3.3 (0.5)	5.1 (0.8)	5.5 (0.3)
200	0.074	0.2 (0.1)	0.25 (0.0)	0.2 (0.0)	0.4 (0.1)	0.5 (0.1)	0.7 (0.1)	1.3 (0.2)	1.1 (0.1)
Pan	....	0.5 (0.1)	1.05 (0.1)	0.4 (0.1)	1.0 (0.2)	0.8 (0.1)	1.5 (0.2)	2.2 (0.3)	1.8 (0.1)
Total.......	....	550.8 (100.0)	523.85 (100.0)	511.4 (100.0)	591.7 (100.0)	665.3 (100.0)	669.9 (100.0)	649.1 (100.0)	1 747.3 (100.0)

* Numbers in parentheses are percentages. † Sample of sand added to chamber.

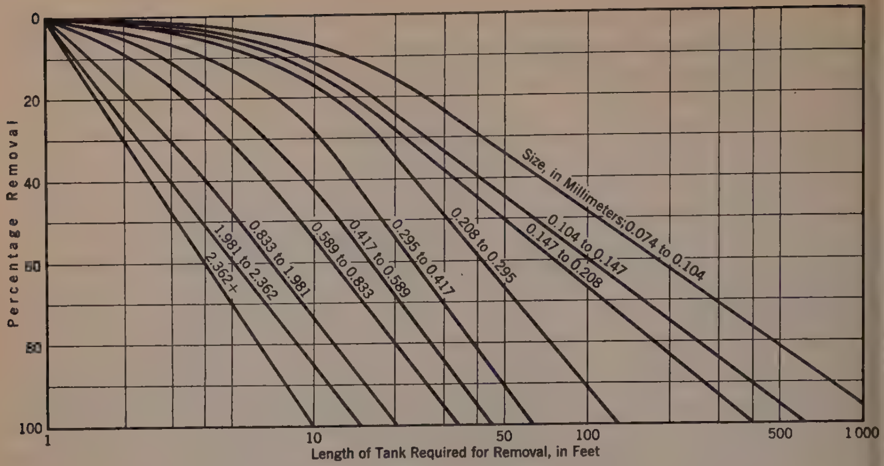


Fig. 7.—Percentage Removal of Sand of Various Sizes, by Grit Chamber, at Grand Rapids, Michigan

Dearborn Test.—Fig. 8 shows the general plan of the grit channel tested at the East Side Plant. The sewage is raised to the chamber by two pumps having a combined capacity of 6.5 cu ft per sec so that it was possible to adjust the effluent weir to obtain an average velocity of 0.9 ft per sec, the average depth being 2.8 ft. The actual velocity during the test, as obtained by float measurements, varied from 0.89 ft per sec to 0.96 ft per sec and averaged 0.93 ft per sec.

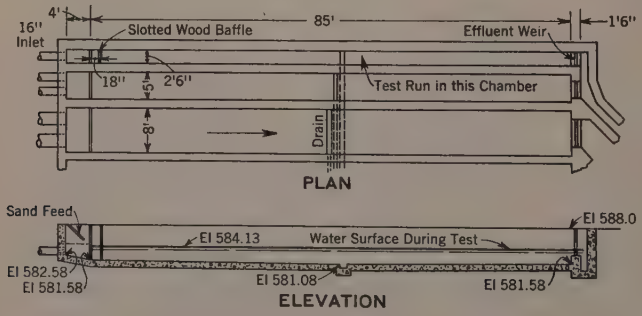
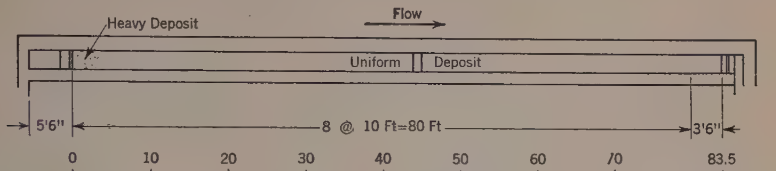


Fig. 8.—Plan of Grit Chamber, Dearborn, Michigan

The high entrance velocity was dissipated by means of two baffles, the first consisting of a solid plank 24 in. high placed at the bottom, the second being a vertically slotted arrangement made of 1-in. by 3-in. strips. However, turbulence persisted in the first 20 ft of the chamber, but beyond that point flow conditions were uniform.

In order to obtain a wide variation in the composition of the sand placed in the chamber, 990 lb of coarse sand, 673 lb of medium sand, and 1 587 lb of fine sand, were fed to the channel on an inclined board so that the drop to the water surface was 6 in. In this test all of each grade of sand was fed continuously, in the following order: Coarse, medium, and fine. In the future, this procedure will be avoided as it gives a stratified deposit which is difficult to sample. The weight and location of the sand deposited in the grit chamber are shown in Fig. 9.



Sample No.	1	2	3	4	5	6	7	8	Total
Boxes of Sand Removed	16.09	14.00	8.00	5.60	2.18	2.52	1.27	1.50	51.16
Pounds of Sand Removed	885	770	440	308	120	140	83	70	2816
	Total Pounds of Sand Placed in Chamber								3250
	Total Pounds of Sand Passing Out of Chamber								434

FIG. 9.—PERCENTAGE AND LOCATION OF SAND DEPOSITED IN GRIT CHAMBER

The sieve analysis of the various samples is shown in Table 2. The computations for percentage removal of sand of various sieve sizes have been platted in Fig. 10 in the same manner as the Grand Rapids test data.

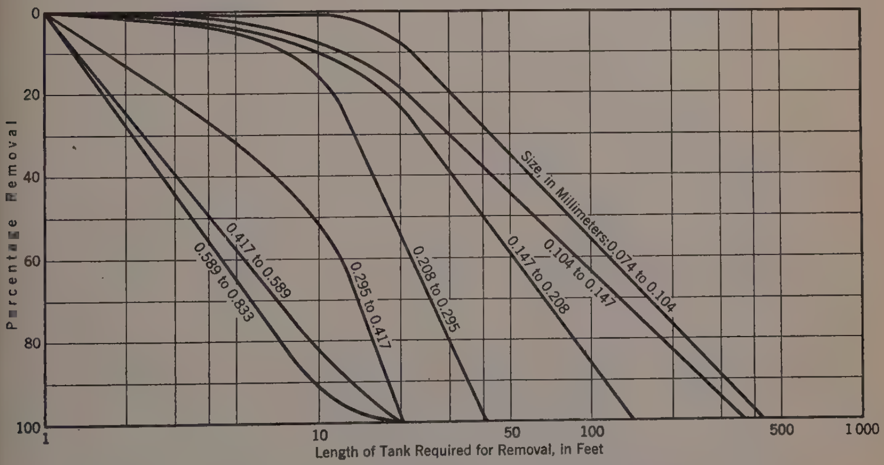


FIG. 10.—PERCENTAGE REMOVAL OF SAND IN TANK 2.8 FEET DEEP, BY GRIT CHAMBER, AT DEARBORN, MICHIGAN

Results of Tests.—The laboratory and field model tests were conducted to enable the prediction of the grit removal to be expected from a tank having an effective depth of 15 ft and a length of 150 ft.[4] Based on a test run of the

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[4] It is recognized that, in general, the term, "grit," is all inclusive and covers all material settling out in the grit chamber. As here used, it refers only to material similar to sand, abrasive in nature, the Detroit grit chamber functioning only to protect the mechanical equipment for sludge handling. Sand embedded in the sludge removed from grit chambers may not have the same mechanical analysis as the sand used in the tests reported on. This, however, has no effect on the removals for a given sand size.

TABLE 2.—SIEVE ANALYSIS OF SAND REMOVED FROM AND ADDED TO GRIT CHAMBER

SIEVE SIZES		MATERIAL, IN GRAMS, RETAINED ON A SIEVE IN ANALYSIS OF SAMPLES NOS.:										
Meshes per inch	Opening, in milli-meters	1	2	3	4	5	6	7	8	9	10	11
Station		0–10	10–20	20–30	30–40	40–50	50–60	60–70	70–83.5	Sand added to chamber		
8	2.362	163.1 (14.1)	..	..	..	..	..	..	..	42.8 (6.8)	126.0 (19.5)	..
9	1.981	60.1 (5.2)	..	..	..	..	..	..	..	9.5 (1.5)	50.7 (7.8)	..
20	0.833	305.8 (26.3)	19.0 (2.2)	..	..	..	..	..	..	55.6 (8.9)	194.1 (30.0)	0.2 (0.0)
28	0.589	149.3 (12.8)	31.2 (3.6)	2.9 (0.3)	1.8 (0.2)	0.2 (0.0)	..	..	..	38.5 (6.2)	55.4 (8.5)	0.6 (0.1)
35	0.417	149.6 (12.8)	59.8 (6.8)	14.0 (1.6)	3.3 (0.4)	0.7 (0.1)	0.9 (0.1)	0.5 (0.1)	..	50.6 (8.1)	43.1 (6.6)	4.0 (0.7)
48	0.295	186.4 (16.2)	217.1 (24.8)	116.0 (13.0)	38.8 (5.2)	13.5 (1.6)	4.7 (0.5)	1.7 (0.2)	3.1 (0.5)	110.0 (17.6)	62.0 (9.6)	30.3 (5.1)
65	0.208	93.4 (8.0)	314.7 (35.9)	404.0 (45.3)	327.2 (44.2)	278.4 (33.2)	139.0 (14.6)	23.6 (3.5)	12.4 (1.9)	137.0 (22.0)	51.8 (8.0)	143.7 (24.4)
100	0.147	38.6 (3.3)	158.3 (18.1)	214.6 (24.1)	215.4 (29.0)	384.3 (45.8)	531.3 (55.5)	361.4 (53.5)	257.0 (39.3)	101.7 (16.3)	39.5 (6.1)	195.4 (33.2)
150	0.104	10.2 (0.9)	49.2 (5.6)	87.8 (9.8)	94.7 (12.8)	83.0 (9.9)	144.6 (15.2)	158.5 (23.5)	195.3 (29.8)	44.0 (7.0)	13.8 (2.1)	114.4 (19.3)
200	0.074	2.6 (0.2)	18.8 (2.1)	39.0 (4.3)	44.3 (6.0)	58.0 (6.9)	91.1 (9.6)	86.6 (12.9)	127.7 (19.4)	18.9 (3.0)	4.8 (0.7)	60.4 (10.2)
Pan	....	2.1 (0.2)	7.9 (0.9)	14.0 (1.6)	16.5 (2.2)	21.1 (2.5)	42.7 (4.5)	42.3 (6.3)	59.7 (9.1)	16.6 (2.6)	7.0 (1.1)	41.0 (7.0)
Total....	....	1 161.2 (100)	876.0 (100)	892.3 (100)	742.0 (100)	839.2 (100)	954.3 (100)	674.6 (100)	655.2 (100)	625.2 (100)	648.2 (100)	590.0 (100)

laboratory model using sand that will pass a 140-mesh sieve and be retained on a 150-mesh screen, Table 3 has been prepared showing the expected percentage removal of sand having a prototype size of 0.2 mm to 0.3 mm. Results based on the field tests are also shown.

TABLE 3.—COMPARISON OF PREDICTED REMOVALS OF SAND, 0.2 MM TO 0.3 MM IN DIAMETER, IN A CHAMBER OF 15-FOOT DEPTH, BASED ON VARIOUS MODEL TESTS

Length of tank, in feet	PERCENTAGE OF SAND REMOVED			Length of tank, in feet	PERCENTAGE OF SAND REMOVED		
	True, small-scale model, laboratory test	Field test, Dearborn, Mich.	Field test, Grand Rapids, Mich.		True, small-scale model, laboratory test	Field test, Dearborn, Mich.	Field test, Grand Rapids, Mich.
(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
Scale	1 : 15	1 : 5.35	1 : 2.5	Scale	1 : 15	1 : 5.35	1 : 2.5
60	72	..	40	120	90	63	65
80	79	37	50	140	94	73	70
100	85	50	59	150	96	77	73

As might be expected the laboratory model indicates a greater percentage of removal than the field models. This can be explained in part as due to the reduced turbulence corresponding to a lower velocity in the small model. The results based on the field tests for this sand size are in close agreement for the greater length values. For other sand sizes, the variation is greater, the Dearborn test generally indicating greater lengths for the same percentage removal. It is assumed that velocity-variation effects were the same in both tests and will be the same in the prototype. Undoubtedly, the distribution in the prototype will be more uniform, with consequent greater removals. Fig. 11 gives the expected removal for a tank of 15-ft effective depth. The removals shown are based on the Grand Rapids test and represent the writer's judgment in interpreting the available data.

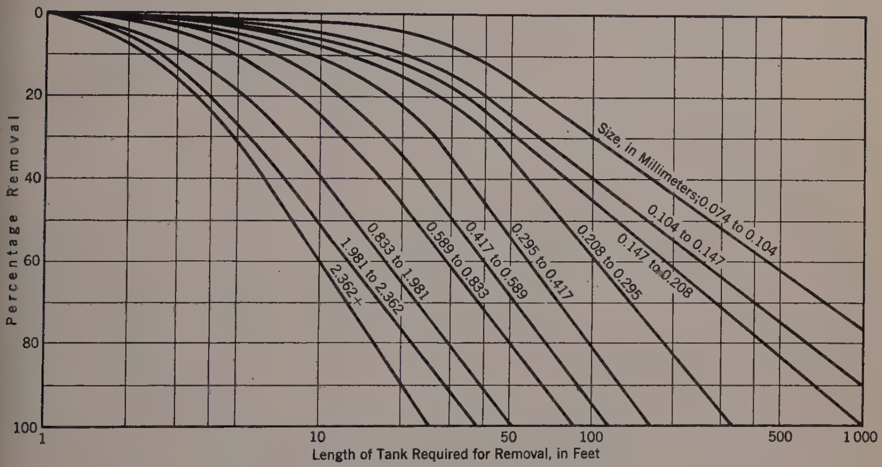

FIG. 11.—EXPECTED PERCENTAGE REMOVAL OF SAND IN TANK, 15 FEET DEEP, BY DETROIT GRIT CHAMBERS

Size of Sand Reaching the Settling Tank.—In setting up the requirements for the grit chamber the Board of Consulting Engineers (Clarence W. Hubbell, M. Am. Soc. C. E., the late Harrison P. Eddy, Past-President, Am. Soc. C. E., and the late John H. Gregory, M. Am. Soc. C. E.) stated that "tests should be run to determine: (1) The possibility of depositing all sand greater than a 0.2-mm effective size; and (2) the length of flow required for such deposition." This may be interpreted as meaning that it would be desirable to deposit all sand having a size greater than 0.2 mm.[5] Table 4 gives a sieve analysis of the sand content of sludge or grit removed from the Grand Rapids grit chamber and from sludge obtained from the final digestion tank. An analysis is included of a catch sample of sand found at the curb of a city street where the soil is clay, with well kept lawns.

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[5] "Grit Chamber Practice": A Symposium, *Transactions*, Am. Soc. C. E., Vol. 91 (1927), p. 495.

Of the sample of grit removed from the digested sludge, 54.5% was retained on a 65-mesh screen, or was larger than 0.2 mm. Analysis was made possible by treating the sludge with acid to remove the organic material. Apparently, this grit does not cause excessive wear on the mechanisms in the plant. Presumably, then, it is not necessary to remove all the grit of this size in a grit chamber. The field tests indicate that the Detroit chambers will not remove all the grit coarser than 0.2 mm.

TABLE 4.—SIEVE ANALYSIS OF SAND OBTAINED FROM GRIT AND SLUDGE, GRAND RAPIDS SEWAGE TREATMENT PLANT AND TYPICAL DETROIT STREET SAND

Sieve mesh	GRIT CHAMBER, IN THE BOTTOM SLUDGE				DIGESTER		TYPICAL DETROIT STREET GRIT	
	10 Feet from Influent End; Material Retained		50 Feet from Influent End; Material Retained		Removed from the Digested Sludge			
	In grams (1)	Percentages (2)	In grams (3)	Percentages (4)	In grams (5)	Percentages (6)	In grams (7)	Percentages (8)
8	0.10	0.3	5.8*	23.2	0.15	0.1	8.1	1.7
9	0.10	0.3	0.9*	3.6	0.10	0.1	1.5	0.3
20	0.55	1.6	2.4*	9.6	1.95	1.4	18.0	3.7
28	0.70	2.1	0.8	3.2	2.15	1.5	17.3	3.6
35	1.10	3.2	0.6	2.4	4.40	3.1	24.8	5.1
48	5.40	15.9	0.9	3.6	19.60	14.0	63.9	13.2
65	11.60	34.2	1.6	6.4	47.95	34.3	107.2	22.3
100	8.20	24.1	2.5	10.0	40.20	28.5	102.7	21.3
150	2.80	8.2	1.75	7.0	13.70	9.7	56.9	11.8
200	1.30	3.8	1.50	6.0	5.20	3.7	31.9	6.6
Pan	2.15	6.3	6.25	25.0	5.00	3.6	50.1	10.4
Total	34.00	100.0	25.00	100.0	140.40	100.0	482.4	100.0

* Large particles of silt cemented together.

In analyzing the Detroit grit chambers, Eugene A. Hardin, M. Am. Soc. C. E., prepared the material in Table 5, based on the foregoing tests and an analysis of an assumed sample of grit (Column (3)), which indicates that the Detroit chamber, 15 ft deep by 150 ft long, will remove 17% more grit than one 100 ft long, and 72% more than one 50 ft long. Comparing this tank with one of equal length, but 6 ft deep: At 150 ft the 15-ft tank removes 19% less than the 6-ft tank; at 100 ft, 26% less; and at 50 ft, 38% less. The economic length of the grit chamber would appear to be not more than 100 ft since the extension of the tank to 150 ft will increase the total removal only 17% and effect this in the range of grit smaller than 0.3 mm in diameter. It was estimated that the increase in length from 100 ft to 150 ft would cost $75 000. This expenditure seemed warranted in Detroit for the following reasons: The tanks were unusually deep, with no similar tanks available for comparison; and the increased length would reduce the solids load on the settling tanks to some degree and would act as a factor of safety insuring a minimum of grit passage to the sludge disposal mechanism. However, the writer believes that a grit chamber, 150 ft long, is not necessary.

Conclusion

It is hoped that this brief description of a method of approach to the grit-chamber problem will result in other similar investigations. Theoretical computations for sand removal in grit chambers are uncertain, due to the difficulty of evaluating the various factors involved, whereas the results obtained from tests are substantially correct and practical in application.

TABLE 5.—Estimated Percentage Removal of Sand in Various Lengths: Normal Velocity, 0.9 Foot per Second

Sieve No.	Diameter of Particles, in Millimeters		Analysis of Detroit grit; percentage retained	Percentage Removal in:					
	From	To		50 Feet		100 Feet		150 Feet	
				Percentage of a given size present	Percentage in Detroit grit	Percentage of a given size present	Percentage in Detroit grit	Percentage of a given size present	Percentage in Detroit grit
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
(*a*) Chambers 15 Feet Deep									
9	2.0	2.4	2.0	100	2.0	100	2.0	100	2.0
20	0.8	2.0	3.7	100	3.7	100	3.7	100	3.7
28	0.6	0.8	3.6	80	2.9	100	3.6	100	3.6
35	0.4	0.6	5.1	68	3.5	95	4.8	100	5.1
48	0.3	0.4	13.2	54	7.1	81	10.7	97	12.8
65	0.2	0.3	22.3	35	7.8	59	13.1	73	16.3
100	0.15	0.2	21.3	28	6.0	45	9.6	55	11.7
150	0.1	0.15	11.8	24	2.8	40	4.7	48	5.6
200	0.07	0.10	6.6	16	1.0	30	2.0	38	2.5
Pan	Dust		10.4	0	0.0	0	0.0	0	0.0
Totals	....	....	100.0	....	36.8	....	54.2	....	63.3
(*b*) Chambers 6 Feet Deep									
9	2.0	2.4	2.0	100	2.0	100	2.0	100	2.0
20	0.8	2.0	3.7	100	3.7	100	3.7	100	3.7
28	0.6	0.8	3.6	100	3.6	100	3.6	100	3.6
35	0.4	0.6	5.1	100	5.1	100	5.1	100	5.1
48	0.3	0.4	13.2	91	12.0	100	13.2	100	13.2
65	0.2	0.3	22.3	67	14.9	91	20.8	100	22.3
100	0.15	0.2	21.3	50	10.6	67	14.3	76	16.2
150	0.1	0.15	11.8	45	5.3	60	7.1	69	8.1
200	0.07	0.10	6.6	34	2.2	49	3.2	57	3.8
Pan	Dust		10.4	0	0.0	0	0.0	0	0.0
Totals	....	....	100.0	....	59.4	....	73.0	....	78.0

A laboratory approach to the problem has been indicated which may be of value, although it is felt that more reliable information is gained by eliminating the velocity factor.

Excellent flow distribution can be obtained by means of bar screens having $\frac{3}{4}$-in. openings. Such screens introduce sufficient loss of head to insure uniform velocity conditions during grit deposition. Grease can be induced successfully into submerged conduits by using a draw-off velocity of 5 ft per sec and conduit roofs having slopes of less than 14 degrees. In large installations, deep grit chambers proportioned so that uniform velocity conditions exist, are more economical to construct, and will perform with the same degree of efficiency as grit chambers of one-half the depth and length and twice the width.

ACKNOWLEDGMENTS

The writer wishes to acknowledge the valuable suggestions offered by Albert Roth, Assoc. M. Am. Soc. C. E., Arthur B. Morrill and Eugene A. Hardin, Members, Am. Soc. C. E.; and, for making the field work possible, he is grateful to James R. Rumsey, M. Am. Soc. C. E., former Sanitary Engineer for the City of Grand Rapids, and Austin Habemehl, former Chief Chemist at the Dearborn Plant.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
PAPERS

ECONOMIC ASPECTS OF ENERGY GENERATION
A SYMPOSIUM

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Note.—Presented at the meeting of the Society and at the Joint Meeting of the Power and Engineering-Economics and Finance Divisions, Pittsburgh, Pa., October 14, 1936. Publication has been made possible by the financial co-operation of the Power Division. Written comments are invited for immediate publication; to ensure publication the last discussion should be submitted by **April 15, 1938.**

PROGRESS IN THE GENERATION OF ENERGY BY HEAT ENGINES

By Geo. A. Orrok,[1] M. Am. Soc. C. E.

Synopsis

The writer gives estimates of the total of power-producing machinery used in the United States as of 1936 and discusses hours of use and kilowatt-hours of work done annually, showing the contrast between 1880 and 1936. The size and type of units with the average economies of various types of machinery are given. The influences of high pressure and vacuum and high temperature are discussed with prediction of economies now in sight. Thermal economy and commercial economy are discussed as well as the cost of installations over the period. Tables are given for the cost of steam power.

The internal combustion engine, Otto, Diesel, and solid injection cycles, are considered and the paper ends with a review of possibilities of other cycles and other mediums of heat supply. The conclusion is that steam will be the most used medium of power generation for the near future.

In 1896 the late Robert Henry Thurston, M. Am. Soc. C. E., surveyed the field of power production and collected data on the increasing use of steam power during the Nineteenth Century.[2] It is interesting to note that his figures for 1896 show only 17 000 000 hp of power-producing machinery in the United States, divided as follows: Locomotive, 11 000 000 hp; marine, 2 000 000 hp; and stationary, 4 000 000 hp. He plotted his figures in a curve, and commented on the trends of the possible tremendous increase in power output which has since taken place.

In 1909 Henry Adams, some time Professor of History at Harvard College, wrote his essay on the Phase Rule which was published posthumously in 1919 as "The Degradation of Democratic Dogma." In this essay Professor Adams speculated as to the great increase in the use of power in the world and tried to apply the second law of thermodynamics to the power situation. His curves show the same characteristics as those of Thurston, but with the well-known Adams pessimism, he carried the curve to its logical conclusion, forgetting the law of diminishing returns with which he was without doubt much more familiar than with the second law of thermodynamics.

Among the authorities who have since contributed to power statistics both in the United States and in other countries may be mentioned, D. B. Rushmore,[3] M. Am. Soc. C. E., F. R. Low,[4] and Professor Thomas T. Reed.[5]

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[1] Cons. Engr. (Orrok, Myers & Shoudy), New York, N. Y.

[2] "The History of the Steam Engine," by Robert Henry Thurston, Appleton, 1899.

[3] "Hydro-Electric Power Stations," by D. B. Rushmore, John T. Wiley & Son, 1917.

[4] Presidential Address, A. S. M. E., Vol. 46, 1924, p. 1535.

[5] *Mechanical Engineering*, May, 1926.

In 1930, the writer brought these figures up to date[6] (for the United States only), and more recently the increases from that year to June, 1936, were shown by Dean A. A. Potter in his paper for the National Resources Committee.[7]

According to Dean Potter, the total horse-power of prime movers in the United States in June, 1936, was divided as follows:

	Horse-Power
Electric central stations	44 670 000
Industrial power plants	20 133 000
Electric railway plants	2 500 000
Isolated non-industrial plants	1 500 000
Mines and quarries	2 750 000
Agricultural prime movers	72 763 000
Automotive	965 000 000
Airplanes	3 500 000
Locomotives	88 000 000
Marine	30 000 000
Total	1 230 816 000

It is to be noted that the transportation services accounted for more than three-quarters of the power-generating machinery shown in Thurston's figures in 1896, while Potter in 1936 credits more than 91% to transportation—thus justifying Kipling's statement that "transportation is civilization." Of the 91% credited to transportation, about 8% use steam as the heat medium, and 92% use oil or gas in internal combustion engines. Of the stationary power plants (except those credited to transportation), Dean Potter schedules 56 684 000 hp in steam plants, 16 075 000 hp in water-power plants, and about 2 000 000 hp in oil and gas engine plants. Roughly, then, the stationary plant installation can be classified as 20% water power and 80% steam power.

When it comes to the actual horse-power-hours, kilowatt-hours, or work done, the figures are a little uncertain. However, it is probable that the 1935 output of the steam and water-power plants alone was not far from the equivalent of 142 000 000 000 kw-hr, of which more than 100 000 000 000 kw-hr was generated by steam. The average per capita use of power, central station plus industrial, is thus on the order of 1 115 kw-hr per yr. The equivalent kilowatt-hours of transportation power, locomotive plus automotive, is estimated conservatively at a somewhat larger amount than the stationary power, say, 150 000 000 000 kw-hr—hence, the total per capita use may be taken as the equivalent of 2 300 kw-hr per yr.

Whether the curve of prime-mover capacity is plotted from 1 800 (Thurston's curve) or from 1 500 (Adams' curve), the ascending segment is steep, and a 10-yr moving average smoothes out sufficiently the cyclic irregularities. (The curve of equivalent kilowatt-hours use follows practically the same law, the growth in use factor being very slow.) There is as yet little sign of the saturation that is evident in population curves, but this may appear in the future, particularly as the trend is toward a stationary population in the next twenty years.

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[6] Rept. on the "Status and Progress in the Art of Power Engineering," *Proceedings*, Am. Soc. C. E., March, 1930, Papers and Discussions, p. 437.

[7] Rept. of National Resources Committee, 1937.

The remainder of this paper omits from consideration the power used for transportation and that part of the stationary power that is generated by water, and is concerned solely with the stationary power-plant installation of 58 700 000 hp of heat engines. In most of this installation (56 680 000 hp), steam is the heat transfer medium.

The average annual use of this installation is about 1 730 hr, therefore, the total annual output of the steam plants is roughly 98 000 000 000 kw-hr. The central station output of 55 000 000 000 kw-hr is generated at an expenditure of about 1.45 lb of coal per kw-hr, while the industrial output of 43 000 000 000 kw-hr requires not far from 3 lb per kw-hr. The average use for both classes of plants is 2.13 lb per kw-hr, or the equivalent of 104 000 000 tons of coal per yr, roughly, 20% to 25% of the total annual coal output of the country.

In 1882, the late Thomas A. Edison considered 10 lb of coal per kw-hr a very good record for the first central station. F. R. Low, in 1891, reported to the National Electric Light Association that only one central station reported an economy as low as 4 lb of coal per kw-hr. His actual words were "250 watts per lb of coal." In 1936 the central station, at Port Washington, Wis., was consistently operated at an economy of 0.86 lb of 13 000 Btu coal per kw-hr—an over-all thermal efficiency of 30.6 per cent. In 1880, the late William Cawthorne Unwin, Hon. M. Am. Soc. C. E., the English authority on steam generation and use, considered 12 lb of Welsh coal per ihp-hr an average record in industrial establishments. This is equivalent of 16 lb per kw-hr. In 1936, industry was averaging about 3 lb per kw-hr of poorer coal and A. G. Christie[8] quotes a figure of 0.93 lb for one large modern industrial plant.

In 1880, in England, Unwin tested a Lancashire boiler and found the efficiency to be a little more than 80 per cent. Two years later, in the United States, J. C. Hoadley tested a return tubular boiler that gave 83% to 85% efficiency with a small air heater and about 79% without the heater. Power engineers have re-introduced the feed-water heater (which Edison used in the first of all central stations), the superheater, and the air heater, or economizer, making them a part of the modern boiler. They have added water-cooled furnaces, invented more than 100 yr ago, but not coming into general use until 1924. They have taken the stoker, invented about 1861, and improved it until it is nearly automatic, and they have perfected the burning of pulverized fuel. Even such fuels as lignite, or brown coal, containing 65% of water, are fired efficiently on a stepped grate. Oil wastes and acid sludges are also burned with high efficiency. Boiler tests currently show efficiencies of between 82% and 92%, and whenever it should become commercially feasible, these values can be exceeded. Year-round boiler-house efficiencies of from 78% to 88% are regularly reported by good operators. The increase in boiler-house efficiencies is necessarily small, because the law of heat transfer is fundamental. Progress has been in two directions: (*a*) Learning how to burn all the fuel at the best condition; and (*b*) adding heat traps to catch all heat not absorbed by the boiler proper.

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[8] *Mechanical Engineering*, September, 1936, p. 539.

Prior to 1900, the heat engine was the steam engine of Watt and his successors. Unwin's figure of 12 lb of coal per ihp per hr (1880) had been reduced in the following 20 yr to less than 1 lb by increase of pressures and compounding, by going back to condensing practice, and by better workmanship. A compound engine with 1 : 7 ratio cutting off at 14% stroke in the high-pressure cylinder would give about 50 expansions, and water rates varying from 11 to 9 lb per ihp-hr were being guaranteed by many builders. Poppet-valves were used if superheated steam was furnished, but the ordinary Corliss valves could take care of 30° to 50° F of superheat. The main difficulty was the demand for large units; low-pressure cylinders as large as 110 in. were made. Sometimes, two low-pressure cylinders of 96 in. each were used, as in the engines of the Interborough Rapid Transit Company, at the 59th Street Power Station, New York, N. Y., where 8 000-kw units were installed. Rolling-mill engines with two 110-in. low-pressure cylinders developed as high as 25 000 hp. All these units were of comparatively low speed, to ensure continuity of operation and low upkeep.

Following 1900, came the steam turbine—in small units at first, but rapidly increasing in size to meet the growing demand for large blocks of power. The maximum size of individual units increased from 5 000 kw in 1903 to 20 000 kw in 1912; 60 000 kw in 1925; 110 000 kw in 1928; and 165 000 kw in 1929. A three-shaft unit of 208 000 kw was installed in the same year. It is now possible to build a larger single generator unit than 165 000 kw, but the demand has not yet developed. The early turbines did not have as good a water rate as the better engines, because high vacuums were not used; the 26 in. used by engines was thought good enough. The adoption of a 28-in. vacuum raised the number of expansions to 100, and the later increase to 29 in. doubled this figure, greatly increasing the efficiency and the unit capacity. To-day, designers strive for an absolute pressure of $\frac{1}{2}$ in. to $\frac{5}{8}$ in. of mercury, and water rates of approximately 9 lb per kw-hr, or 6.75 lb per hp-hr are common, even at ordinary pressures and with temperatures below 750° F.

Meanwhile, the Power Industry has learned the value of high pressure and high superheat. Pressure has been increased to 600 lb per sq in., then to from 1 200 to 1 400 lb, then to 1 800 lb, and, in Europe, installations using steam at the critical pressure (3 200 lb) are in operation. In temperature, 825° F is now standard, 932° F (500° C) is contemplated, and at least one unit has been in operation for some time at 1 000° F. Potter lists 16 stations carrying pressures between 710 and 1 840 lb; 25 stations between 500 and 700 lb; and 174 stations between 300 and 500 lb.

Table 1 shows the increase in thermal efficiency since Watt's time (1770). The first four entries are from a paper on "The Commercial Economy of High Pressure and High Superheat."[9] In this paper and the discussion that followed, the regenerative cycle, old in engine practice, was stressed, and since that time bleeding has become general practice. The predicted economies have been obtained. As many as six stages of feed-water heating have been used, and the feed-water is now heated nearly to the saturation temperature. Steaming economizers have been used, but the practice does not seem to be increasing. The most modern plants still fall short of introducing all the heat to the medium at the maximum temperature, although in the mercury vapor installations a larger proportion of the heat is introduced at approximately that point.

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[9] *Transactions*, A. S. M. E., Vol. 44, p. 1119.

TABLE 1.—INCREASE IN THERMAL EFFICIENCY SINCE 1770

Year	Prime mover	Pressure, absolute	Temperature, in degrees Fahrenheit	Carnot cycle, actual percentage	British thermal units per kilowatt-hour	Thermal efficiency, percentage
1770	Newcomen engine	15	212	2.7	416 000	0.82
1810	Watt engine	30	300	9.17	61 000	5.60
1900	Nordberg engine	215	400	20.8	19 200	17.75
1922	Turbine	265	650	35.0	18 000	19.0
1935	Turbine*	1 245	825	58.0	11 166	30.6
1935	Turbine†	1 245	825	58.0	10 850	31.5
1935	Mercury vapor turbine	140	950	61.0	10 500	32.5
	Proposed mercury vapor turbine	240	1 025	63.5	9 500	36.0

* Port Washington, Wis., 8-month record, *Mechanical Engineering*, November, 1936, p. 697.

† Port Washington, Wis., record for single month, *Mechanical Engineering*, November, 1936, p. 697.

The cost of central stations has varied but little since Mr. Edison built the Pearl Street Station, in New York City. The writer's records, in more or less detail, cover more than 150 plants built between 1882 and 1936. The costs include land, buildings, foundations, water tunnels, coal and ash handling, boiler and turbine-room machinery and auxiliaries, the switching arrangements, and outgoing feeders to the building line. The average cost of these plants is substantially $100 per kw of capacity. Potter quotes figures collected by the Federal Power Commission, for 80 plants built since 1920, which average $108 per kw of capacity, while L. L. W. Morrow[10] lists 16 plants built since 1927 at an average cost of $114 per kw installed.

The average cost for the 250 plants in these three lists (with possibly some duplicates) is about $103 per kw of installed capacity. The installations are of many types—large and small, high- and low-pressure, light, power, and railway plants, with a small proportion of industrial plants. The lowest cost is about $60 per kw, and the highest, approximately $225 per kw. Potter lists 5 plants that cost more than $150 and 4 less than $70. Morrow's variation is from $82.50 to $145.00 per kw.

As a general rule, large plants cost a little less per kilowatt than small plants. High pressure plays little part; of the 1 400-lb plants reported by Morrow some were in the lower range of prices and some in the higher range. In favored locations, high-pressure stations have been built for as little as $82.50 per kw, and it is probable that this figure might be considerably smaller with later designs.

In the earlier years building costs were low, while machinery costs were high. Switchboards were simple and cheap. In later years, the electric costs have risen from 5% to as much as 25% of the total cost. (It should be noted that the cost of switchboard safety devices and other electrical equipment mounts rapidly with the size of the connected system.) Building costs, including cost of foundations, have increased from 10% to 40% of the total. Land has varied from 0.5% to 2.5 per cent. On the other hand, costs of machinery—boilers, turbines, piping, and auxiliaries—have decreased from 80% to 35% of the total.[11]

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[10] *Electrical World*, November 23, 1935.

There is much diversity of opinion as to the cost as well as the desirability of "out-door" stations, and this is likely to continue until more actual experience has been obtained with the design, construction, and operation of this type of plant. Meanwhile, the convenience of operation and maintenance of the enclosed plant under severe weather conditions is likely to prevent much experimentation with the new design.

Central stations using steam as the heat medium are now in operation giving yearly economies, coal burned to kilowatt-hours sent out, of as low as 11 000 Btu per kw-hr—a thermal efficiency of 31 per cent. By taking full advantage of high pressure and temperature, carrying regeneration to the limit, and using at least one reheating stage, the thermal efficiency might be raised to about 37%, or the economy to about 9 200 Btu per kw-hr. With the mercury-vapor cycle, about the same economy might be obtained ultimately. Plants with such high efficiencies would be complicated in design and difficult of operation, as well as costly. They would require much better materials than are now obtainable, and are not even contemplated at present.

A steam plant running at 10 000 Btu per kw-hr, however, may be anticipated in the near future. The new heat-resistant materials, increased knowledge of the more common metals and alloys, the recent developments in boiler and turbine design, and the demand for more efficient plant, will bring it about.

The same economy is also in sight with the mercury vapor plants. A single month's record of 10 400 Btu per kw-hr was reported from the mercury plant, at Hartford, Conn., in 1936, and designs have already been made for plants with economies even lower than 10 000 Btu per kw-hr.

But are these very efficient plants justified? What is the relation between thermal efficiency and commercial efficiency? Thermal efficiency is easily measured. Fuel bills are plain, and present-day sampling and calorimetry are trustworthy. Station watt-meters are standard instruments and quite accurate. The two ratios, coal per kilowatt-hour and British thermal units per kilowatt-hour, are well known by all operators.

Commercial efficiency is a different story, for it involves many factors subject to conflicting interpretations. The cost of the installation may be book cost, replacement cost, or depreciated cost, or even cost minus some part charged off to steam heating, general offices, or rentable property. When this cost has been delimited, the percentage of fixed charges must be determined. This usually includes interest, reserve for renewals, taxes, and insurance, lumped together as 12.5% to 15% of the prime cost; and, then, enters the use factor of the installation. The average cost of stations is, say, $100 per kw. Fixed charges at 12.5% are then $12.50 per annum per kilowatt installed. If the use of the installation is 3 000 hr per annum, the fixed charges are 0.417 cts per kw-hr; if the use is 8 000 hr, the fixed charge is 0.156 cts.

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[11] "Rôle of the Civil Engineer in Power Development," by the late Ira W. McConnell, M. Am. Soc. C. E., *Proceedings*, Am. Soc. C. E., January, 1928, p. 155; and the discussions by Messrs. P. Junkersfeld, Geo. A. Orrok, Joel D. Justin, and F. W. Scheidenhelm, published in *Proceedings*, Am. Soc. C. E., March, 1928, p. 923; J. A. Sargent, M. Am. Soc. C. E., *Proceedings*, Am. Soc. C. E., August, 1928, p. 1917; and the closing discussion in *Proceedings*, Am. Soc. C. E., for April, 1929, p. 977.

Operating charges also vary with the use factor, but not nearly to the same extent as fixed charges. With coal at the prices which ordinarily maintain on the Eastern seaboard, operating charges per kilowatt-hour may be expected to be less than fixed charges for uses of 4 000 hr, or less, per yr. Under conditions like those in the vicinity of Pittsburgh, Pa., the point of intersection may be as high as 7 000 hr. The total cost of power at the station wall is, of course, the sum of the fixed and operating charges. Hence, its magnitude always depends on the hours of use. In this connection see Table 2. It will be noticed that even with coal at $5 per ton, the fixed charges run from 36% of the total cost at 8 760 hr of use to 73% at 1 750 hr. Commercial efficiency increases with the use factor, and each plant presents a unique problem in which the prime cost of installation must be balanced against hours of use and fuel cost to secure the most favorable combination.

TABLE 2.—COST OF STEAM POWER*

Use factor, percentage	100	80	60	40	30	20	10
Hours of use	8 760	7 008	5 256	3 504	2 628	1 752	876
Fuel per kilowatt-hour, in pounds	0.773	0.777	0.786	0.804	0.821	0.857	0.964
Fixed charges, in cents per kilowatt	0.143	0.178	0.238	0.357	0.475	0.714	1.429
Other charges, except fuel	0.08	0.08	0.08	0.08	0.08	0.08	0.08
Total, all charges except fuel, in cents per kilowatt-hour	0.223	0.258	0.318	0.437	0.555	0.794	1.509
Cost of coal, in cents per kilowatt-hour:							
Coal at $1.00 per long ton	0.045	0.0347	0.035	0.0358	0.0366	0.0382	0.043
Coal at $3.00 per long ton	0.1035	0.1041	0.105	0.1074	0.1098	0.1146	0.129
Coal at $5.00 per long ton	0.1725	0.1735	0.175	0.1790	0.1830	0.1910	0.215
Total cost, in cents per kilowatt-hour:							
Coal at $1.00 per long ton	0.2575	0.2927	0.353	0.4628	0.5916	0.8322	1.552
Coal at $3.00 per long ton	0.3265	0.3621	0.423	0.5444	0.6648	0.9086	1.638
Coal at $5.00 per long ton	0.3955	0.4315	0.493	0.616	0.738	0.985	1.724

* Conditions assumed: Cost per kilowatt of installation, complete, $100; no reserve capacity; fixed charges, 12.5%; plant economy, in British thermal units per kilowatt-hour, $10\,500 + \frac{30\,000}{\text{Use factor in percentage}}$; and heat value of coal, 14 000 Btu per lb.

The Power Industry has long known that the road to commercial economy lies in the extension of the hours of use of a given installation. Edison, Crompton, Wordingham, and other early authorities have made this fact plain to every maker of power. Balanced loads, steady running, and diversified services are sought by all, for power cannot be stored except at great expense. Unfortunately, however, the tastes and habits of users of power lead to a wide variation of load in industries and power supplies. In spite of this, however, load factors have grown. Edison, at Pearl Street, was well pleased with a 12% load factor. To-day, system load factors may go as high as 40%, while certain industries maintain a load factor of 70% or more; and the trend is toward a still larger use. With better load factors, the use of higher pressures and temperatures will increase, bringing higher efficiencies and a more general distribution of the benefits attained.

Improvement in the design and efficiency of the internal combustion engine has been steady and continuous, but because of the nature of the problem the advance has not been nearly so spectacular. Ever since gasoline has been available, Otto-cycle engines have been able to develop a brake-horse-power-hour on 0.6 lb of fuel; high-speed, high-compression aviation engines now give a brake-horse-power-hour on 0.4 lb of fuel.

The best Diesel engines of the World War years used 0.36 lb of oil per bhp per hr. Two-cycle Diesel engines did not do quite so well, and 0.4 lb of fuel was a good record for them in those days. Solid-injection engines of modern design do a little better than the earlier air-injection engines. The best record that has come to the writer's attention is 0.326 lb, attained by a two-cycle solid-injection Diesel. None of these figures includes the oil used for lubrication, a portion of which is burned in the cylinder and adds to the developed heat.

Solid-injection two-cycle engines are now the standard for all sizes, although four-cycle engines are still used in sizes up to about 2 000 hp, and air-injection designs are used where low-quality fuel is to be burned. The range between 0.32 lb and 0.4 lb of fuel is small, and the heat use ranges between 6 300 and 8 000 Btu per bhp per hr, or, putting the conversion factor at 700 watts to allow for generator efficiency, 9 000 to 11 400 Btu per kw-hr for the prime mover alone. Of the 60% of the heat lost in the exhaust and jacket, about three-fourths may be recovered; the main difficulty in the way of doing so is the cost and complexity of the apparatus required.

The range of thermal economies for the oil-engine plants is much the same as that for steam plants, but the lower limit is easier of attainment. At best, the internal combustion engine would appear to have an advantage of about 10% in thermal economy. On the other hand, the installation cost is usually much higher than for steam, the average cost per kilowatt being more than one-half again as large. No very large oil-engine stations have been built, the one at Vernon, Calif. (35 000 bhp, in five units) being the largest as of 1936. About 2 000 hp has been developed in the double-acting cylinder, and twelve cylinders on one shaft have been proposed. The large peak-load engines (Copenhagen), run at high piston speeds (more than 1 200 ft per min, so high that a long life is problematical). The best figures on internal combustion engine costs and operation are to be found in the reports[12] of the Oil Engine Power Cost Committee of the American Society of Mechanical Engineers.

The steam engine, as developed by Watt, his competitors and successors, made the Nineteenth Century the century of steam. Parsons, Delaval, Rateau, and Curtiss, and their successors, in the Twentieth Century have carried on their work. Otto, Clerk, and Diesel, following other lines, have brought about a tremendous development in transportation. Meanwhile, the binary-vapor proposals of the last hundred years have been put into material form by Emmet in the mercury-vapor turbine, which has already equalled the economy of the Diesel engine. All these developments lead one to speculate on what other forms of prime movers may be developed in the near future.

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[12] *Transactions*, A. S. M. E., Vol. 57, p. 69.

On the basis of Milliken's electron theory and Rutherford's smashing of the atom a number of modern scientists and engineers have predicted that the next step will be atomic power—although Milliken's final summary on this question[13] is that power from atomic degradation can never be generated in commercial amounts.

In at least two locations (Llarderello, Italy, and Sonoma County, California), steam power is now generated by the heat of volcanic rocks, and Charles A. Parsons has proposed a well 12 miles deep to utilize the internal heat of the earth for the same purpose. The engineering difficulties of the latter proposal would be enormous, as the deepest wells to-day are less than 2 miles in depth.

In 1936 Dr. Abbott, of the Smithsonian Institute, at Washington, D. C., exhibited his solar engine, which utilizes Ericson's mirror system and is three to four times as efficient as former engines of this type. However, his apparatus is not comparable in size to Shuman's plants at Tacony, Pa.,[14] and Cairo, Egypt. The Tacony Plant, under test, developed 32 hp when the sun shone.

None of these proposals meets the criterion of a commercial power plant which must generate power when wanted, as wanted, and at a cost a little less than that at which it can be produced to-day. Engineers, scientists, and inventors are looking for a new source of power, convenient, plentiful, and cheap; but until that source appears it is certain that a major part of the world's power will be generated in heat engines using steam as the heat medium.

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[13] *Science*, September 28, 1928.

[14] *Engineer* (Lond.), July 5, 1912.

HYDRO-GENERATION OF ENERGY

BY FRANK H. ROGERS,[15] ESQ.

SYNOPSIS

The installed capacity of hydro-electric plants has been increasing almost continuously since 1900. Concurrently, the maximum size and horse-power of individual units have grown by leaps and bounds. The limit in efficiency was closely approached by 1920, and the most important event in the history of turbine design since that time is the development of the propeller-type turbine.

This paper reviews briefly the important design features of modern high-head, medium-head, and low-head turbines; discusses the importance of laboratory testing (with special reference to low-head installations); points to six important mechanical improvements in turbine design made since about 1925; and concludes with an examination of possible future developments, such as the pumped-storage plant and the use of the propeller-type turbine for higher heads.

Various aspects of the progress in the hydro-generation of energy in the United States since the beginning of the century are indicated in Fig. 1.

As for installed capacity, it will be noted that from 1900, when the first developments at Niagara Falls were undertaken, there was a rapid increase until 1922, at the rate of about 300 000 hp per yr. Then the slope of the curve rose to a still higher rate of about 800 000 hp per yr from 1922 to 1931. In the next four years the increase in installed capacity fell off considerably, during 1935 amounting to about 100 000 hp. In 1936, however, a great increase occurred, about 1 500 000 hp capacity being installed, of which probably 800 000 hp actually went into operation before the close of the year.

The demand for larger units has been met by the use of better materials and new construction methods. The highest point on the unit horse-power curve indicates the 115 000-hp Boulder

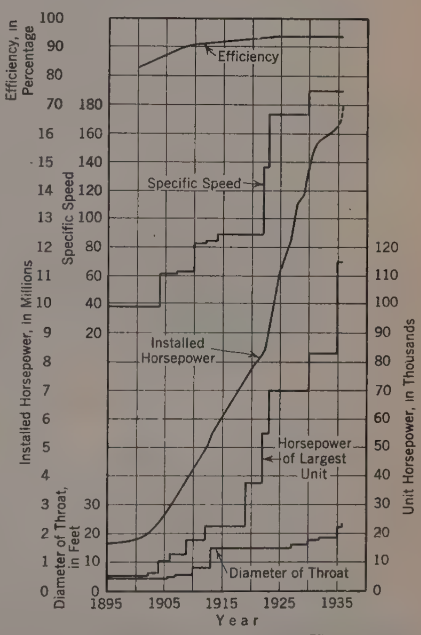

FIG. 1.—PROGRESS CHART OF THE HYDRAULIC TURBINE IN THE UNITED STATES, 1895–1936

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[15] Chf. Engr., I. P. Morris Div., Baldwin-Southwark Corp., Philadelphia, Pa.

Dam turbines the most powerful units in the world. As for physical size (diameter of throat), the largest units yet constructed are those for the Bonneville Power Plant, in Oregon.

Since about 1910 a large majority of the turbines installed have been of the vertical-shaft type. This design has permitted the building of much larger units than would have been feasible with a horizontal setting. The development of the Kingsbury thrust-bearing about that time was the most important contribution to the success of the vertical unit.

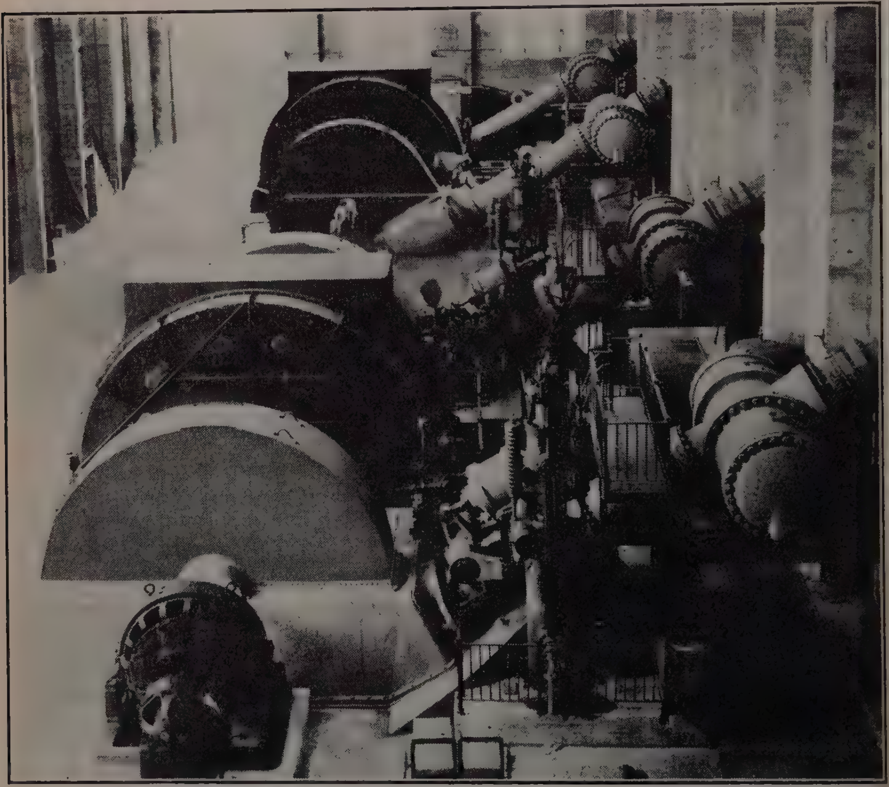

Fig. 2.—Big Creek No. 2-A Development, Southern California Edison Company, with Two 70 000-Hp Impulse Wheels

Little increase in efficiency has been secured since 1920, as the possible limit was closely approached at that time.

The increase in specific speed in 1922 is, however, noteworthy. In that year the large-sized propeller-type turbine, which permitted the economical development of large low-head plants, was introduced in the United States. This development is the most important in the recent history of turbine design, and this paper will devote considerable space to the problems involved in the design of this type of unit.

First, however, the progress made in the design of high and medium-head turbines will be reviewed briefly.

High-Head Turbines

The classification, "high-head turbines" is arbitrarily taken to include all units designed for heads greater than 300 ft. It comprises both turbines of the Francis type, and impulse wheels. For heads of 1 000 ft and higher the latter are used almost exclusively in spite of their lower efficiency and lower specific speed as in that head range cavitation limitations become critical, and the stresses involved in the casing and head cover would be excessive if the Francis type unit were used.

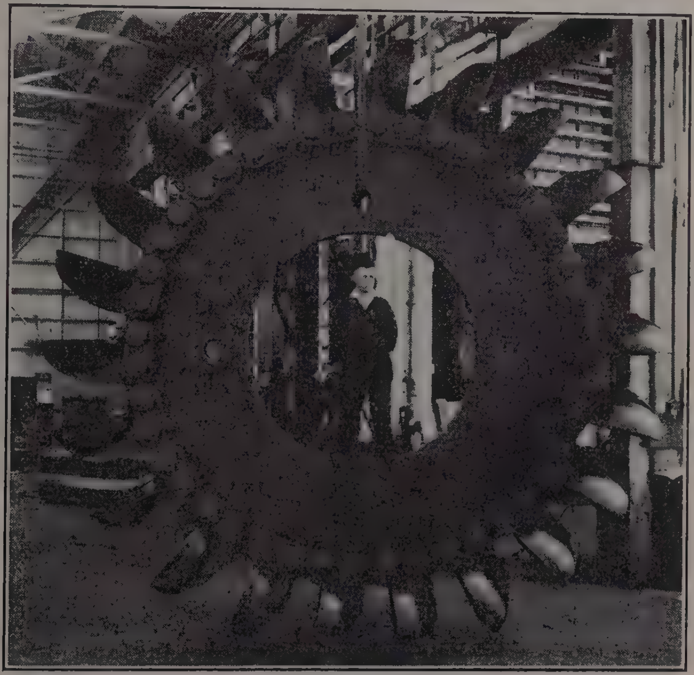

Fig. 3.—Runner, 162-Inch, for One of Big Creek No. 2-A Wheels

Impulse turbines are usually of the horizontal-shaft, single or double-runner, and single or double-nozzle type. They have been built in the United States in units as large as 70 000 hp, and for heads up to about 3 000 ft. Fig. 2 shows the two 70 000-hp impulse wheels of the Big Creek No. 2-A Development of the Southern California Edison Company. These units are of the two-runner, single-nozzle type, and operate under a head of 2 200 ft, at a speed of 250 rpm. One wheel of one of these units, with the buckets assembled, is shown in Fig. 3. The pitch diameter of the runner is 162 in. This unit is not only the largest in physical dimensions, but also the most powerful unit yet constructed for installation in this country.

Since about 1925 impulse-wheel efficiencies have been considerably improved by research in bucket and nozzle design. As an example, in Fig. 4 are shown one of the older elbow-type nozzles with relief nozzles below, and a straight-flow nozzle of improved design first developed in 1926. The water passageway in the latter, being perfectly straight, produces a jet with a minimum of turbulence and, consequently, of higher efficiency, and permits somewhat higher approach velocities to be used.

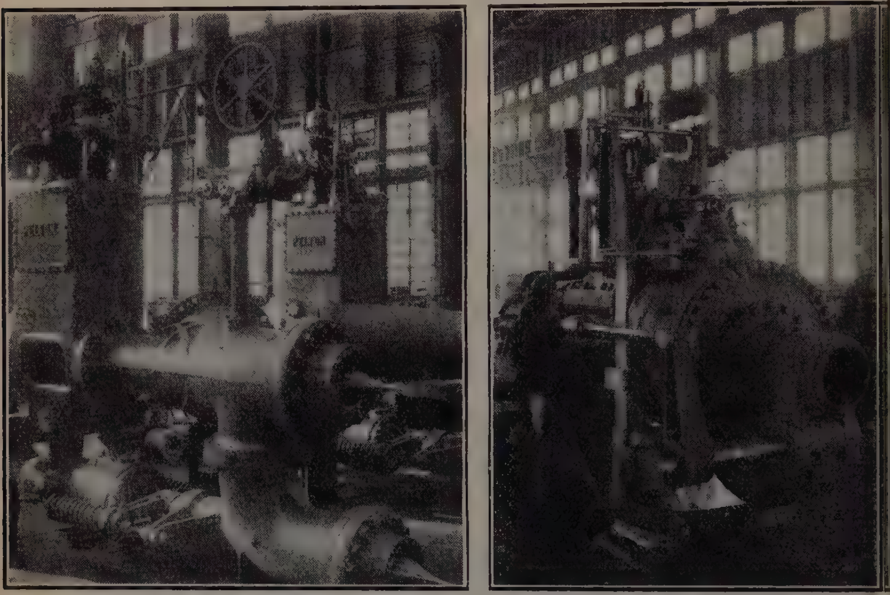

Fig. 4.—Impulse Wheel Nozzles; Left: Elbow Type; Right: Improved Straight-Flow Type

Reaction or Francis-type turbines have been used for heads up to about 850 ft. As examples of high-head Francis units the following may be mentioned:

30 000-hp turbine for the Big Creek Plant No. 8 of the Southern California Edison Company; head, 680 ft.
33 000-hp turbines for the Santeetlah Plant, of the Tallassee Power Company (now the Carolina Aluminum Company); head, 660 ft.
35 000-hp turbine for the Oak Grove Plant, of the Portland Electric Power Company; head, 850 ft.
49 000-hp turbine for the Waterville Development, of the Carolina Power and Light Company; head, 755 ft.
115 000-hp turbines for the Boulder Dam Plant, of the United States Bureau of Reclamation; maximum head, 590 ft.

The Boulder Dam units will operate under a wide range of head (420 to 590 ft), the average net head being 530 ft. At 420 ft each unit is guaranteed to deliver 90 000 hp, and at 480 ft and higher, 115 000 hp. Fig. 5 shows two of these turbines completely assembled in the shop. The casing has a 10-ft intake diameter, and was shop-tested to a pressure of 500 lb per sq in. One of the cast-steel runners, with an intake diameter of 14 ft 3 in., is shown in Fig. 6.

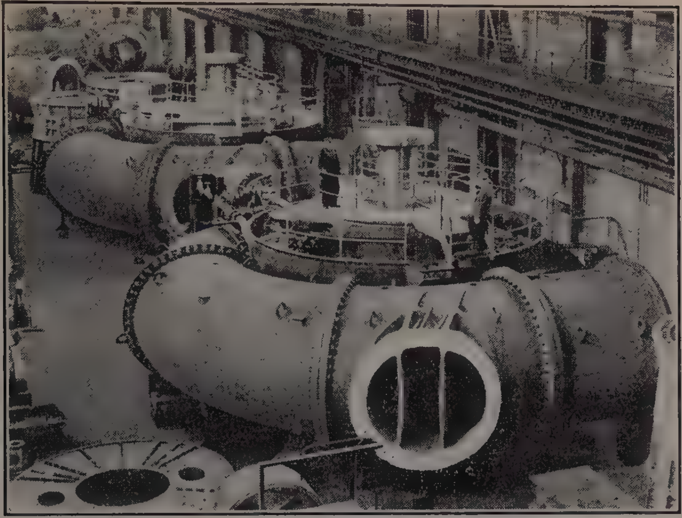

FIG. 5.—SHOP ASSEMBLY OF TWO 115 000-HP TURBINES FOR BOULDER DAM PLANT, OF THE UNITED STATES BUREAU OF RECLAMATION

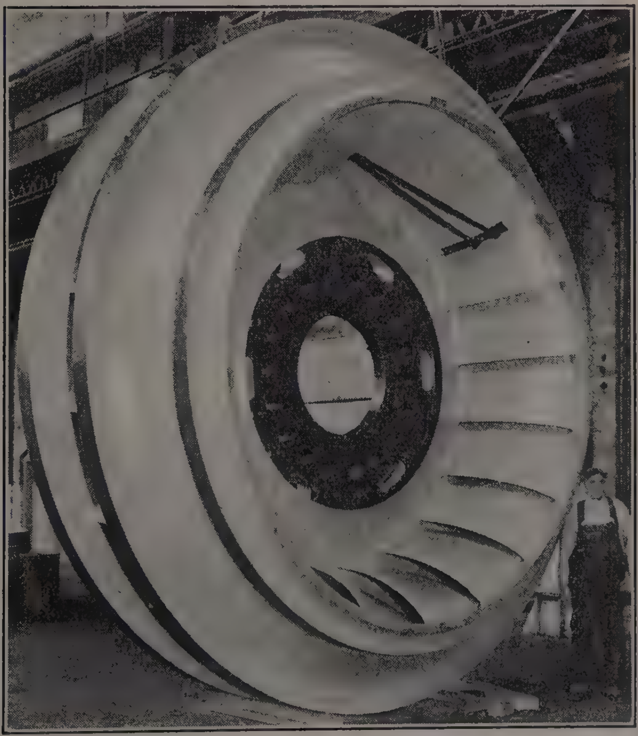

FIG. 6.—RUNNER, 171 INCHES IN DIAMETER, FOR 115 000-HP TURBINE AT BOULDER DAM

In high-head units especially, it is important to hold the leakage at the runner seals to a minimum. Any increase of the seal clearance due to wear materially affects the efficiency of the unit. A very close clearance can be used without danger of seizing due to contact, by adopting for the stationary seal an insert of soft metal such as Parson's white brass. Such a design (Fig. 7) is used in connection with two of the units of the Boulder Plant.

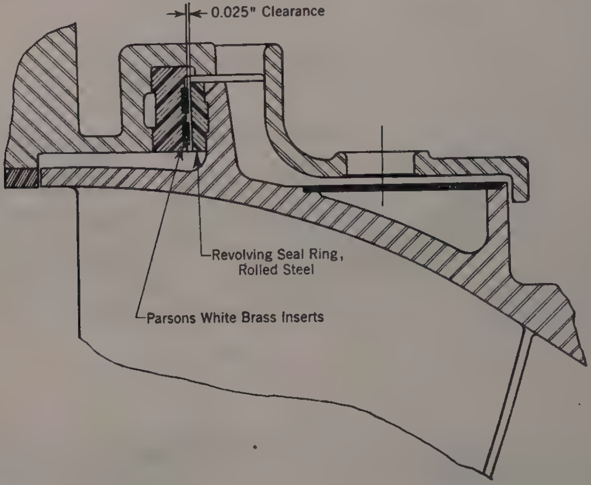
0.025" Clearance
Revolving Seal Ring, Rolled Steel
Parsons White Brass Inserts

Fig. 7.—Seal Ring Design for Two of the Boulder Dam Units

Medium-Head Turbines

For heads ranging from about 100 ft to 300 ft Francis turbines of the vertical-shaft, single-runner type are used. For the lower heads within this range the casings are of plate steel, and for the higher heads they are of cast steel. Noteworthy installations of this type include:

Seven 54 000-hp turbines—head, 89 ft—installed in the Conowingo Development of the Susquehanna Power Company, on the Susquehanna River, Maryland.

Two 66 000-hp turbines—head, 165 ft—installed in the Norris Dam Plant, of the Tennessee Valley Authority, on the Clinch River, Tennessee.

Three 70 000-hp turbines—head, 213 ft—installed in Station No. 3, of the Niagara Falls Power Company, on the American side of the Falls.

Two 83 000-hp turbines—head, 310 ft—installed in the Diablo Plant, of the City of Seattle, on the Skagit River, Washington.

Fig. 8 is a sectional elevation of one of the Conowingo units, with its butterfly valve. The runner has a throat diameter of 16 ft 2 in., and is made in four sections secured together by a crown plate and band. The casing, of steel plate, has an intake diameter of 27 ft. The butterfly valve is probably the largest ever constructed. During periods of low flow, this installation is a peak-load plant and the units are shut down for a considerable part of the time. For this reason, leakage past the butterfly valves is an important item, and in order to reduce it to a minimum these valves are provided with a special rubber tube sealing device. When the valve is closed, water under pressure is admitted to the tube, expanding it through a slot in the valve housing so that the rubber is in contact with the outside periphery of the valve disk.

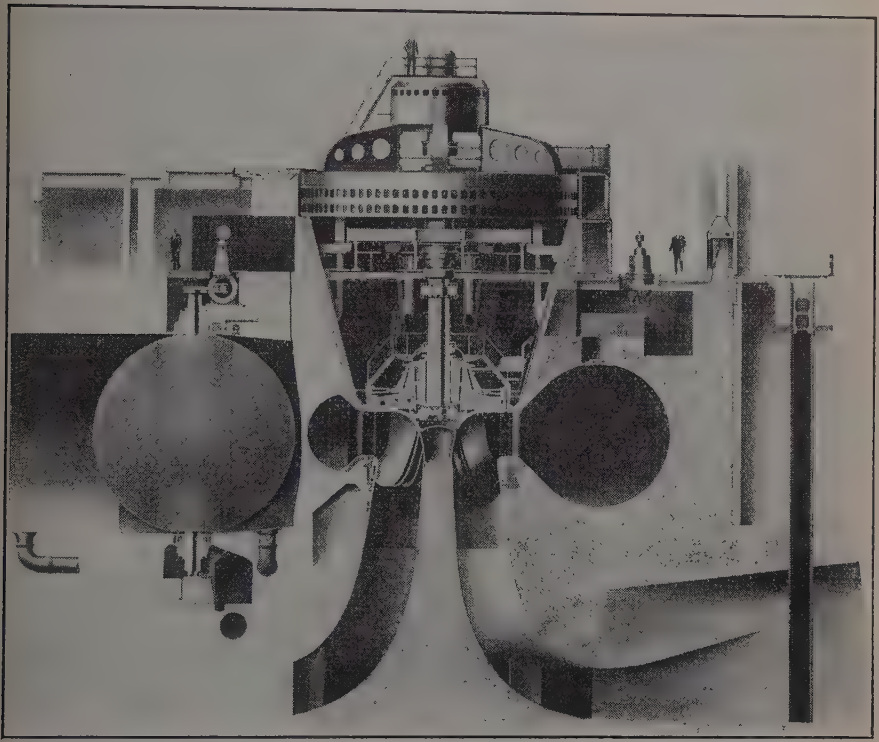

Fig. 8.—Sectional Elevation of 54 000-Hp Turbine, Conowingo Development, Susquehanna Power Company

Fig. 9 shows the shop assembly of the plate-steel casing and cast-steel stay-ring of the Norris Dam turbines. The intake diameter of the casing is 17 ft 8 in. Fig. 10 is an interior view of the stay-ring, casing, and penstock of the installed unit, and clearly indicates the field riveting at the casing joints and the connection to the stay-ring.

The two-bearing unit with umbrella-type generator is an excellent design adopted in a number of installations. Fig. 11 shows one of the two 31 000-hp units of this type in the Morony Development, of the Montana Power Company, at Great Falls, Mont. Particular attention is called to the excellent design of the water passages. The penstock is straight from the intake gates to the casing, thus reducing to a minimum the losses at the intake and through

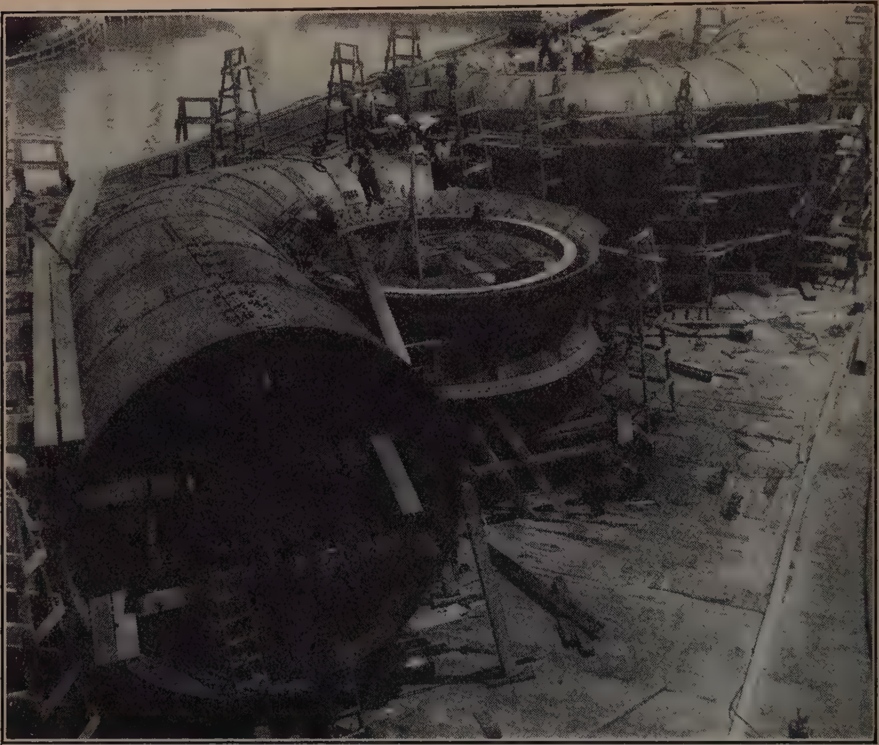

FIG. 9.—SHOP ASSEMBLY OF SPIRAL CASINGS AND STAY-RINGS OF 60 000-HP TURBINES FOR NORRIS DAM PLANT, TENNESSEE VALLEY AUTHORITY

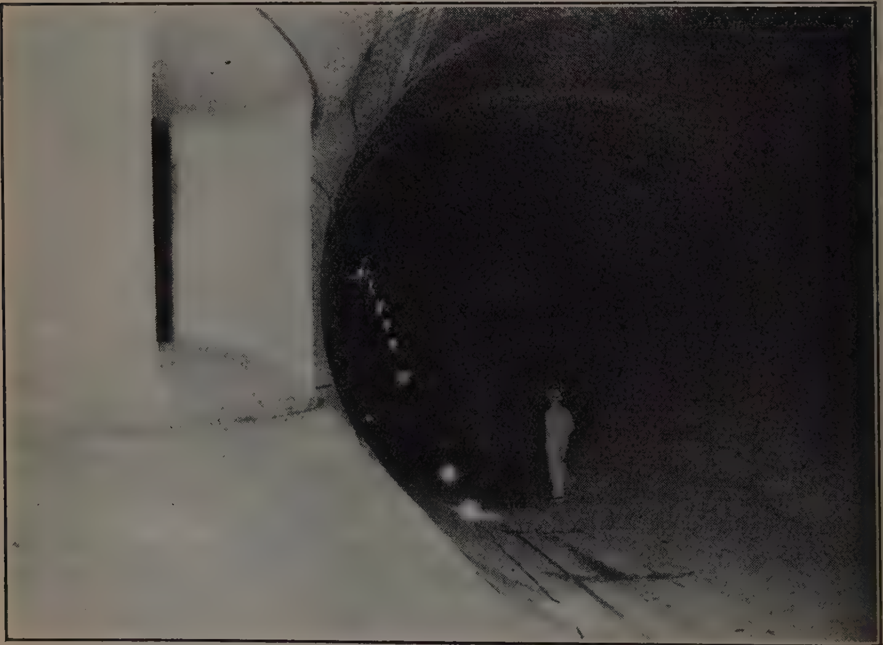

FIG. 10.—INTERIOR VIEW OF STAY-RING, CASING, AND PENSTOCK, NORRIS DAM PLANT

the penstock. Turbine efficiency in the field test was 93%, and over-all efficiency from head-water to tail-water was 92.6 per cent.

Low-Head Turbines

The greatest advancement in hydro-generation of energy since 1920 has been due to the development of the high-speed propeller-type turbine. The increase in specific speed from about 90 for the Francis-type runner to 150 and higher for the propeller type has made possible the economical development of the large low-head installation, primarily by permitting the use of the lower-cost, higher-speed generator and reducing the cost of the power-house superstructure.

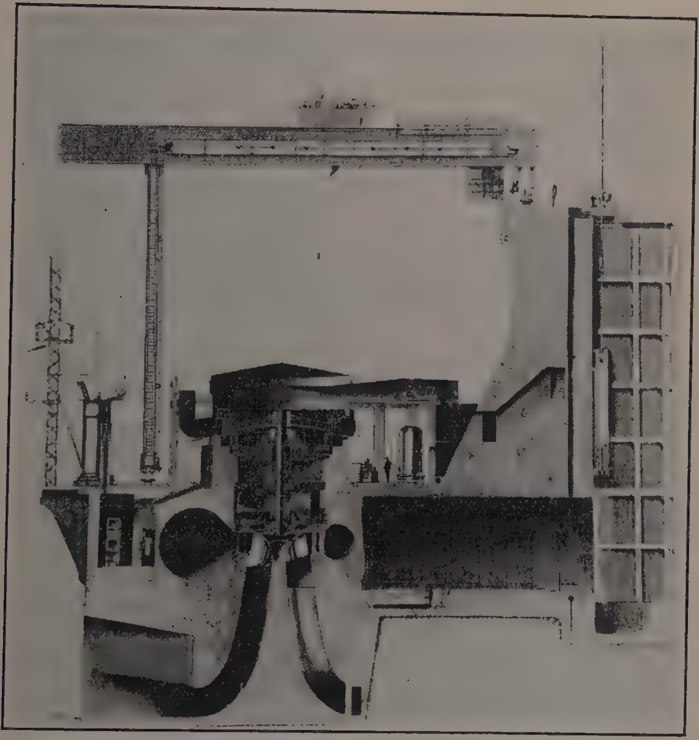

Fig. 11.—Two-Bearing Unit, with Umbrella-Type Generator

The radical difference in size and shape of the propeller runner as compared to the Francis runner is shown in Fig. 12, illustrating four runners of different specific speeds. These runners are drawn to the same scale, and, under the same head, each runner would develop the same horse-power, but at a speed proportional to its specific speed. The first type would be suitable for heads up to 300 ft; the second, for heads up to 100 ft; and the last two, for heads up to 60 ft. It may be asked why the high-speed runner shown at the bottom cannot be used under the higher heads and thus obtain higher speeds for the generators. If this were attempted the velocity through the runner would be very high, and the absolute pressure would fall so low that vapor-filled cavities would form, the water would separate from the blades, and the resulting cavitation would not only reduce the power and efficiency, but cause pitting of the runner.

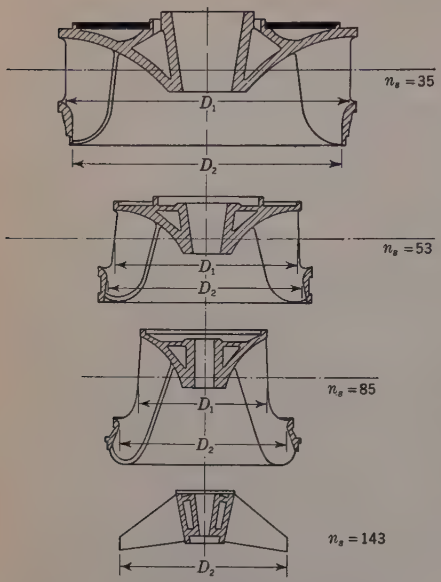

FIG. 12.—COMPARISON OF RUNNERS OF EQUAL POWER, BUT OF DIFFERENT SPECIFIC SPEEDS

It is a general characteristic of turbine-runner design that, as the specific speed of the runner increases, the horse-power efficiency curve becomes steeper and the part-load efficiencies decrease. This characteristic is shown for the four runners of Fig. 12 in Fig. 13. It will be noted that for high-head installations where low specific speed runners are chosen, flat horse-power efficiency curves are obtained, whereas for low-head plants, where high specific speed runners are used, steep horse-power efficiency curves result. If it is necessary to operate the fixed-blade high-speed runner at part loads, a rapid falling off in efficiency will occur.

This inherent weakness in the characteristics of the fixed-blade propeller turbine was overcome by the development of the adjustable-blade runner known as the Kaplan type (after Victor Kaplan, of Austria, who first suggested it). In this type of unit the runner blades and guide-vanes are automatically and simultaneously adjusted in accordance with the load demand to provide

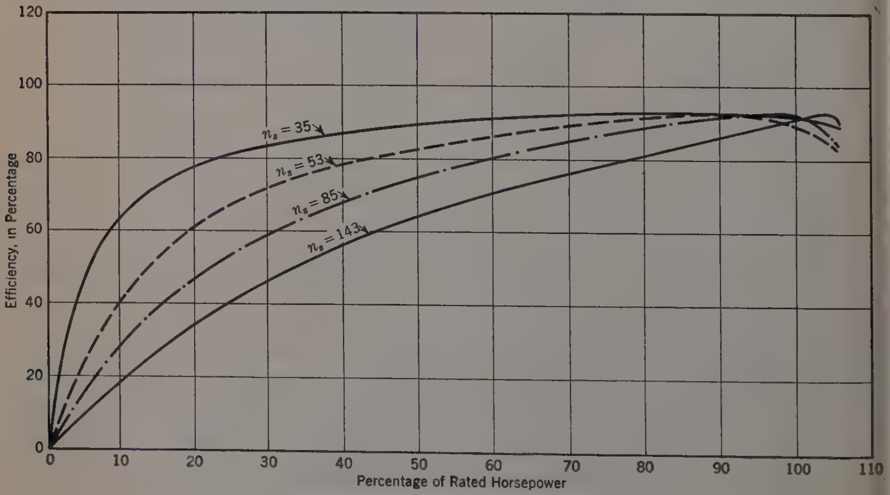

FIG. 13.—PERFORMANCE CURVES FOR RUNNERS OF FIG. 12

the best possible hydraulic relation between guide-vane opening and runner-blade position.

Fig. 14 shows a typical horse-power efficiency curve for the Kaplan type turbine. Each of the dotted curves shows the performance of the unit if operated as a fixed-blade runner at the blade angle indicated. By automatically moving the runner blades at the same time as the guide-vanes the unit can be operated at the peak of each separate efficiency curve, so that the envelope curve (shown solid) becomes the real operating curve. The Kaplan-type turbine thus permits the adoption of high speed, yet maintains a high efficiency over a wide range in load.

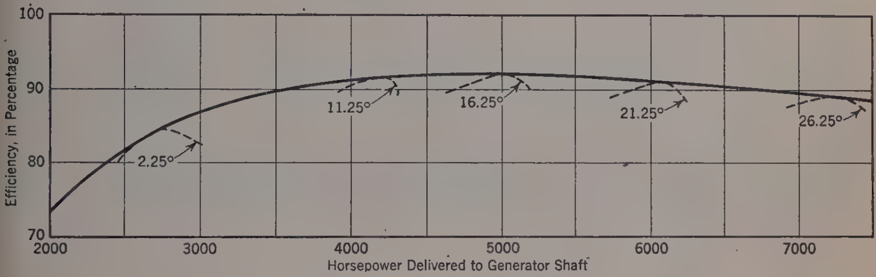

FIG. 14.—TYPICAL PERFORMANCE OF KAPLAN-TYPE TURBINE

The following installations will illustrate the fixed-blade, propeller-type turbine:

Four 13 500-hp turbines—37-ft head—installed in the Louisville (Ky.) Plant of the Louisville Hydro-Electric Company.

Four 21 000-hp turbines—32-ft head—installed in the Rock Island Plant, of the Puget Sound Power and Light Company, Columbia River, Washington.

Two 45 000-hp turbines—48-ft head—installed in the Wheeler Dam Plant of the Tennessee Valley Authority, on the Tennessee River, Alabama.

Among the Kaplan-type installations may be mentioned:

Two 7 250-hp turbines—23-ft head—installed in the London and Marmet Stations of the Kanawha Valley Power Company, on the Kanawha River, in West Virginia.

Six 42 000-hp turbines—55-ft head—installed in the Safe Harbor Plant of the Safe Harbor Water Power Corporation, on the Susquehanna River, in Pennsylvania.

Two 48 000-hp turbines—43-ft to 56-ft head—being installed in the Pickwick Development of the Tennessee Valley Authority, on the Tennessee River, in Tennessee.

Two 66 000-hp turbines—69-ft head—being installed in the Bonneville Development of the Federal Government on the Columbia River, Oregon. These turbines, with a runner diameter of 23.3 ft, are the largest as well as the most powerful units of the Kaplan type as yet constructed in the United States.

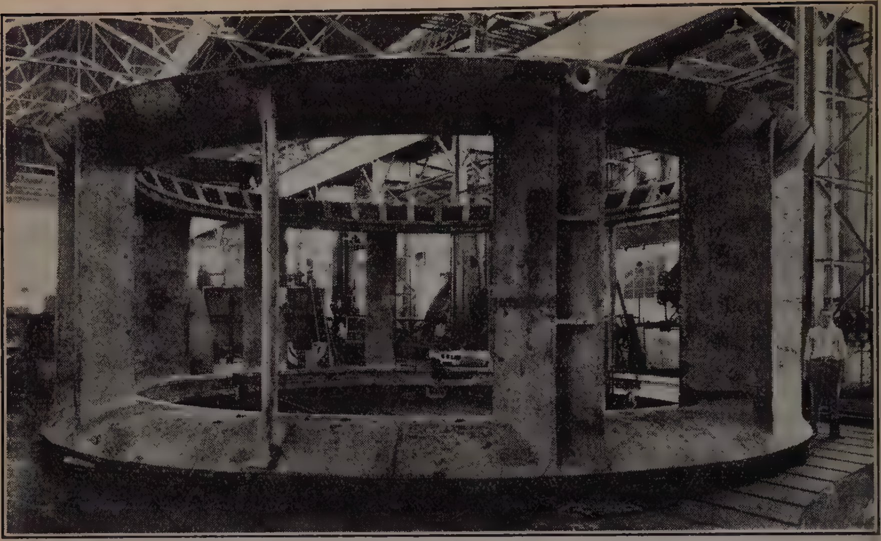

Fig. 15.—Stay-Ring for 45 000-Hp Propeller-Type Turbine for Wheeler Dam Plant, Tennessee Valley Authority

Fig. 15 shows the assembled stay-ring for one of the 45 000-hp Wheeler Dam units, and Fig. 16, the assembled runner for the same turbine.

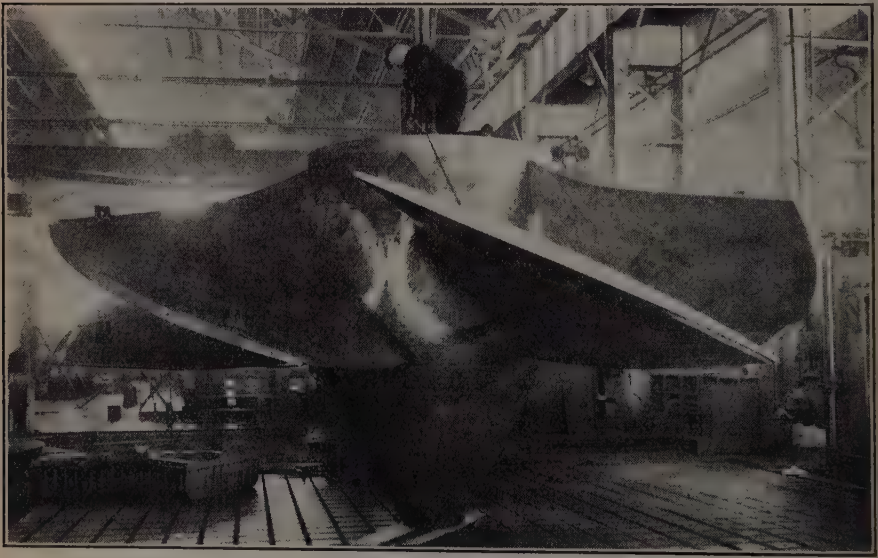

Fig. 16.—Shop Assembly of 22-Foot Fixed-Blade Propeller Runner for 45 000-Hp, Wheeler Dam Turbine

Fig. 17 is a sectional elevation of one of the 42 000-hp Kaplan-type turbines in the Safe Harbor Plant. At the time the first four units were installed (1931) they were the largest and most powerful Kaplan units in this country, the runner diameter being 18.33 ft.

LABORATORY TESTING

One of the important factors in the progress of turbine design and operation is the laboratory research carried on by all the turbine manufacturers. Models exactly homologous to the large units, including the intake, casing, and draft-tube, are tested over a wide range in power and speed. The model runners are usually 16 in. in diameter. Changes can readily be made in the runners, guide-vanes, casing, and draft-tube to obtain the best possible combination, before starting construction of the large units; and from the test data secured in the laboratory the performance of the large unit in the field can be accurately predicted. For the large low-head installations the intake, casing, and draft-tube are usually built by the power company in the concrete substructure, and the most efficient and economical design for the particular conditions at each

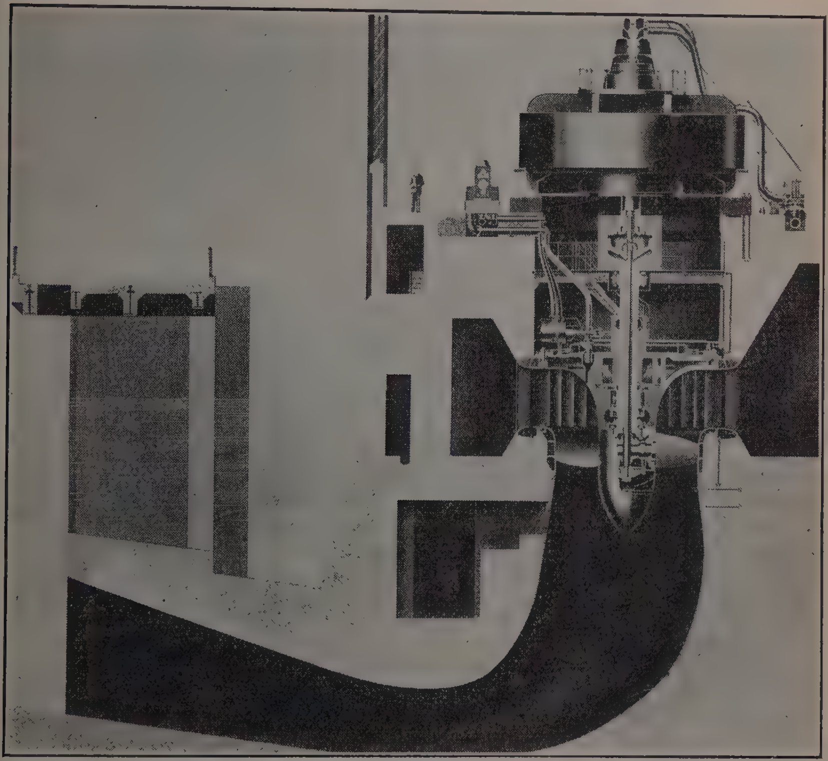

FIG. 17.—SECTIONAL ELEVATION OF 42 000-HP TURBINE, SAFE HARBOR PLANT, SAFE HARBOR WATER POWER CORPORATION

site can be determined from the laboratory test made in advance. Fig. 18 shows a model unit of the Boulder Dam turbines, and Fig. 19 shows the results of the laboratory tests made on it.

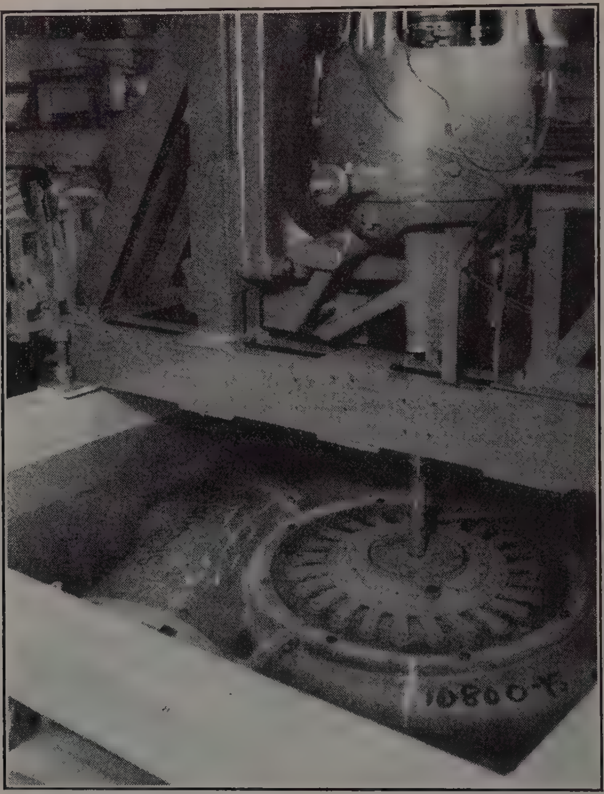

FIG. 18.—MODEL OF BOULDER DAM TURBINE

Since the development of the high-speed runner the question of cavitation has assumed far greater importance than formerly, and much research has been devoted to it. Cavitation may be defined as the formation of vapor-filled cavities at points on the runner blade, or at any point within the water stream which disturbs the continuity of flow and causes the water to leave the blade surface. Cavitation reduces turbine efficiency; and due to the collapse or

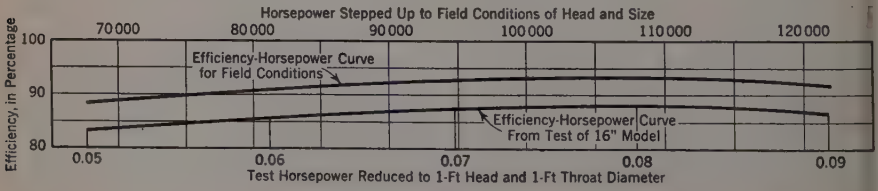

FIG. 19.—TEST RESULTS OF BOULDER DAM MODEL

"implosion" of the cavities where the flow reaches points of higher pressure, causes pitting of the runner or adjacent parts.

At a given elevation above sea level and a given temperature, the absolute pressure available is the water barometric pressure. At the runner discharge this pressure is reduced: (1) By the elevation of the runner above tail-water; (2) by the velocity head; and (3), locally, by the curvature of the flow lines. This third element is dependent on the runner design, particularly on the proportional magnitude of the blade area and blade spacing, and may differ materially on two runners of the same specific speed. Hence, for any given runner, it is necessary to determine the correct elevation with respect to tail-water, so as to assure sufficient absolute pressure at the discharge of the runner blades under the required head in order to avoid cavitation.

For this purpose special cavitation laboratories have been built by several of the turbine manufacturers, and by the Pennsylvania Water and Power Company, at Holtwood, Pa., and the Shawinigan Water and Power Company, at Shawinigan Falls, Ont., Canada. In these laboratories the elevation of the tail-water can be varied while the total head on the model turbine is maintained constant. As the tail-water is lowered, observations are made of the discharge, power output, and efficiency; the break in these curves indicates the cavitation limit.

The cavitation coefficient, σ, is defined by:

$$\sigma = \frac{H_b - H_s}{H} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

in which H_b = height of barometric water column; H_s = greatest elevation of runner above tail-water at which cavitation does not occur, with the turbine operating at a given percentage of its rated capacity. (If cavitation begins while the runner is still below tail-water, H_s is negative and represents the least depth of runner below tail-water at which cavitation does not occur.) H = total effective head on the turbine. Values of σ must be determined over the entire range of power and head through which the turbine is to be operated. In fixing the elevation of the runner in the field a factor of safety should always be allowed; that is, the value of σ used in the field should be well above the actual value determined in the laboratory.

Studies have been made of the behavior of various installations in operation, with particular reference to the avoidance of pitting, and several investigators have prepared curves to serve as a general guide to the selection of plant σ values. Such curves, for both the Francis and propeller-type turbines, are shown in Fig. 20.[16] The data shown have been collected and brought up to date by Professor L. F. Moody. The solid curves show the minimum values of plant σ recommended by the writer. The curve for propeller turbines gives the recommended limit for fixed-blade propeller units, but for Kaplan turbines σ values about 10% higher should be used. The curves are presented only as an approximate method of determining the elevation of the runner for a proposed installation; the final elevation should be fixed from the results of the cavitation tests made on a model of the runner to be used, or from field experience in homologous installations.

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[16] In Fig. 20, "Rogers and Moody, 1925," refers to paper entitled "Inter-Relation of Operation and Design of Hydraulic Turbines," by F. H. Rogers and L. F. Moody, *Engineers and Engineering*, Engrs. Club of Philadelphia, 1925; "Rogers and Sharp, 1935," refers to paper entitled "45 000 HP. Propeller Turbine for Wheeler Dam," by F. H. Rogers and R. E. B. Sharp, *Mechanical Engineering*, August, 1935; "Scoville, 1936," refers to discussion by J. D. Scoville of paper entitled "Cavitation Testing of Model Hydraulic Turbines and Its Bearing on Design and Operation," by L. M. Davis, *Transactions*, A. S. M. E., May, 1936, p. 323; and "Scoville Discussion, Cavitation Tests," refers to test points derived from preceding reference.

MECHANICAL IMPROVEMENTS

Since about 1925 there have been many mechanical improvements in turbine design. Among them at least six are deserving of special mention:

(1) The use of stainless steel for those parts subject to mechanical wear and erosive action of the water is increasing. For example, stainless steel castings or forgings are used for the shaft sleeves of water-lubricated bearings and for the stuffing-boxes. Plates of stainless steel are used in some units to protect the top and bottom surfaces of the guide-vanes, for distributor plates, and on the periphery of propeller-type runner blades. Those parts of the runner-blade surfaces and throat rings most subject to pitting are in some cases protected by pre-welding with stainless steel.

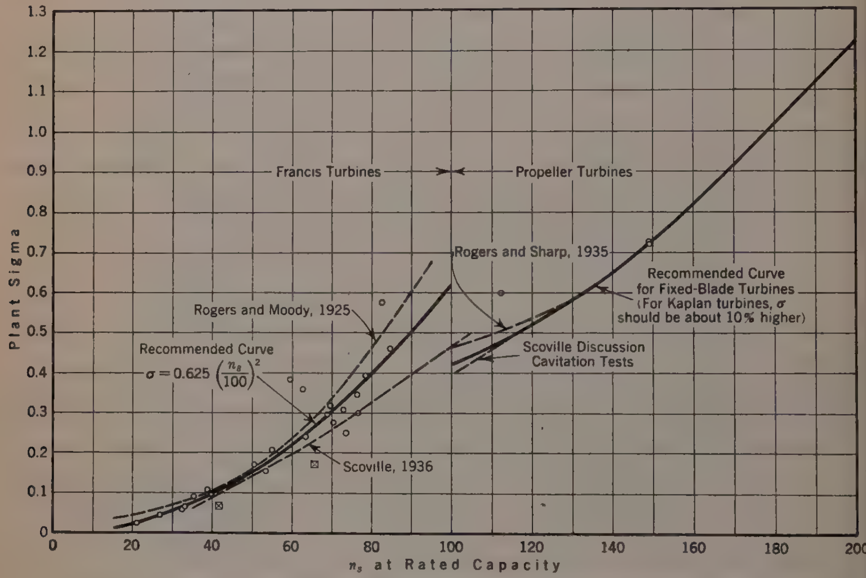

FIG. 20.—VALUES OF σ FOR USE IN DETERMINING APPROXIMATE ELEVATION OF RUNNER IN PROPOSED INSTALLATION

(2) Welded-steel construction has been used in some installations, for the guide-vanes, head covers, and stay-rings, in order to assure a more homogeneous metal than is obtainable with steel castings.

(3) Water-lubricated rubber-lined bearings have been used successfully in a large number of noteworthy low-head installations. This type of bearing was first used in one of the exciter units at the Holtwood Station of the Pennsylvania Water and Power Company, in 1924, and by 1936 about sixty of them were in use or under construction. They have proved superior to the usual

lignum-vitæ type in wearing qualities, particularly where abrasive matter is present in the lubricating water. Compared to the oil-lubricated, babbitted bearing, the rubber type is superior as regards operation, accessibility, and simplicity. It eliminates the inaccessible stuffing-box below the bearing, permitting the runner to be placed close to the bearing. The small lubricating pumps and water-drainage pumps required for the oil bearing are also unnecessary.

(4) The machining of blade surfaces of a propeller-type turbine (one of the units of the Safe Harbor Plant) has recently been tried. By this means it is possible to reproduce more accurately the curve and shape of blade desired than by the usual chipping and grinding to templates. It is not as yet possible to determine the actual benefit obtained by machining as regards either performance or avoidance of pitting.

(5) An adjustable-blade propeller turbine has recently been developed in which the runner blades are automatically adjusted by water flowing through the turbine. No inter-connection is provided between the runner blades and the governor, which actuates the guide-vanes in the customary manner. The runner blades are pivoted on roller bearings, and inter-connected through segmental gears and racks to a dash-pot piston enclosed in a cylinder that forms the upper part of the runner hub. The water flowing past the blades sets up a hydraulic moment tending at all times to open them. This moment is balanced by a reactive device, utilizing springs, weights, or balance piston, which tends at all times to close the blades. Experiments have shown, it is reported, that if the runner blades are properly proportioned and pivoted, they will assume the correct position with relation to the guide-vanes to maintain best efficiency for each load. A unit of this type, rated at 7 600 hp (23-ft head, 90 rpm), was installed in 1936 in the Marmet Plant, of the Kanawha Valley Power Company, in West Virginia.

(6) Automatic air-valves are now used in a large majority of turbines. They are operated automatically by an attachment to the turbine-gate mechanism, so that the admission of air is progressively reduced as the gates open and finally ceases at a predetermined gate position. (If pressure exists under the head cover, air may have to be introduced under pressure.) These valves improve operating conditions at small loads, and in the fixed-blade propeller-type turbines their effect is noticeable even at loads approaching normal load. The admission of air at speed-no-load condition is also of considerable benefit in synchronizing units. When the generators are operating as synchronous condensers, a great saving in power can be effected by admitting compressed air to depress the water until it clears the runner.

Possible Future Developments

At present, the Rocky River Development, in New England, is the only pumped-storage plant of any appreciable size on this Continent. However, the demand for reserve power and peak-power service, particularly near large load centers, has resulted in many studies of similar projects.

In the existing plant the motor-driven pump and the turbine-driven generator are separate units; but if and when any of the present proposals materialize, special turbine and pump equipment will be desirable to meet their requirements. Accordingly, at least one manufacturer has already designed and tested two model pump-turbine units which combine in one piece of apparatus the pumping and generating elements. This arrangement not only makes one machine do the work of two, but, in addition, does away with the elaborate hydraulic connections required when separate pumping and generating units are used. The adoption of such equipment will materially reduce the cost of pumped-storage plants.

The pump-turbine unit which has been developed is of the Francis type, with volute casing and movable guide-vanes surrounding the runner. The curves in Fig. 21 show the performance of the two model units as obtained by laboratory tests. (Both models had a diameter at the intake of about 18 in. and a throat diameter of about 12 in.) The high-head model has a specific speed, when generating, of about 26, and would be suitable for heads up to about 300 ft. The medium-head model has a specific speed of about 42, and would be suitable for heads of about 100 ft. It will be noted that when pumping, maximum efficiencies of about 85% and 87% were secured on the high-head and medium-head units, respectively; and that when generating, the maximum efficiencies were about 89% and 88%, respectively. In view of the dual performance required of the runner, these efficiencies are indeed excellent.

The two speeds required to give as nearly as possible the best efficiency for generating and pumping, respectively, are shown on the curves (Fig. 21). This two-speed requirement may be met by the use of a two-speed type generator; one of the electrical manufacturers has carefully studied the requirements of such a machine and foresees no difficulty in building it. In some cases, however, it may be preferable to adopt a constant speed intermediate between the two values, and to sacrifice a small differential in efficiency.

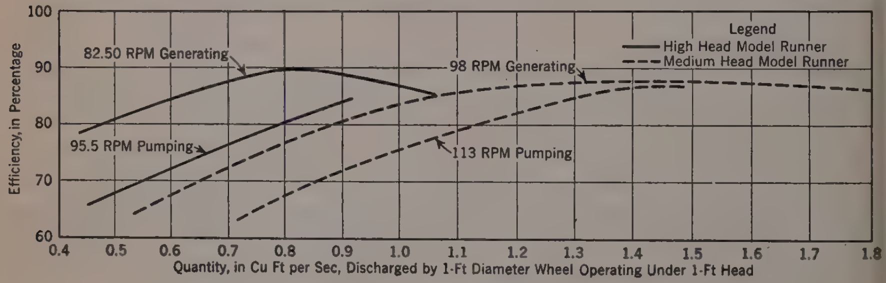

FIG. 21.—PERFORMANCE OF MODEL PUMP-TURBINE UNITS

Another promising field is the development of the propeller-type turbine for higher heads. The highest head to which this type has been applied in the United States is 69 ft (at the Bonneville Development, in Oregon). However, on the Shannon River, in Ireland, a Kaplan-type turbine has been installed to operate under a head of 106 ft. For these higher heads, runners with lower specific speed and large blade areas are required, in order to obtain low values of σ and thus avoid the necessity of locating the turbines at excessive depths below tail-water elevation. Undoubtedly, such runners will be developed to fill the gap in specific speed, between the present high limit of the Francis runner (85) and the low limit of the propeller-type runner (125). The economic advantage of higher speed as well as the better operating characteristics of the Kaplan turbine will certainly warrant the necessary research work along these lines.

With the inter-connection of large systems, automatic load control and frequency control have increased in importance. For any particular load on a station there is a most favorable division of power output between the separate units to secure minimum water demand or maximum over-all efficiency; and this optimum distribution between units can be maintained continuously and accurately only by automatic control. The same principle can be extended to load division between stations in a system. As to frequency, every station in a system must be controlled within very narrow limits. Further, if electric clocks are to operate satisfactorily, the cumulative errors caused by slight frequency changes must be compensated with a high degree of accuracy. Various devices have been developed to meet all these requirements, but further improvements and refinements in this type of equipment will undoubtedly be necessary to meet the severe demands of the large interconnected systems.

IMPROVEMENTS IN THE UTILIZATION OF ENERGY

By Joel D. Justin,[17] M. Am. Soc. C. E.

Synopsis

Recent development in the increased use and decreased cost of electrical energy is outlined herein. The paper traces the development of long-distance transmission, of regional power companies, of inter-connection between companies and of the co-ordination of steam and hydro-electric power development and shows the effect of these factors on the utilization of electrical energy.

Attention is directed to peak-load hydro-electric plants as a means of reducing the total cost of energy in a system, including both steam and hydro-electric plants.

Before the World War, power requirements of American communities were met, to a large extent, by local power companies. The major domestic use of electrical energy was for lighting. Its use in the home for refrigeration, washing machines, vacuum cleaners, etc., was relatively insignificant.

To-day, not only have all the old uses greatly increased, but additional uses for electricity in the home are constantly being found. The consumption per domestic customer has increased from an average of 268 kw-hr per yr in 1914 to 770 kw-hr for the year ending July 31, 1937.

Changes in Use of Electrical Energy

In 1914, manufacturing plants, to a considerable extent, owned and operated their own sources of power. American industry in that year purchased from central stations only 15% of its power requirements; and produced the remaining 85% in its own individually owned power plants. In 1936, by contrast, industry purchased about 53% of all the power it used and made in its own plants only 47 per cent.

Before the World War, a considerable percentage of all electrical energy produced was used in city and interurban electric traction; but now many of the interurban lines—and the trolley lines as well—are gone, and the electric traction load, except in the larger cities, is a relatively small percentage of the total load. There is, however, some indication that, in time, its place may be taken by the electrification of some of the steam railroads.

To-day more electrical power and less man power is being used by industry per unit of product. Electro-chemical industries, which are heavy users of energy, are being further developed. In the commercial field, there has been an increased use of electrical energy for better and more extensive lighting; and among the new uses of electricity, air-conditioning is rapidly assuming importance. A single large office building may require a motor installation of 3 000 or 4 000 hp for air-conditioning purposes alone. Stores and restaurants are finding it economically advisable to air-condition. If the present trend continues, within the next few years the office building or hotel that is not air-conditioned will be an obsolescent, low-rental structure. Air-conditioning of homes, although not yet an important factor in the increase of electrical load, will probably become so in time. The demand for it is beginning to appear.

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The increase in the variety of uses of electrical energy, the increase in the amount produced, and the decrease in the unit price, have been made possible by marked technical improvements in generation; and by a great advance in the adopted methods of utilization, whereby the maximum economic advantage of the generating facilities has been secured.

In Fig. 22 is shown the annual electrical energy output of privately owned public utilities for the period 1912 to 1936.

DECREASE IN PRICE OF ELECTRICAL ENERGY

The increase in output of electrical energy has been paralleled by steady reductions in its price to the consumer. In 1914, the price of electrical energy for domestic use averaged 8.2 cts per kw-hr; in the year ending June 30, 1936, the average price was 4.51 cts per kw-hr, a decline of 45 per cent. By contrast, in 1935 the cost of living was approximately 40% higher than in 1914.

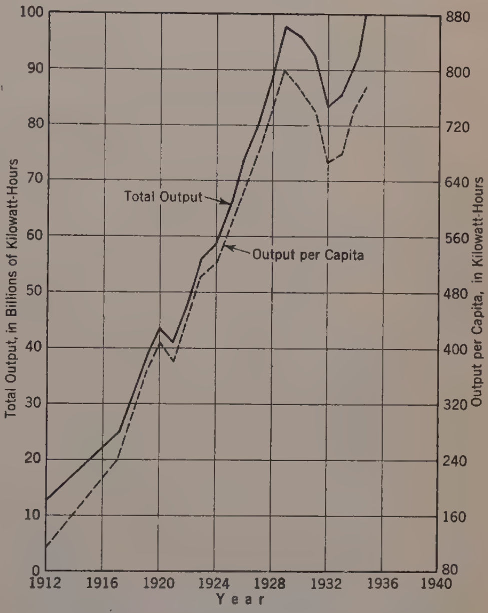

FIG. 22.—ELECTRICAL ENERGY OUTPUT OF CENTRAL STATIONS IN THE UNITED STATES, 1912–1936

Only the privately owned public utility plants reporting to the Edison Electric Institute are included in the foregoing comparison. However, such plants account for from 94% to 95% of the total output of the United States.

CAUSE OF DECLINE IN PRICE OF ELECTRICAL ENERGY

Although Government regulation has been the cause of some individual rate reductions, the fundamental major cause of the decline in the price of electrical energy is economic.

Management, striving for greater total profits, worked for and obtained ever-increasing sales which, in turn, had a material effect in reducing costs. With the reduced costs, lower prices became possible. Lower prices stimulated greater use, and greater use permitted the introduction of more technical improvement. An endless chain whose links are technical improvement, decreased unit cost, decreased unit price, increased use, and increased total profits, was thus set in operation and has functioned successfully, with slight interruptions, to the present time. It will continue to operate to the advantage of all concerned (limited only by an apparently remote saturation point and the law of diminishing returns), unless one of the links in the chain is broken.

COMMUNITY POWER COMPANIES

Prior to the World War there were relatively few regional power companies and their development was in its infancy. In general, each community or small group of communities was served by a local power company. Loads and plants were relatively small and transmission distances with a few exceptions were comparatively short. A 5 000-kw generator was considered a large unit in those days, and a 50 000-kw plant was a big one, such plants, for the most part, being confined to the larger centers of population. As there was not much inter-connection, the reserve requirements of individual systems were high, and it was not unusual for power companies to have installations equal to 150% or even 200% of their peak load for any given year.

STEAM COMPANIES AND HYDRO-ELECTRIC COMPANIES

Before the World War power companies might have been divided into two classes: (1) Those utilizing steam as their main source of power, often referred to as steam companies; and (2) those utilizing water power as their main source of power, often referred to as hydro-electric companies. The useful hydro-electric installation plus the necessary machine reserve for any given water power company was limited to that for which there was always sufficient stream flow available to produce enough power to serve the peak load. Even so, the hydro-electric companies, at times, would have available energy which they could not use on their load curves. This surplus, which was produced at off-peak periods and during seasons of high stream flow, was sold to neighboring steam companies. This policy required the construction of inter-company transmission lines, and enabled the steam companies to shut down some of their steam generating units, at times, and thus save fuel.

IMPROVEMENT IN LONG-DISTANCE TRANSMISSION

The opportunity for utilizing low-cost surplus hydro-electric energy at important industrial centers, served largely by steam-generated electric power, put a premium on the development of the long-distance transmission of electrical energy. At the same time, it was found that some sources of hydro-electric power could be developed economically to serve primarily a distant load center, provided the cost of transmission was low enough.

These factors were largely responsible for the development of long-distance transmission. Voltages were stepped up from 66 000 to 110 000 and then to 220 000, and with the increases in capacity which these high-tension lines permitted, the cost of transformation, regulation, and control was decreased per unit of energy transmitted. Technical improvements in the generation of power by both steam plants and hydro-electric plants also led to material decreases in cost.

Regional Power Companies

A rapid growth in sales followed, and the "set-up" of the community power company proved to be an inefficient means of handling the increasing business with its extensive distribution over wide territories, and its need for rapidly increasing capital expenditures and for co-ordinated operation between the various units.

Consequently, regional power companies became more prevalent. Community power companies joined together or were absorbed to form integrated and co-ordinated systems, many of which served territories hundreds of square miles in area, with populations of several million people. The resulting economies, as indicated by the decreasing price of the product, have been very material. During the years of the great depression the tendency was retarded, but is again becoming evident with a return of more normal conditions.

Inter-Connection

Inter-connection developed coincident with the growth of regional power companies. By inter-connection, as herein used, is meant the tying together, by means of high-tension transmission lines, of two or more independently operated power systems. The economies of inter-connection ensue from savings in investment in additional generating facilities and from savings in operating expense. Against such savings must be set the increased fixed charges and operating expense for the additional transmission facilities.

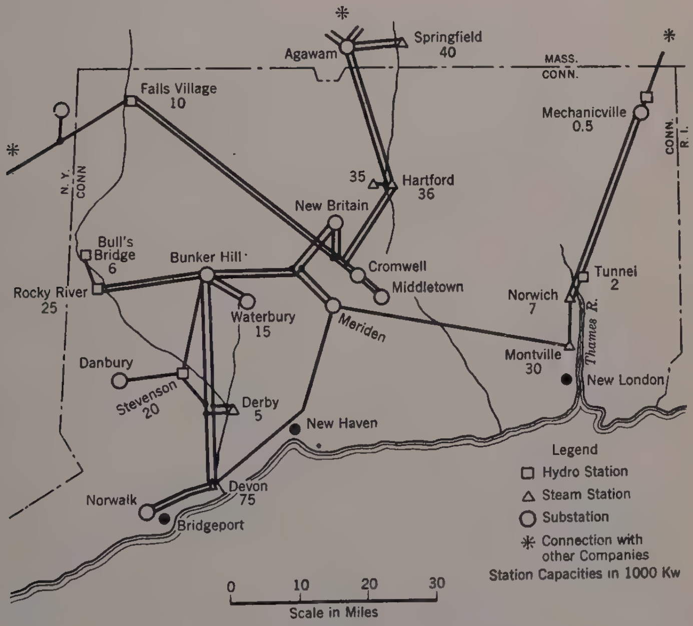

Fig. 23.—The Connecticut Valley Power Exchange

Extensive economies have been attained through many of these inter-connections. On the other hand, there have been some in which the increased annual cost of the inter-connection has exceeded the annual savings so far attained thereby. Every proposed inter-connection should be carefully analyzed to determine its potential economies.

Between 1920 and 1930, there was a rapid growth in inter-connection. In some cases very large inter-connected systems (like that shown in Fig. 23[18]) were built up, and industrial centers with a combined load of 1 000 000 kw, or more, were tied together with high-tension transmission lines. Territories covered by such systems sometimes exceed 20 000 sq miles and include populations of several million people. The power companies forming such inter-connections or power pools have entered into agreements for the purpose of operating the combined system to secure maximum economy in capital expenditures and in annual operating costs. This inter-connection of independently operated properties made little progress during the depression and even in 1937 had not been greatly revived.

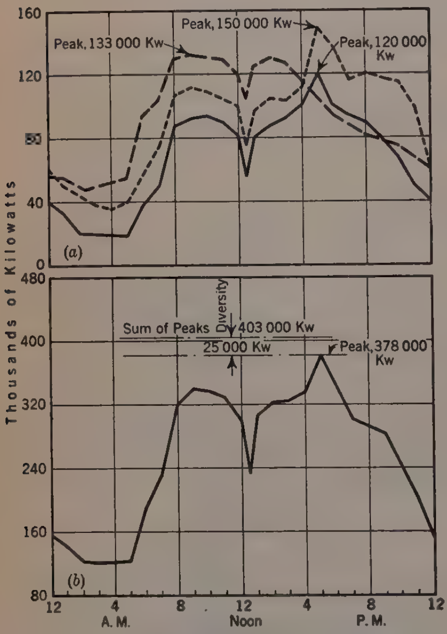

FIG. 24.—DIVERSITY OBTAINED BY INTER-CONNECTION: (*a*) PEAK DAY LOAD CURVES OF SEPARATE COMPANIES; (*b*) PEAK DAY LOAD CURVE AFTER INTER-CONNECTION

SAVINGS DUE TO LOAD DIVERSITY THROUGH INTER-CONNECTION

The load characteristics of various companies may differ materially. The load curve of a company serving a metropolitan district is very different from that of a company serving small centers and rural areas, and the load curve of a company serving an area in which heavy industries predominate will probably be very different from either of the first two. Thus, one company may have its seasonal peak in October, whereas another near-by company may have its annual peak in December. There may also be a material diversity in the daily load curves, one company having its daily peak load much earlier in the day than the other; and even where companies serve communities of a similar character, there is almost always some "incidental" load diversity—that is, a diversity for which there appears to be no special reason but which, nevertheless, occurs year after year.

If companies having load curves like those just discussed are inter-connected and operated as a single economic entity, either with or without identity of ownership, it is obvious that the combined system will not require as much installed capacity as would be the case without such inter-connection. Fig. 24 shows the load diversity obtained by a typical inter-connection of three companies.

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[18] Adapted from an illustration in "Power Supply Economics," by Joel D. Justin and William G. Mervine, John Wiley & Sons, Inc., New York, N. Y., 1934.

RESERVE CAPACITY SAVINGS DUE TO INTER-CONNECTION

Assume that two hypothetical power systems are considering inter-connection. Each has an annual peak load of 100 000 kw, and each is served by a steam plant having an installed capacity of 120 000 kw, consisting of six 20 000-kw units, one unit in each plant being in reserve. It will be assumed also that the total load diversity between the two companies is 20 000 kw. Then, if the two companies are inter-connected and operated as an economic entity, the combined load that must be served will be 100 000 + 100 000 − 20 000 = 180 000 kw. In this case, experience has determined that the required reserve must be equal to 10% of the peak load, or to the capacity of the largest unit, whichever is the greater. Of the peak load 10% is 18 000 kw, and all the units are 20 000 kw in size. Therefore, the required reserve is 20 000 kw, and the required capacity is 180 000 + 20 000 = 200 000 kw. The actual total installed capacity, however, is 240 000 kw. Thus, by inter-connecting the two companies, which separately had only enough capacity to serve their peak loads in the given year with the required reserve, a surplus capacity is made available to take care of a future growth of 40 000 kw in the demand.

As each of these companies, at the time of inter-connection, was faced with the necessity of installing an additional 20 000-kw unit, the saving due to inter-connection from this cause would be the capital sum of, say, 40 000 × $130 = $5 200 000, or a saving in fixed charges of, say, $5 200 000 × 0.14 = $728 000 per yr. In addition, there would be a saving of the fixed portion of the operating cost on the two 20 000-kw units which, because of inter-connection, it was not necessary to install. This might amount to as much as $100 000 per yr additional.

SAVINGS IN FUTURE CAPITAL EXPENDITURES WITH INTER-CONNECTION

Due to inter-connection and operation as an economic entity, material savings may be expected in future capital expenditures. Continuing the hypothetical illustration, it will be noted that, whereas 20 000 kw was probably an economic size of unit for a system with a 100 000-kw load, a unit twice as large would probably be more economical for a system with a peak load of 180 000 kw.

Therefore, when the growth in the load of the combined system requires additional capacity, the requirements may be met by installing units of 40 000-kw capacity each. The capital cost and operating cost of such units would be considerably less than those of the 20 000-kw units which the companies were in the habit of installing prior to inter-connection. The additional annual saving might be material.

SAVING IN ALLOCATION OF PLANTS TO LOAD THROUGH INTER-CONNECTION

It will be assumed that the base load of the individual companies (that is, the load which is continuous throughout the 8 760 hr of the year) is 20% of the peak load. The base load of the combined system will thus be 40 000 kw. The several units of any system practically always vary materially in efficiency, largely because of the difference in their age. It will be assumed that one of the two companies has three units which are much more efficient than any of the other units in either system (all units having a capacity of 20 000 kw).

With the combined load due to inter-connection, these three units could operate on a much higher annual capacity factor—that is, they could operate more hours per year at full capacity. The less efficient units would operate correspondingly fewer hours per year. Such operation would reduce the average unit production cost of the combined system, producing an additional annual saving. Fig. 25 shows the allocation of units to the load demand for a typical day, before inter-connection, for the company possessing the three highly efficient units. It is evident that, after inter-connection, the base load would be twice as great as that indicated in Fig. 25 and the three efficient 20 000-kw units would then be able to operate at an annual capacity factor approaching 100 per cent.

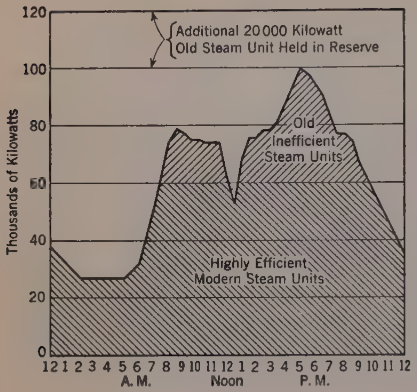

FIG. 25.—ALLOCATION OF PLANTS TO LOAD, BEFORE INTER-CONNECTION (FOR COMPANY WITH THREE HIGHLY EFFICIENT UNITS)

ANNUAL SAVINGS DUE TO BETTER USE OF HYDRO-POWER WITH INTER-CONNECTION

Many engineers think of steam and hydro-power as competitive sources of electrical energy. Occasionally, they are competitive, but over wide areas of the United States they are more usually complementary. It has been found that a system whose power supply is derived from both steam and hydro-power plants in proper proportion will have a lower over-all cost of power supply than a system otherwise identical but served solely by steam capacity.

Assume the same two hypothetical companies as before, but for one of the 20 000-kw steam units of one of the companies substitute a hydro-electric plant with an installed capacity of 30 000 kw. Further, assume that stream flow and pondage conditions are such that, at time of peak load, this plant can be relied on for a firm capacity of only 20 000 kw when operating on the load curve of the one company alone. (Firm hydro-electric capacity may be defined as that part of the total installed capacity of a hydro-power plant which, under existing conditions of load, stream flow, and pondage, is capable of performing the same function in serving the part of the load curve allocated to it, that an alternative steam plant could perform.)

As the magnitude of a load curve increases, there is a lesser amount of energy in the upper part of the load curve for any given number of kilowatts in the peak of the load. Consequently, when the two companies are inter-connected, it is found that all the installed hydro-electric capacity of 30 000 kw is firm capacity on the combined load curve.

Fig. 26 illustrates this growth of firm hydro-electric capacity. In order to plot such a diagram two preliminary steps are necessary: First, a "load-duration curve," with loads as ordinates and time as abscissas is plotted. Areas under the curve thus represent energy. By planimetering, or otherwise, energy for various numbers of kilowatts, measuring down from the peak, is determined. Second, a "peak percentage curve" is plotted using the information thus obtained. For this curve, the ordinates are the percentage of total load (measuring down from the peak) and the abscissas are the percentage of total energy for the period down to this given point. Using this curve and knowing the peak load for the given year, the energy required to serve any given part of the load may be obtained. After the peak percentage curve is plotted, the curve of Fig. 26 can be constructed readily. In some cases it may be necessary to investigate other periods than the peak-load week.

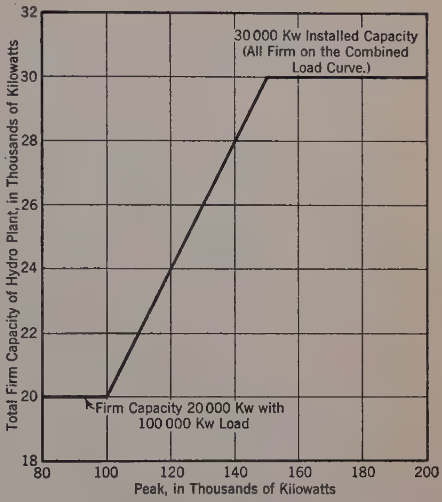

FIG. 26.—INCREASE IN FIRM CAPACITY OF 30 000-KW HYDRO-ELECTRIC PLANT WITH INCREASING SYSTEM LOAD

From the preceding discussion it is evident that by inter-connection the firm capacity of the hydro-electric plant has been increased by 10 000 kw, and that the effective capacity of the combined system has also been increased by 10 000 kw (in addition to the 40 000-kw increase previously mentioned). The corresponding capital saving of, say, $1 300 000, would accrue at such time as the companies would otherwise have had to install at least that much additional capacity.

The inter-connection of the two systems also gives an opportunity to utilize more nearly all the energy that the hydro-electric plant is capable of generating with the existing installation and the available stream flow. Without inter-connection, the base load on which the hydro-electric plant can operate continuously at periods of ample stream flow is only 20 000 kw, and, consequently, during a considerable part of the time, a large part of the available hydro-electric energy must be wasted. On the other hand, when the systems are inter-connected, the base load becomes 40 000 kw, and when ample stream flow is available the hydro-electric plant can operate continuously on the base without waste of energy.

The improvement in the utilization of available hydro-electric energy throughout the development period of the regional power companies has been very marked. Many individual power companies, with a high percentage of hydro-electric installation, had hydro-electric utilization factors of from 35 to 60 per cent. (The utilization factor is the ratio of the energy actually produced by a hydro-electric plant to the energy that could have been produced by the same plant during the same period, if there had been a market for it.) Now that these companies have been absorbed into large regional power systems, and the plants are inter-connected with load centers served largely by steam, the annual utilization factors of the same hydro-electric plants have risen in many cases to more than 90%, and, in a few cases, to practically 100 per cent.

Utilization of Peak-Load Plants

Supplying energy for the peak load of any power system is always very expensive as compared to the average cost of electrical energy. It is not unusual for the top 20% of the annual load curve to contain only from 0.5 of 1% to 1% of the total annual energy output of the system. Thus, a plant serving this part of such a load curve might have an annual capacity factor of from $1\frac{1}{4}$% to 2 per cent. As the average annual capacity factor of the system as a whole may be 35%, it is evident that the cost of providing the peak-load service may be a great many times the average total cost of energy.

In many systems, peak-load service is performed by the more antiquated steam plants in the system. Their high cost of energy production does not greatly matter because they produce very little energy. However, the maintenance and operating cost, which it is necessary to incur in order to have them ready to operate when required, and the cost of keeping them in "hot reserve" for a part of the time, are frequently high.

Peak-Load Hydro-Electric Plants

The possible improvement in the utilization of energy and the reduction in the total cost of power supply, through the use of peak-load hydro-electric plants in large regional power companies and in inter-connected systems, does not always receive the attention it deserves. The operating and maintenance cost of a hydro-electric plant is low as compared to that of a steam plant. Further, the hydro-electric plant can be put on the line at a moment's notice, thus making feasible a reduction in the amount of "hot" reserve that must be carried at the steam plants of the system.

Hydro-electric plants with ample pondage and a very high ratio of installation to available stream flow are particularly suitable for performing this peak-load service. For base load service the steam plant is generally a much cheaper source of energy. A high installation ratio usually means a low over-all cost per kilowatt of capacity, since the major part of the total investment is usually in the power plant itself ($50 to $70 per kw of installation) and the investment per kilowatt in dams and flowage is relatively small.

The economic advisability of using hydro-electric plants of the peak-load type for performing peak-load service is indicated in Fig. 27. For comparative purposes, the total unit cost of the energy output of a relatively modern steam plant, using cheap fuel, is shown by Curve *A*. Curve *C* gives the capital cost of an alternative hydro-electric plant for peak-load service, based on actual and fairly typical experience, and Curve *B* gives the total unit cost of its output. The principal assumptions are indicated in Fig. 27.

The decline in the capital cost of the hydro-electric plant per kilowatt of capacity with a decrease in the annual capacity factor actually reflects the fact that the average cost per kilowatt for a hydro-electric plant generally decreases rather rapidly with an increase in installed capacity. Although the incremental cost of steam-plant capacity is also frequently much less than the average, this holds only until the predetermined maximum capacity of the plant is reached; further, the tendency is not nearly so marked as in the case of hydro-electric installations.

It should be noted that for an annual capacity factor of 5% the total cost of energy from the hydro-electric plant would be 1.7 cts per kw-hr, while at the steam plant the total cost per kilowatt-hour would be 4.0 cts. For lower annual capacity factors the discrepancy is still greater. Although the comparison is quite typical it would vary materially for any individual case.

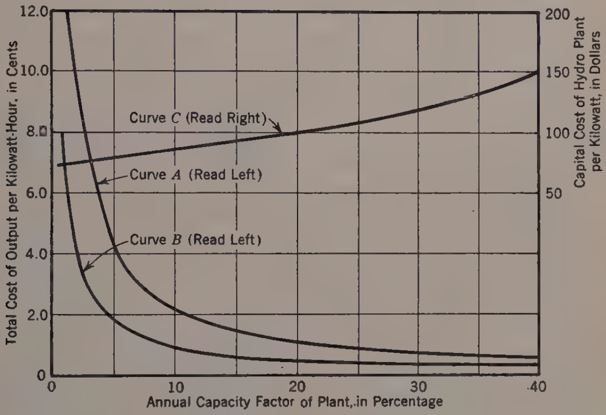

FIG. 27.—COMPARISON OF COST OF ENERGY FROM STEAM PLANT AND HYDRO-ELECTRIC PLANTS ON PEAK-LOAD SERVICE

It is not usual, however, to use new steam plants for peak-load service, as their low cost of energy production usually makes it advisable to allocate them to the base load or, at any rate, to some point lower down on the load curve. For old steam plants on peak-load service, interest charges on the investment in them should not be considered in comparisons such as the foregoing. In most cases, the fixed part of the annual operating cost for old steam plants is high enough to make up for the omission of interest charges, with the result that the comparison is frequently not any more favorable than that indicated in Fig. 27.

Run-of-river hydro-electric plants, when provided with ample pondage, are also used for peak-load purposes during periods when the stream flow is low; provided, of course, that the relation of low-water stream flow to load is such that they will be firm capacity on the load curve. It is believed that peak-load hydro-electric plants will find an increasing application, as there are many systems in which their more extensive use would prove economic. Their development, however, is limited by the availability of suitable sites near large load centers. The cost of a long transmission line, added to that of the plant itself, might more than offset the economies.

PUMPED-STORAGE HYDRO-ELECTRIC PLANTS

The pumped-storage hydro-electric plant is simply a special case of the peak-load hydro-electric plant, the main difference being that for the pumped-storage plant no source of water supply is required. In such installations the cost of spillway dams is minimized or eliminated. All that is required is a site for a pond or reservoir in the hills, another one at lower elevation, and a sharp drop between. In its simplest form (Fig. 28[18]) the pumped-storage plant consists of a head-water pond, penstocks, a power-house with turbines and pumps and a tail-water pond. "Make-up" water to take care of seepage and evaporation may be furnished by a brook or may be pumped from a distance. At peak-load periods, when the plant is in demand, water flows from the head-water pond through the turbines, generating power. During low-load periods, when the plant is not in demand, low-cost off-peak steam-generated energy or surplus hydro-electric energy flows to the plant and pumps the water back into the head-water pond, and the plant is then ready for another peak-load period. In Europe, there are thirty or forty such plants in successful operation, functioning as storage batteries for various power systems, but in

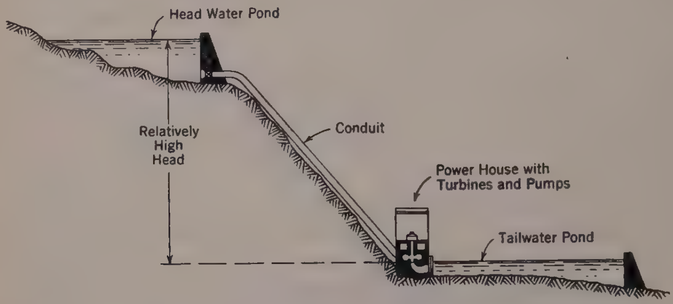

FIG. 28.—TYPICAL PUMPED-STORAGE HYDRO-ELECTRIC PLANT

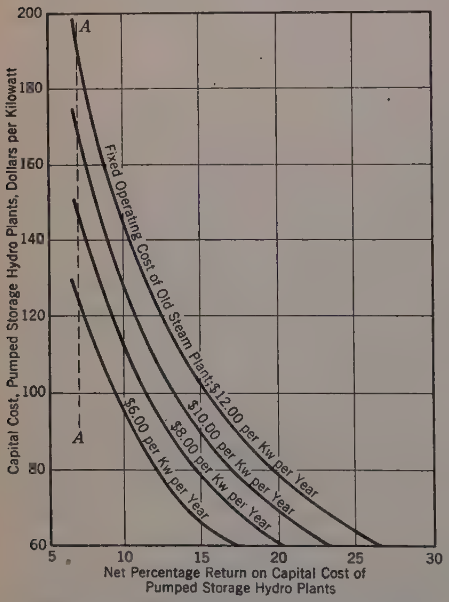

FIG. 29.—ECONOMIC LIMITATIONS OF PUMPED-STORAGE PLANTS TO REPLACE OLD STEAM PLANTS WITH 15% ANNUAL CAPACITY FACTOR

America there is only one—the Rocky River Development of the Connecticut Light and Power Company, near New Milford, Conn., with a capacity of 24 000 kw. In that plant, incidentally, the tail-water is a river instead of a pond.

Where suitable sites exist near large load centers, pumped-storage plants offer a means of reducing the cost of peak-load service and, hence, the total cost of power supply. In some such cases it will be found economical to use them to replace the antiquated steam plants already performing peak-load service. In order to make such a change economically desirable it is necessary that the total annual cost of the proposed pumped-storage plant, including interest on the investment, should be less than the annual cost of the old steam plant without considering interest on the investment. Fig. 29[18] gives curves indicating to some extent the economic limits in the application of pumped-storage hydro-electric plants for this purpose. The principal assumptions are:

Old Steam Plant:

Fixed charges	0
Renewals and replacement	\$2.00 per kw per yr
Taxes and insurance	\$2.50 per kw per yr
Incremental cost of energy	5 mills per kw-hr

Pumped-Storage Hydro-Electric Plant:

Fixed charges	(See Fig. 29)
Over-all efficiency	60%
Operation and maintenance	\$1.00 per kw per yr
Renewals and replacements	1% on capital cost
Taxes and insurance	1% on capital cost
Off-peak energy for pumping	2.2 mills per kw-hr

In Fig. 29, the point where Line *AA* cuts each curve gives the maximum permissible cost for the pumped-storage hydro-electric plant, if the net return on the investment is to be 7 per cent.

As there are many situations where the fixed part of the operating cost of old steam plants is within the range indicated in Fig. 29, it is evident that there is room for a further application of pumped storage in the United States.

Pending Developments

In a rather general manner, this paper has discussed means of utilizing to the greatest economic advantage various technical developments in the electric-power field. With the resumption of industrial activity at an accelerated pace further technical advances may be expected.

One such advance that might produce radical improvements in the utilization of electrical energy, is long-distance direct-current transmission. A system of this type is already in experimental use between the Schaghticoke Hydro-Electric Plant of the Niagara-Hudson System and Schenectady, N. Y. Some electrical engineers are quite optimistic about its possibilities. They visualize the possibility of obtaining thereby the economic transmission of electric power to much greater distances and at considerably less cost than is possible with alternating-current transmission and transformation. The development of huge Government power plants remote from the load centers provides an additional inducement for such a development.

However, even without such startling developments as this, much will still be accomplished in the way of a better and more economical utilization of existing power-supply resources. In the existing inter-connections and power pools it has been found that, where there is an identity of ownership, the inter-connected properties come much closer to taking advantage of all the economies that are theoretically possible. Hence, with the resumption of industrial activity there will doubtless be a resumption of further inter-connection and consolidation on a territorial basis to the advantage of consumers and power companies alike.

COST OF GENERATION OF ELECTRIC ENERGY

BY PHILIP SPORN,[19] ESQ.

SYNOPSIS

This paper is a general discussion of the subject of the cost of electric energy, with particular emphasis on the cost of generation and the related items of transmission and distribution. It covers the economics of steam power *versus* water power and shows that, in most cases, present-day steam power can be produced more cheaply than present-day water power. It shows that there is no economy in long-distance transmission of electric energy and that sound economics require the location of the generating plant near the load. The cost of distribution is presented briefly, emphasizing the point that this item is the major cost in expenditure for electric service.

The paper concludes that the trend in cost of producing, transmitting, and distributing electric energy is downward—although no sharp dip is to be expected—and that future power requirements will be met by steam plants located economically with reference to loads.

INTRODUCTION

The cost of generating electric energy is only a small part of the total cost of electric service, which must also include transmission and distribution. Current technical and political-economic discussion on the cost of electric service is largely out of focus because this fact is not fully realized. More particularly is it neglected in much of the discussion on the cost of the generation of hydro-electric power by governmental agencies. In many cases, it is assumed that the apparently lower cost of generation of hydro-electric power can and will result in a much lower cost of electric energy to the consumer.

This, of course, is not the case. Hydro-electric power is not necessarily cheap power, and, in most cases, is not competitive with steam power. Nevertheless, it cannot be denied that the cost of generation is an item, and sometimes an important item, in the total cost of power.

The writer, in this paper, will attempt to clarify various factors in the cost of generation by different types of prime mover. It must be borne in mind that the costs are presented from a general viewpoint. The problem, of necessity, is complicated, and, within the scope of this paper, only rough studies and outlines are possible. The main object, however, has been to determine the approximate boundaries and trends of generation cost.

PRESENT COSTS OF ELECTRICAL ENERGY

The sources of energy used commercially to-day are falling water—that is, hydro-electric power—and fuels, either in steam plants or in internal-combustion plants. Internal-combustion plants, however, have such limited application in so far as central stations are concerned that they will not be discussed herein.

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[19] Vice-Pres. and Chf. Engr., Am. Gas & Elec. Co., New York, N. Y.

Investment Cost of Hydro-Electric Power.—Hydro-electric power has received a great deal of attention since 1933, particularly because its development has been pushed vigorously in connection with various governmental projects; and the impression has spread that, as it involves only the harnessing of "natural" power, its cost cannot be equalled by any other method. That the actual facts seldom warrant any such assumption is known to a few, but certainly is not generally realized. Most of the favorable hydro-electric sites have already

TABLE 3.—HYDRO-ELECTRIC PLANT INVESTMENT COSTS

(Data abstracted from Yearly Reports of Federal Power Commission)

Project No. (1)	Name of company (2)	Plant (3)	CAPACITY, IN HORSE POWER: Primary* (4)	Present (5)	Ultimate (6)
1 025	Safe Harbor Water Power Corp.	Safe Harbor	25 707	255 000	510 000
432	Carolina Power and Light Co.	Waterville	23 760	139 500	139 500
82	Alabama Power Co.	Mitchell	21 760	72 000	93 000
618	Alabama Power Co.	Jordan	30 400	146 800	180 000
349	Alabama Power Co.	Martin	37 750	135 000	180 000
459	Union Electric Light and Power Co.	Bagnell	15 000	201 000	268 000
346	Northern States Power Co.	Blanchard	5 900	18 000	24 000
135	Portland General Electric Co.	Oak Grove	41 400	88 000	88 000
637	Chelan Electric Co.	Chelan	54 000	68 000	136 000
619	Feather River Power Co.	Bucks Creek	33 100	67 000	67 000
487	Pennsylvania Power and Light Co.	Wallenpaupack	8 148	54 000	54 000
516	Lexington Water Power Co.	Saluda	63 600	87 000	260 000
20	Utah Power and Light Co.	Soda Springs	19 820	21 000	70 000
659	Crisp County, Georgia	Crisp County	4 080	7 500	16 000
382	Southern Cal. Edison Company	Borel	3 809	14 400	14 400

Project No. (1)	Stated cost, in dollars (7)	COST, IN DOLLARS, PER KILOWATT OF: Primary capacity (8)	Installed capacity (9)	FIXED CHARGES, IN DOLLARS,† PER KILOWATT OF: Primary capacity (10)	Installed capacity (11)	INVESTMENT COST, IN CENTS, PER KILOWATT-HOUR OF OUTPUT AT: 90% load factor* (primary capacity) (12)	40% load factor‡ (installed capacity) (13)
1 025	24 995 111.74	1 350	136	145.23	14.62	1.840	0.417
432	13 764 856.51	807	139	86.75	14.94	1.003	0.426
82	10 646 056.96	673	206	72.35	22.15	0.017	0.632
618	13 047 334.50	600	124	64.50	13.33	0.818	0.380
349	17 551 299.53	650	181	69.87	19.46	0.886	0.555
459	36 024 117.21	3 350	250	360.12	26.87	4.567	0.767
346	3 451 175.25	816	266	87.72	28.59	1.112	0.816
135	9 546 593.24	322	151	34.61	16.23	0.439	0.463
637	11 067 055.53	285	227	30.64	24.40	0.388	0.696
619	9 564 549.87	404	199	43.43	21.39	0.551	0.610
487	9 070 137.00	1 555	234	167.16	25.15	2.120	0.718
516	21 661 510.44	476	348	51.17	37.41	0.649	1.067
20	3 560 872.19	251	236	26.98	25.37	0.342	0.724
659	1 267 016.11	432	235	46.44	25.26	0.589	0.721
382	3 147 501.23	1 150	305	123.62	32.79	1.568	0.936

* Power available 90% of the time.

† 10.75% of costs shown in Columns (8) and (9).

‡ Assuming that all the installed capacity is firm at 40% load factor. This assumption is necessary since Federal Power Commission data do not show actual annual kilowatt-hour output at all of these sites.

been developed, and any analysis of existing developments is likely to present a too optimistic picture of what is available at the remaining sites.

An analysis of fifteen hydro-electric plants is shown in Table 3. The data in Column (1) through Column (7) are taken from various Annual Reports of the Federal Power Commission, and are used to compute the entries in the remaining columns. The assumptions made are stated in the footnotes.

An examination of Table 3 discloses: First, the extremely high cost per kilowatt of primary capacity even among the lowest-cost projects of the fifteen analyzed; and, second, the wide variation between the highest and lowest costs. The unit costs on the basis of primary capacity are perhaps somewhat higher than they would have been if additional capacity beyond the primary had not been installed. However, the cost per kilowatt on an installed-capacity basis is still very high, running from $124 to $348. The fixed charges show variations from 0.342 ct per kw-hr to 4.56 cts per kw-hr on a 90% load-factor basis, and from 0.380 ct per kw-hr to 1.067 cts per kw-hr on a 40% load-factor basis.

All this definitely shows: (1) That hydro-electric energy, far from being "free," involves fixed charges that, in many cases, are well in excess of the fixed charges and operating costs of steam power; (2) that the costs are not predictable from general experience and knowledge of the art of generation, but depend on such factors as head, topography, stream flow, and location; and (3) that hydro-electric plants, in general, as a source of primary power, are out of the economic range. The true economic function of hydro-electric power is, without doubt, to supplement steam power. This point is covered in the paper by Mr. Justin.

Operating Cost of Hydro-Electric Plants.—The cost of operating hydro-electric plants represents only a small portion of the total cost, the preponderant portion being the fixed charges on investment, as previously discussed. Operating costs at a particular plant are more or less a fixed item and substantially independent of the kilowatt-hours produced. A recent study of operation costs of plants of various sizes is summarized in Table 4. It will be seen that the median value on the basis of the annual cost per kilowatt installed varies from $2.80 to $0.68, depending upon the size of the plant. The maximum and minimum values are $8.00 and $0.50 per kw per yr, respectively. Based upon an annual load factor of 40%, this represents a kilowatt-hour cost of from 0.8 mill to a low of 0.19 mill.

TABLE 4.—OPERATING COSTS OF HYDRO-ELECTRIC PLANTS

Installed capacity, in kilowatts	OPERATING COSTS, IN DOLLARS, PER KILOWATT OF INSTALLED CAPACITY PER YEAR			Median value of operating costs, in mills, per kilowatt-hour at 40% annual load factor
	High	Low	Median	
Up to 10 000	8.00	0.90	2.80	0.80
10 000 to 25 000	3.18	0.70	1.40	0.40
25 000 to 50 000	3.00	0.60	0.98	0.28
50 000 to 100 000	1.50	0.60	0.75	0.21
100 000 to 200 000	1.65	0.50	0.68	0.19

Steam Power Investment Costs.—The general range of costs of steam plants has been fairly well established. L. W. W. Morrow found[20] that in one group of sixteen plants built between 1924 and 1927, the range in cost, including step-up sub-station equipment, was from \$101 to \$180 per kw, the average being \$139 per kw, and in another group of sixteen plants, built since 1927, the range was from \$82.50 to \$145 per kw, with an average cost of \$114 per kw. From other data in possession of the writer on an even greater number of plants, the range of costs for steam plants built prior to 1932 was found to be from \$85 to \$140 per kw of installed capacity, with the greatest number of plants comprising the greatest amount of installed capacity falling within the range of from \$100 to \$110 per kw. There are some "sports" below and above the limits of \$85 and \$140. One plant is reported to have cost less than \$70 per kw, but this was an unusual situation; in a power system supplied in the main by hydro-electric plants, economy in steam generation was properly sacrificed to some extent in order to obtain the maximum capacity at the minimum cost. Also, in this as in a number of other very low-cost installations, the so-called "framework" features that normally are associated with a steam-power development were omitted.

The wide variation in costs thus found makes it apparent that no rational deduction can be drawn as to the "average" cost per kilowatt of steam plants. Therefore, in preparing Fig. 30, the writer has used capital costs per kilowatt of \$100, \$115, and \$130, as being representative of the costs of existing steam power plants. The annual fixed charges forming part of the total cost data in Fig. 30 were based on these capital costs. No separate investment values are plotted in Fig. 30, the two sets of data actually plotted being operating costs and total costs.

Steam Power Plant Operating Costs.—The operating costs of a steam electric power plant, unlike those of a hydro-electric power plant, form a large part of the total cost of generating. This is illustrated by Fig. 30. In this diagram the three upper curves, A_M, B_M, and C_M, show the total cost of generation at annual load factors from 10 to 90 per cent. These curves were obtained by adding the annual fixed charges per kilowatt-hour calculated at a rate of 12.75% to the annual operating costs per kilowatt-hour. An analysis of steam power plant operating costs—fuel, labor, and maintenance—discloses that for the same load factor the variations in cost per kilowatt-hour are just as wide as the investment cost.

The central station industry of the United States operates approximately 22 000 000 kw of steam generating capacity. Of this amount, more than 11 000 000 kw were installed in the years between 1923 and 1930. In that period great strides were made in the art of steam power generation, affecting the efficiency in the use of fuel and labor. This 11 000 000 kw of capacity, being the newest and presumably the most efficient, is undoubtedly generating the bulk of the power sold to-day. However, it would be just as fallacious to say that the cost in the latest plant is representative of operating costs as it would be to say that the costs in the earliest plant were representative. The answer lies somewhere in between. Curve M is an attempt to show the average operating cost and was derived after study of the costs of many representative plants.

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[20] *Electrical World*, October 27, 1928, and November 23, 1935.

The curves in Fig. 30 show clearly the effect of load factor on costs. At low load factors, say 30%, operating costs constitute only a little more than 33% of the total cost (using $115 as the capital cost), while at 80% load factor, operating costs make up 50% of the total cost.

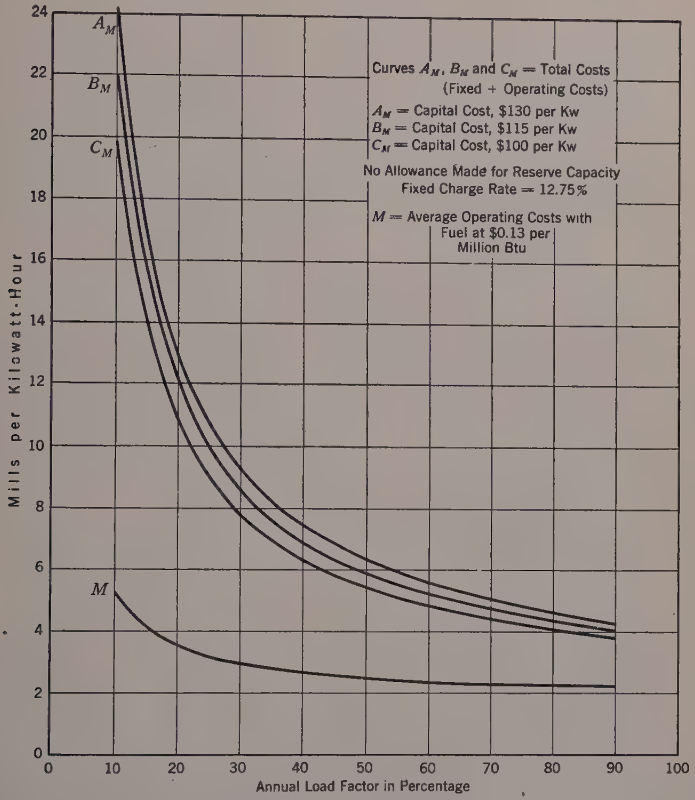

FIG. 30.—GENERATING COST OF STEAM-ELECTRIC STATIONS. THESE CURVES SHOW BASE-GENERATING COSTS AT GENERATING STATION

Cost of Transmission.—One of the most misunderstood factors in the cost of power is the cost of transmission. Those who have studied the problem closely and thoroughly understand the economics of power supply have been for the most part unable to find any sound economical basis for long-distance transmission of power. In general, power is most economical when generated nearest the load, and beyond a certain distance transmission definitely is not an economical proposition. Except in isolated cases, such as those where fuel is not easily available and sources of hydro-electric power are remote from the load, long-distance transmission of power is not practical. Hence, in economically developed supply systems transmission has been used largely as a means of integrating relatively small loads into reasonable blocks in order to take advantage of their diversity and to make possible economical generation within a limited load area.

Unfortunately, the collection of specific data on this subject is extremely difficult. An attempt is made herein, however, to present some definite data, even though they are approximate.

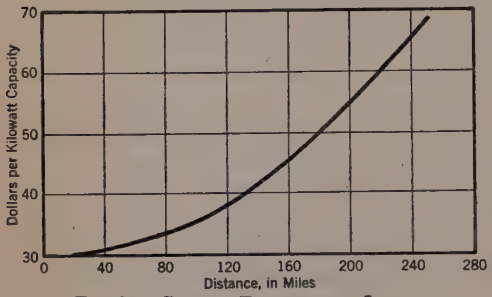

FIG. 31.—COST OF TRANSMISSION LINES

In Fig. 31 the cost of transmission lines, in dollars per kilowatt of capacity, is plotted against distance. Two points in connection with this curve are worthy of note: First, the tendency of the cost per kilowatt to flatten out as distance is materially decreased (this is due to the heavy cost of terminal equipment); and, second, the rapid climb of the curve, at distances of more than 200 miles, to costs approximating those of modern steam plants.

In Fig. 32 there is plotted a series of curves showing cost of transmission, in mills per kilowatt-hour, against transmission distance, in miles, for load factors of from 30 to 90 per cent. Fixed charges, taken at 12.25%, include 0.75% for operation and maintenance. The interesting point in connection with these curves is the rapid increase in the cost of transmission with increase in distance and, particularly, with decrease in load factor. Thus, at a load factor of 50% and a distance of 250 miles, the cost of transmission per kilowatt-hour is approximately that of the operating cost of generation alone in existing modern steam plant practice (see Fig. 30).

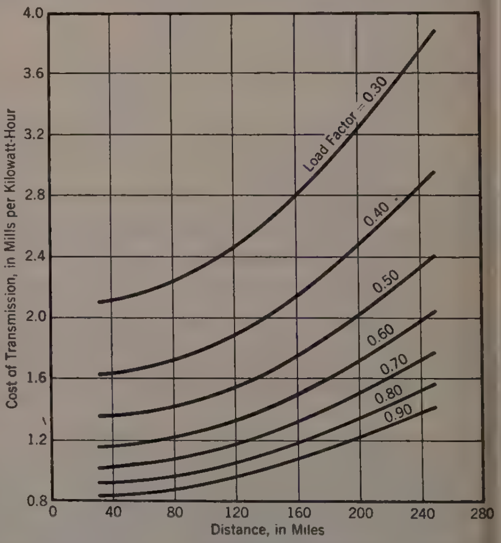

FIG. 32.—COST OF TRANSMITTING ELECTRICAL ENERGY, INCLUDING CHARGES ON TERMINAL EQUIPMENT

In Fig. 33 average freight rates per net ton of coal are plotted against airline distances. These rates are based on published freight tariffs as of October, 1936, for the shipment of coal from typical coal-producing areas to coal-consuming destinations. From these data, calculations have been made of the cost, in mills per kilowatt-hour, of transmitting energy, either as coal by freight or as electric energy. The assumption is made that coal costs $1.50 per ton and that 1 lb of this coal will produce 1 kw-hr, and that fixed charges on transmission facilities are 12.25 per cent. Losses of 10% for the transmission of electric energy have been included. The results are shown graphically in Fig. 34.

It will be noticed that except for very high load factor operation at distances of 140 miles, or more, freight transmission is more economical than electrical transmission. At load factors of 50% and less, the economy of freight transmission is particularly great. As distances are shortened a point is reached where the choice between freight and electrical transmission cannot be made solely by comparing transmission costs, since the location of the steam station (with regard to condensing water, in particular) is a major consideration: but where choice is at all free, it is obvious that it should be in almost every case in favor of freight as against electrical transmission. The actual economic distance will vary, of course, somewhat with local conditions, particularly as regards the facilities for supplying condensing water. In this connection, it is

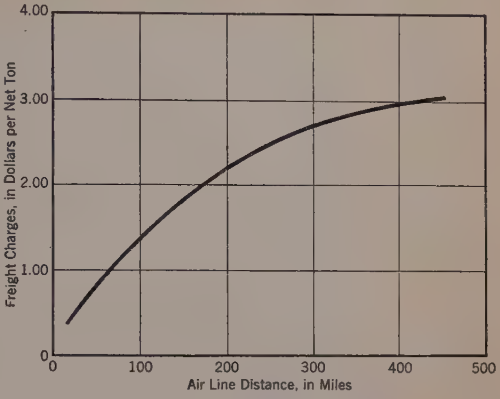

Fig. 33.—Coal Freight Rates

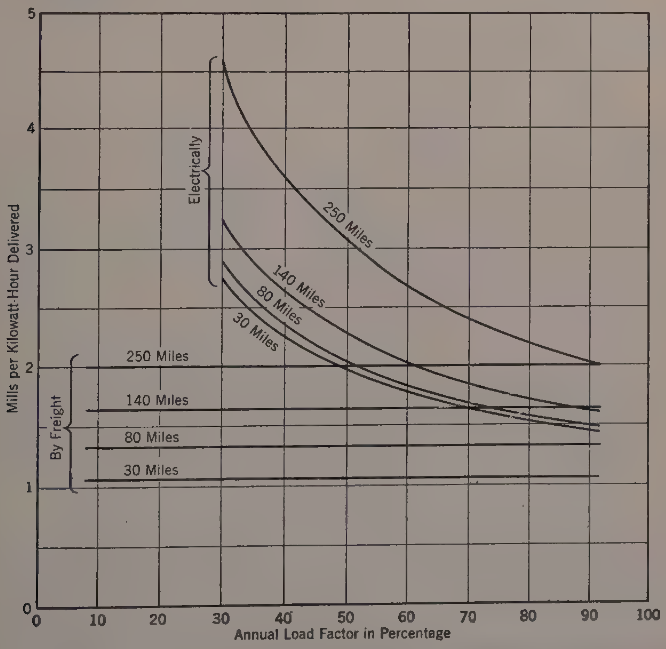

Fig. 34.—Comparison of Cost of Transmission of Energy by Freight (Coal) and Electrically

interesting to note that on the American Gas and Electric Company's interconnected system, extending over nine States and serving about 1 200 comparatively small communities, the actual mean weighted distance of transmission under normal conditions is less than 60 miles—notwithstanding that conditions over a large part of the area (Ohio and Indiana) are extremely unfavorable from the standpoint of availability of condensing water. This definitely illustrates the fact that although inter-connection may be economically sound, long-distance transmission has no such solid economic foundation.

Cost of Distribution.—A discussion of the cost of distribution is beyond the scope of this paper. However, the point needs to be definitely impressed that, at least for residential and general factory use, the cost of distribution is the major cost of energy, despite many recent improvements in distribution facilities. In Fig. 35[21] is shown a nest of curves giving the cost of distribution, in cents per kilowatt-hour (excluding the cost of energy), for different customer densities and for average customer use of from 200 to 2 000 kw-hr per yr. It will be seen that as customer use increases these costs come down to a fairly reasonable figure, running to a low at 2 000 kw-hr of from 1 ct to 2 cts, depending upon the customer density. However, as the customer use decreases, this cost may run to as high as 10 to 20 cts per kw-hr, at very low customer densities and customer use.

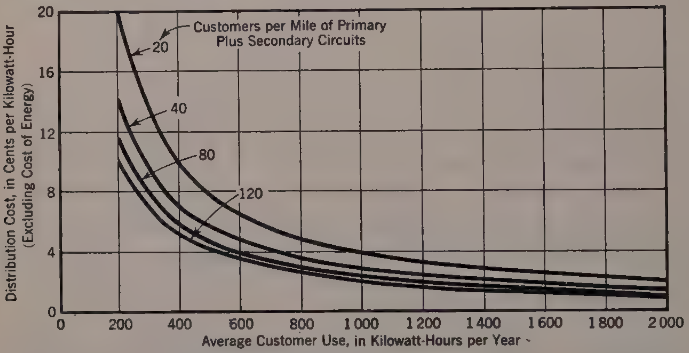

FIG. 35.—RELATIONSHIP BETWEEN CUSTOMER USE AND COST OF DISTRIBUTION FOR VARIOUS CUSTOMER DENSITIES

Total Cost of Generation.—By taking the total cost of generation as the cost of generation at the source, plus the cost of transmission to the load center, the present total cost of generation by steam plants can be obtained for any particular situation, from a combination of the data given in Figs. 30 and 32, within fairly definite limits. On the other hand, it must not be forgotten that in some respects these data have been obtained by simplifying the problem considerably beyond what is practical in an actual system.

For instance, if any electric power system is to furnish adequate and reliable service, it must provide reserve generating equipment. In Fig. 30, in developing the fixed charge item of cost, no allowance was made for reserve capacity.

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[21] "Rational Distribution of Electric Energy and Gas," by Norman R. Gibson, M. Am. Soc. C. E., World Power Conference, Washington, D. C., 1936.

In actual practice on an isolated power system with only one plant, as much as 50% of the installed capacity (100% reserve) may be required to insure continuous service. This may even double the cost of the firm capacity in the plant, that is, the capacity available for sale. Even in an inter-connected system having many generating plants, some part of the installed capacity is needed for reserve. The actual amount may vary from 15% to 30%, depending on the size of the units, the magnitude of the load served, and a number of other factors. This reduction in reserve is obtained by building inter-connecting transmission lines, and their cost must also be considered in any discussion of generating costs.

Likewise, in an actual system, transmission and distribution are frequently so closely interwoven—both lines and equipment being commonly used to perform a dual function—that an exact allocation of equipment and costs between transmission and distribution frequently becomes most difficult, if not impossible.

Nevertheless, making allowance for the simplification used, it will be noticed that for steam generation the outstanding factors are: (1) The fixed charges; (2) the production costs; and (3) the cost of transmission. At very light load factors the first item is the predominant one. As load factors rise, the production cost item comes more nearly in line with the fixed charge item, becoming approximately 60% of the total cost of power at 50% load factor and approximately 50% at 70% load factor. In large inter-connected systems serving a diversified load, annual load factors are on the order of 55 per cent. In isolated systems, serving small communities, where the load is principally that of residential customers, annual load factors are much lower, averaging about 35 per cent. The transmission cost at 50% load factor for a distance of 200 miles may substantially equal the cost of production, and for greater distances may well exceed it.

It is extremely difficult and perhaps unfair to attempt a direct comparison of costs between a single hydro-electric development and a steam plant. On the other hand, this assertion itself discloses the weakness of the typical hydro-electric development, since as a general rule no hydro-electric plant is self-sufficient. This being the case it is impossible to determine its value without considering the position it takes on the load curve, and its effect on costs of other producing sources. In general, this is a point that must be thoroughly considered before a decision as to economic feasibility of a particular hydro-electric project can actually be made.

Another way of stating this is that the hydro-electric plant does not have as much flexibility as regards load factor as the steam plant. The latter can be designed for use at high load factor, and assigned, successively, to positions on the load curve of lesser and lesser use as more economical steam plants are developed. Such shifting is impossible in the case of a hydro-electric plant, which, in general, can be developed economically only for a certain position on the load curve. Its economics are adversely affected when it is used on any other basis. In this connection the data on transmission costs shown in Fig. 32 are pertinent. When proper evaluation of the cost of transmission is made, it is obvious that the economics of hydro-electric generation are bound to be less and less sound as fuel sources are made available within reach of main load centers and as the more economical hydro-electric sites are exhausted.

Future Costs of Generation of Electrical Energy

Trends in Steam Generation.—From the analysis of costs previously given, it appears that one of the most important factors in lowering steam plant generation costs is the reduction of investment costs. The trend in cost of generating equipment has been definitely downward, although admittedly the trend curve has not had a very steep slope. However, there is a definite promise in the direction of, first, higher steam temperatures; and second, the use of higher speeds (which will eventually mean more efficient use of materials).

Installation of larger steam generators, in particular, offers material opportunities for reducing costs in the future. The economics of these large units, however, is contingent upon increasing the availability factors of steam generating units. Recent installations have shown availabilities as great as 94.5%, but a definite step will be made when steam generating equipment can be consistently designed and operated at availability factors of 95 per cent.[22]

Another factor tending to reduce generation cost is superposition in existing steam plants. As a general rule, however, superposition has to be handled in fairly large blocks if it is to be economical. In other words, it is of value primarily where the base loads are heavy and the load factors high. Fortunately, increased co-ordination in the generation programs of inter-connected systems can bring about an economical application of super-position in many cases where an independent program may not prove entirely feasible.

Trends in Hydro-Electric Installations.—There are no present indications of any developments that would make future hydro-electric installations more attractive economically than those already in existence. If anything, the natural trend will be toward less economical installations. The most economical sites were developed long ago, and those that have been exploited recently have, in general, been less attractive economically than those that have preceded them. Nature created only one Niagara and that has been developed.

Trends in Transmission.—The art of transmission has been improved to a remarkable degree since the World War. Continuity standards have been improved; stability limits of alternating-current transmission lines have been extended; and the lightning hazard has been brought under control. Nevertheless, a great many difficulties in the path of completely reliable transmission have not been removed, and the costs as shown herein are still high and non-competitive with other forms of transmission at long distances. Some publicity has been given to direct-current transmission, but at present the idea is still very much in the experimental stage and its economics is totally unknown. In general, although it may be stated that improvements in transmission may be expected in the years to come, no major changes are indicated at the present time.

Trends in Distribution.—The reductions in the distribution system costs made in recent years have resulted from intensive application of increased knowledge, and from the use of engineering methods and economic principles in design. Progress has been slow, but steady. There is no reason to believe that progress in the near future will be revolutionary, but there is every reason to believe that gradual progress will eventually result in lower costs than exist at present (1937).

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[22] "The Value of Proper Furnace Equipment to Power Plant Economy," by M. K. Drewry, Mid-West Power Conference, Chicago, 1936.

Trends in Over-All Cost of Electrical Energy.—The future over-all costs of electrical energy, whether in generation or in generation plus transmission and distribution, can definitely be expected to be downward, although no rapid downward steps are indicated. It appears definite that future power requirements will be met most economically by steam plants, located at a reasonable distance from load centers and equipped with large units, with inter-connecting transmission to integrate and combine small loads and to reinforce and "back up" neighboring systems. As for steam-plant design, further progress along present lines—particularly in the expansion of the present limits of pressure and temperature, and the raising of the availability factor of both steam and generating units to make possible a minimum of duplication of facilities—will bring the optimum results. Here, however, as in other lines of endeavor, it will be necessary to rely upon "the inevitability of gradualness."

SOCIAL IMPLICATIONS OF TECHNOLOGICAL TRENDS IN THE POWER INDUSTRY

BY RALPH E. FREEMAN,[23] ESQ.

SYNOPSIS

The general effects of trends in energy generation on the level of living, unemployment, overproduction, and population distribution, are treated in this paper. A plea is made for a clear distinction between the business point of view and that of the social reformer.

The purpose of this paper is to discuss some of the economic aspects of the technical matters dealt with in the preceding papers.

To begin with, two generalizations may be drawn: First, the costs of energy generation are coming down, and in spite of many obstacles are likely to be lowered still further in the future; and, second, changes are taking place in the relative efficiency of generating energy by different methods, and the trend away from earlier methods still proceeds, moving in the direction of large inter-connected steam and hydro-electric generating plants and, to a less degree, under favorable conditions, in the direction of Diesel power.

It is hardly necessary to dwell upon what is the most obvious result of cheaper power, namely, increased economic well-being. One need only compare the standard of living of to-day with that which prevailed before the discovery of the energy latent in steam to realize the economic benefits of more efficient generation of energy. When a unit of energy is produced with less expenditure of human and material resources, the amount of consumable goods at the disposal of the community is augmented. The effect upon the individual depends, of course, upon the size of the population and the distribution of wealth; but, with no changes taking place in the number of people or in the manner in which wealth is shared among them, cheaper energy means greater material well-being for the individual members of the community.

In the minds of many people in the United States, however, the growing productive capacity of an economic machine driven by cheaper power has raised some terrifying possibilities. They fear a chronic condition of glut due to the incapacity of the markets to absorb the increased output. They believe that, largely as a result of improvements in the generation of energy, the United States is entering upon a new phase of development, another industrial revolution. They claim that, whereas once the country suffered from a scarcity of goods, to-day it is overflowing with plenty. The old economic problem was how to produce enough; the new one is how to dispose of all that can be produced.

This dread of superabundance is groundless. The scientists and engineers who are engaged in raising the efficiency of the power machine system need not be disturbed. Overproduction is an economic phenomenon, not a physical one. Overproduction is unprofitable production; surplus capacity is capacity which cannot be utilized without loss. The goods now glutting the markets of many countries would immediately vanish were they offered for sale at low enough prices. At some price or other buyers could be found for all the wheat the United States is capable of growing, and for all the output of all its steel mills running at full capacity. That this drastic procedure is not feasible is chiefly because of the relatively high prices the farmers and steel-makers have to pay for their materials, supplies, equipment, etc. Congested markets are the result of disjointed prices, and these, in turn, are attributable to unbalanced relationships between one industry and another.

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To dispose thoroughly of the naive conception of a world that is spilling over with abundance would take much more space than is allotted to this paper. The delusion of plenty would be harmless were it not for the fact that it obscures the fundamental nature of the economic problem. It leads people to think of this problem in terms which clearly imply that a solution is to be found in an expansion of money incomes or in a contraction of the volume of goods produced. When people get the notion that depressions are attributable to under-consumption which can be avoided merely by distributing to the poor an increased supply of paper money, they are easily mobilized in support of ill-conceived currency, pension, and "social-dividend" schemes. When business troubles are regarded as the result of excess production, there is an agitation for reduction of output by drastic curtailment of working hours, by checking the introduction of technical improvements, and other restrictive measures.

Conditions may exist that call for additional media of exchange or less productive capacity in certain industries or even for a shorter work-day; but the fact that at times such measures may be beneficial does not imply that the basic cause of economic disturbance lies in a scarcity of money or in a superfluity of goods. At other times there may be too much money available or, in certain industries, the output of goods may be too small. Indeed, as a general rule, the ultimate source of disorder can be traced to unbalanced relationships between one industry and another that are quite independent of the total volume of output or of the total amount of currency in use. Overproduction, in other words, is a symptom of economic disease rather than its cause.

Another important economic result of generating energy at reduced cost is the further displacement of human labor in industry. Since the Eighteenth Century, when the invention of the steam engine enabled Man for the first time to make non-human energy work for him at his chosen time and place, human physical labor as the power supply of industry has been virtually eliminated in many fields. In other types of production, however, the elimination of human power awaits the invention of new machines to handle refractory materials and to deal with complex assemblies of parts, or it awaits the arrival of that relation between labor cost and energy cost which will make it profitable for the producer to substitute machine power. There is no doubt that a cheapening of non-human energy will hasten this substitution, especially if wages remain at their present levels or if, as seems likely, they continue to advance.

On the whole, such a development would be socially beneficial. Where mechanization is still in the half-and-half stage, where invention and research have absorbed only partly the old handicraft processes into the new energy system, a great deal of power has still to be provided by the human element. This makes the worker virtually a part of the machine. He is compelled to perform semi-automatic operations which are stupefyingly monotonous, often at a pace set by the machine which demands a nerve-wracking degree of strained concentration. When the difficulties in the way of complete automatism are overcome, the daily lives of many workmen will be relieved of much deadening drudgery.

Experience with the effects of reductions in the cost of energy generation in the past indicates that there is no need to fear a permanent addition to unemployment because of such developments. Cheaper non-human energy, although it displaces labor as the physical power supply, increases the demand for supervisory and other forms of what may be termed mental labor. Moreover, it enables men and women to leave the mechanized industries and enter occupations of the human service type, trade, finance, amusement, and various kinds of clerical and professional work. There is a real problem, of course, in the re-employment of the man thrown out of a job; there are obstacles to the movement of labor from one occupation to another which are serious, particularly in the case of highly specialized workers; but history clearly demonstrates that when a thousand men are displaced by an improvement in energy generation an additional thousand men are not thereby permanently placed on the relief rolls. A compensatory demand for labor arises from the enlarged social income made available by the reduction of power cost.

In addition to the economic problems arising from the trend toward cheaper generation of energy, there are many interesting economic implications in the changes that are taking place in the relative efficiency of generating energy by different means.

During the Eighteenth and early Nineteenth Centuries the direct application of steam energy brought about far-reaching transformations in the psychology of the worker and his relation to his employer. It resulted in a concentration of production because the increasingly complicated and expensive machines required greater capital and larger productive units; and since the machine compels men to limit themselves to certain branches of production, it resulted in greater specialization. It brought mass production that destroyed the individual character of the product and created uniformity in the wants of consumers. Mechanization resulting from steam power not only extended the period of production, introducing additional intermediate stages, but also increased the proportion of capital goods in the economic structure, and both these factors have enlarged the range of economic fluctuations. Mechanization has enhanced overhead costs relatively to variable costs, thereby introducing rigidities into the price structure which further accentuate industrial booms and depressions.

Are the recent changes in energy generation likely to affect any of these conditions? Consider, for example, the centralization of production in large plants in congested urban centers.

Steam power, if not converted into electrical energy, must be used where made. It is available chiefly, therefore, to the large manufacturing plant which produces power on a large scale; but electrical power can be distributed to points at a distance from the generating station, and made available at low cost to small producing units. Progress in electrification, therefore, may remove the advantage that steam power gives to large manufacturing units.

This applies also to the development of the Diesel engine, in which the economical size is not such as to favor the large generating unit. If cost conditions in a given industry and locality permit the profitable use of Diesel engines of relatively small capacity, small plants may be able to compete successfully with large ones. Whether such situations are now common, or whether there is a trend in this direction, is a matter for competent engineers to determine.

As to whether or not developments in energy generation will in fact lead to a producing system of small plants dispersed over the country, it is not yet possible to make a safe prediction. Although steam power was one of the influences which massed industry into urban centers, the concentration of production is not due to power considerations alone. Marketing facilities, the proximity of suitable labor and raw materials, and various other factors are important in determining the location of industrial plants. Where such factors as these constitute an important part of the cost of production, the electrical development is not likely to effect a great deal of decentralization.

In 1935, Mr. Daniel B. Creamer published the results of a study of population movements, entitled "Is Industry Decentralizing?"[24] This *Bulletin* discloses the major trends in the location of manufacturing industries in the United States from 1899 to 1933. It presents evidence of a moderate amount of decentralization—not so much a scattering of factories up and down the length of the land, but rather a spreading of industries from the great urban centers to their suburbs. Mr. Creamer indicates that, in many cases, particularly in the textile and boot and shoe industries, this movement has been due primarily to the possibility of hiring cheaper labor in new locations. On the other hand, it is to be observed that this migration of industry would have been impracticable in many instances had it not been possible to secure the necessary power in the new locations at reasonably low rates. In other words, although it is doubtful whether power costs in most industries are large enough to be a determining factor, they become of great importance in conjunction with other factors.

There is no doubt that a thoroughgoing diffusion or dispersion of industry would create another revolution in American social life, and in recent years the movement in that direction has given rise to an agitation that the Government should do something to encourage it. This agitation has been reflected in recent governmental policy. Various resettlement proposals and power projects have won support from those whose imaginations have been caught by the vision of a de-urbanized world. In many ways the ideal is attractive. There is, however, a danger that projects may be initiated that are out of harmony with fundamental economic and engineering principles, and that this will result in financial loss not only to the supporters of the new projects, but also to those enterprises with which those projects compete. Financial cost should not be the only guide in these matters; but before public subsidies are expended, careful engineering and economic investigation should be made to determine the cost of the expected social benefits.

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[24] "Study of Population Re-Distribution," *Bulletin No. 3*, Univ. of Pennsylvania Press, 1935.

As long as the economic regime is not centrally planned and controlled, productive activities must conform in large measure to the dictates of prices and costs. To ignore the monetary signals is to invite chaos. It must be assumed that, in the main, prices are reliable guides for the organization of the national economy. It must be assumed that changes in the amount of money offered by buyers (which influence prices) reflect changes in the wants of the community and that these wants should be satisfied. It must be assumed that technical changes (which also influence prices) reflect changes in the physical cost of production and that Society should produce its goods and services with the minimum expenditure of human effort and materials, economizing those forms that are relatively scarce.

This is the method by which, in the main, the economic activities of the United States are organized. It is the regime under which the people have advanced to their present condition of wealth. Moreover, it can be demonstrated that in many instances a departure from this means of organizing the nation's productive resources has reduced the efficiency of the economic machine. The attempts of private monopolies or agencies of Government to set prices at variance with fundamental technical or demand conditions have often been followed by serious maladjustment.

Of course, it is permissible to go behind the price structure to the ultimate conditions of physical cost and human demand and inquire whether prices reflect accurately and faithfully the relative scarcity of resources and the wants of the community as a whole. Thus, it may be argued, on the technical side, that relative prices reflect the relative scarcity of the material means of production in the present and take insufficient account of the future. In the case of energy generation, it has been argued that the use of petroleum involves the depletion of resources that cannot be replaced, whereas the harnessing of falling water is open to no such objection. Should the effects of costs and prices as determined under a private profit-seeking regime, therefore, be modified, and hydro-generation of energy be subsidized in the interests of conserving natural resources? In putting such a policy into effect much caution would be required. It is hazardous to base social plans upon future technical developments which cannot be clearly foreseen.

There is also the question of the social cost of changing techniques. When one method of energy generation displaces another suddenly and over a wide field, obsolete plant is likely to be created, and men are thrown out of work. Idle men and idle machines constitute a cost that is not taken into account in the price system, but it is a cost, nevertheless. Should this social cost be mitigated by governmental action regulating the rate of technical change or imposing a financial burden on the source of disturbance? Changes in the technique of energy generation may affect wide areas of the country and large sections of the population. Should one class or one region bear the cost of technical progress? What should be done about it?

Prices reflect the demand of those who have money, but they do not accurately express the wants or desires of the community as a whole. The price of electrical energy, for example, may be too great to enable an important group of consumers to enjoy its advantages. Should the farmers be given especially low rates to permit a more extensive use of electric power on the farms? Should the Government subsidize power projects to foster agricultural development? Should power projects be constructed by the Government in agricultural regions where private enterprise finds it unprofitable to operate? Questions such as these cannot be excluded from a discussion of the economics of energy generation.

In current controversy, however, there is much confusion and evasion of issues. Lower rates may be advocated on the ground that a greater consumption will result and that enlarged output and sale will permit lower unit costs. That is an economic argument valid within the exchange-price system; but when lower rates are advocated in order to benefit certain types of consumers or to stimulate certain other industries—that is an argument that may be valid on human or social grounds but is outside the economic plane of prices and costs. In considering the problem, these two types of approach are not always kept clearly in mind. Moreover, because accurate quantitative measurement and determination is much easier on the cost-price level than on the social level, many people (scientists and engineers in particular) refuse to admit the validity of the latter. The result is that in discussing these matters there is either a confusion of the issues involved or no meeting of minds by those who participate in the controversy.

ELECTRIC POWER IN ECONOMIC PERSPECTIVE

By B. Alden Thresher,[25] Esq.

Synopsis

The bitterness of political controversy over public utilities has obscured the real determinants of the problem, which are technical and economic. The industry might be conducted successfully under any one of many types of organization, public, private, and mixed. Regulation in the United States has placed an undue emphasis on valuations of physical property quickly outmoded by obsolescence. Both the public and private sectors of the industry are seeking the same basic solution under forms which are superficially different.

The long and bitter controversy over the regulation and control of electric power has obscured certain aspects of the problem which are of abiding and fundamental importance. The issue between public and private operation, between more regulation and less regulation, between the Securities and Exchange Commission and the holding companies—in fact, the long, running fight which has accompanied the entire period of growth in this industry—is, when seen in true perspective, a mere incident in a larger development. That larger development consists of the simultaneous growth of supply and demand. On the supply side is cheap and abundant energy flexibly distributed and readily controlled. On the demand side are marked changes in industrial organization connected with the use of cheap power in industry, and even more spectacular changes in social habits as the retail consumer adjusts himself to power in the home.

The progress of the Power Industry is not so dependent on the winds of political change as many seem to think. Behind the surface controversy over forms of organization and regulation lie the real determinants of the problem—the technical limitations of economic production on the one hand, and the social and industrial nature of the demand for electric power on the other. It is easy to take sides in the current controversy, but taking sides is a mark of the pre-scientific stage of the problem. One might, for example, deplore the abuses of monopoly, detailing the long catalogue of evils that unbounded financial peace has brought upon the industry; or one might elaborate the evils of public control, and adduce examples of the "dead hand" of Government throttling private initiative. Either of these arguments would be in the current fashion; but neither would be to the point. The real issues lie deeper. They arise out of the nature of the industry itself. They will persist under public or private control, or under a mixture of the two; and they will be settled not by the choice between these rival principles, but by the economic logic of the Power Industry itself. More important than the choice between public and private operation is the choice between a stunted power industry and a well-developed one, economically adjusted to the society in which it operates.

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Political scientists will disagree with this statement. They will argue that the issue between collectivism and individualism is fundamental and insistent. Constitutional lawyers will disagree. They will hold that the issue is between two systems of government, two opposed concepts of the tenure of rights legitimately acquired. Those in active control of the industry and immersed in its current controversies will disagree. They see their interests threatened, and few indeed are the men who can take the long view when their investments are at stake. Politicians will disagree, for the man who has to be re-elected is the last man in the world to cultivate perspective on anything.

It has often been stated that the fluctuations of the business cycle are roughly parallel in widely separated countries. Depression overtakes nations whose economic policies are widely different from each other. Similarly, when the forces of recovery are ripe, recovery appears, whether the Government policy of the moment be of the interventionist type or whether private business be left with little control.

In the development of the electric power industry throughout the world an analogous effect is evident. The industry has followed the same general cycle of technical development and consumer acceptance in all countries, although the legal, financial, and political forms under which it has developed differ widely.

In France, as in the United States, private ownership is the rule, public ownership the exception. Interestingly enough, the French power enterprises have managed to survive and attract capital without any of the so-called constitutional safeguards of which so much is made in the United States. The Parliament legislates without restraint in the matter of regulation.

In Sweden, public and private enterprise have managed to co-exist amicably. As Mr. O. C. Hormell has put it:[26]

> "While the state owns and operates an increasingly large proportion of the generating and distributing plants, the state has not taken (or been able to take) for itself any marked privileges or advantages with regard to its power development, over and above those granted to private enterprises."

In Sweden, as in Great Britain, a nation-wide distribution system yields the economies of inter-connection, and permits the public authority to control and direct power development, yet avoids the rancorous rivalry that has marred the relations between public and private agencies in the United States.

In Germany, the long-established tradition of municipal socialism in public utilities, with its roots in the Nineteenth Century, has not prevented a healthy development of private enterprises. In fact, one finds, in the post-war period, public enterprises accepting private capital and a mixed directorship.[27] In Switzerland, likewise, there is an interesting mixture of Federal, cantonal, municipal, and private ownership. The lion and the lamb have lain down together in a way that surprises Americans accustomed to the Corcyran fury of their own power controversy.

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[26] Rept., New York State Committee on Revision of the Public Service Commissions Law, 1930, p. 468.

[27] *Loc. cit.*, p. 406.

This brief summary, which might be greatly extended, suggests that there are more fundamental economic forces at work than are contemplated in the terms of the controversy as it is usually presented. Power can be distributed successfully under a wide variety of legal and economic forms. Where there is a steady and growing demand for a service, and the technical means and the labor and capital are available, the process of production somehow gets itself organized. Whether this organization takes the form of public or private enterprise is not such a fundamental fact as one might suppose. Either form, for survival, must adapt itself to basic economic realities of the industry. In all probability, both forms will persist side by side for many years, and each will gain through the presence of the other. Frequently, the claim is made that regulation has broken down. It would be more accurate to state that the concepts on which regulation in the United States is based, have never caught up with the swift development of that technical and social complex, the "Power Industry."

The economic problem involved may be clarified by asking what characteristics in any industry facilitate a smooth and continuous adjustment of supply and demand, so that at any moment an approximation to economic equilibrium can be maintained. The following ideal conditions may be advanced as making for a smooth and prompt adjustment to economic changes:

(1) The commodity sold should be reasonably uniform and standardized.

(2) It should be sold largely to ultimate consumers, rather than to producers of other goods.

(3) It should be capable of storage.

(4) Entrance into the industry should be free and should not require a heavy investment.

(5) Abandonment of production or reduction of output should be possible without heavy loss.

(6) (A corollary of Conditions (4) and (5).) Cost should be predominantly direct, rather than of an overhead nature.

(7) The industry should be in a state of steady, but not too rapid, growth.

(8) Technical progress should not be so rapid that obsolescence becomes a serious problem.

(9) The surrounding economy should be operating with an approximately stable price level.

These conditions are obviously at variance with the actualities of the electric power industry. The commodity sold is not primarily energy, but a combination of energy, usually in small amounts, with varying amounts of readiness-to-serve, and the services of a distribution plant whose importance economically far outweighs that of the energy itself. Storage is not practicable on a major scale. The very nature of the industry prohibits freedom of enterprise in the full sense, and the fixed plant is enormous in proportion to annual gross business. Downward adjustments of output are difficult, and are likely to entail heavy loss. The tendency for overhead costs to increase, as compared with direct costs, is exceptionally marked. The rate of technical development has been so rapid as to impose a heavy burden of obsolescence on the industry.

All these departures from the theoretical ideal may be classified under two heads: First, rigidities in the market structure, centering upon the problem of overhead cost allocation, and leading to a condition of natural monopoly enforced by the technical conditions of production; and, second, dynamic elements; that is, rapid qualitative change in technique and in demand.

Some examples of the rigidity caused by the difficulty of cost allocation may be mentioned briefly. First, there is the definition of the unit or commodity sold. Most problems of rate structures involve this basic question. It may be seriously questioned whether an energy unit is an appropriate one. The rich variety of multi-part rates, the attempts to separate stand-by and readiness-to-serve elements, and customer costs and distribution costs, indicate that the commodities sold often consist chiefly of elements of access and availability.

More important still is the group of maladjustments arising out of heavy overhead costs, which, in turn, are traceable to the predominance of fixed plant in the total investment. The most obvious problem under this head is that of depreciation and operating cost. In a body of fixed plant which is undergoing rapid expansion the effect of inadequate depreciation reserves can be long postponed. As long as expansion continues, retirements continue abnormally light in proportion to total investments, and true costs may long be under-estimated. In this respect a day of reckoning is surely in store for many companies otherwise conservatively managed.

It is in the matter of capital charges and rate of return that controversy has been most acute. The history of public utility valuation contains a lesson which is important not only for the power industries, but for many others as well. In this controversy both sides have taken the line of expediency and immediate self-interest, rather than that of long-range wisdom.

In 1898, when the case of Smith *v.* Ames was decided at the end of two decades of falling prices, consumers, represented by William Jennings Bryan, argued that valuation of utility property should follow reproduction cost. This, under the circumstances, led, of course, to a lower rate base. Producers (in that instance, the railroad companies) clung tenaciously to the higher original cost of construction, as representing their idea of justice.

In the long upward swing of the price level to 1920 these positions were sharply reversed. The growing utility companies went on record in great detail as favoring reproduction cost, both in practice and in theory. It came to be assumed almost as a law of Nature (so short is human memory) that a reproduction-cost rate base is always higher than the original cost.

Another complete reversal of positions might, therefore, have been expected in the price liquidation in the years following 1930. This reversal, however, was only partial and incomplete. The constitutional guaranties which seemed to hedge the reproduction-cost concept, gave to the producing interests a different kind of advantage from that arising from mere price changes.

In an industry in which costs are falling rapidly, any device that delays a normal price readjustment may be expected to offer great temporary advantage to producers. The constitutional guaranty with which the Supreme Court has hedged property devoted to public use supplied just such a device. Public clamor was partly quieted by partial rate reduction, but largely resisted by resort to the Courts. A fair return on the valuation was increased by hypothetical calculations, which assumed reproduction, at great expense, of plant which no one could logically wish to reproduce. This is not to attack the motives of the power companies. They acted in good faith in seeking legitimately an advantage that was legally and morally theirs. Their attorneys were not hypocritical in pressing for the last penny. More accurately, they were honest, but short-sighted.

As the matter has turned out, theirs was a Pyrrhic victory. The rate reductions, which were resisted so strenuously, have mostly been conceded, not in deference to any legal right of the consuming interest, but rather out of sheer expediency. The technical and economic situation in the industry has swept aside a body of legal principle. It is not that the legal principle was unjust or reactionary; in fact, it represented an honest effort by able men to do justice to all parties in a complex situation. The error lay in the failure to take account of the economic effect of technical progress in an industry in which technical progress has rushed ahead by leaps and bounds. The year 1930 found the power companies strongly entrenched in an unassailable legal position, honestly won. The year 1936 found them conceding more ground than they had gained; and they were doing so in response to economic and technological forces stronger than any body of legal rights.

This aspect of the problem must be left for a moment to consider other types of rigidity that distort the free play of market forces. The Power Industry, in common with transportation, is a proper field for class price, or, more properly, price discrimination. It should be understood that this is not a term of disparagement. Discrimination may be defined as "differences of price not attributable to differences in cost."

Discrimination is possible: (1) When there exists a monopoly, or at least a sufficient approach to monopoly to prevent competitors from breaking down a system of class prices; and (2) when the commodity sold is not susceptible of storage and resale (railroad freight rates are a classic example, and rate differences between main classes of electric power loads illustrate the same principle). What the public often fails to appreciate is that such discrimination actually benefits not merely the class of customers serving the low rates, but even the persons who apparently are discriminated against as well. In so far as a low rate brings in business not otherwise obtainable, it allows the burden of overhead to be distributed more widely, and hence prevents the high-rate business from carrying an even higher rate.

Finally, as a typical result of heavy overhead cost, one finds the phenomena of diversity and inter-connection, which have played a large part in power development. Here, again, the situation is completely at variance with that which prevails in the sale of most commodities. It is necessary to reckon not merely with the normal economies of large-scale production, but with those differences in timing that make it possible to improve the central station load factor by connecting loads of different characteristics. Both the class-price principle and the diversity principle make for a degree of economic solidarity among power customers that is not equalled in any other industry. The price Jones pays for a pair of shoes or a set of tires depends upon how many other people are willing to buy shoes or tires, but the price which Jones pays for electric energy depends not merely upon the number of other customers, but upon what time of day they use their power. Jones, furthermore, may actually benefit by having somebody else buy power cheaper than he does—although Jones would probably never be convinced of this, even if all the rate experts and economists in the world should try to demonstrate it to him.

The peculiar economies of load diversity, over and above those of mere large-scale generation of power, have led to regional inter-connection by several different approaches. It early became apparent that diversity was advantageous within a single community. To bring it about also between communities and regions has been one of the major objectives of consolidations and holding company developments since before the World War. The same logic underlies private contractual interchange agreements like the Connecticut Valley Power Exchange. The British "Grid" provides still another variant, with a public inter-connection system interposed between private power producers and retail distribution systems of which some are public and some private. The Grid thus serves as a stimulant to unified development of a national system, but leaves that system predominantly in private hands. It serves also as a regulative device by interposing the public authority as a middle-man, instead of, as in the United States, either a competitor or a policeman. Still another variant is found in the regional projects under wholly public auspices, like the Tennessee Valley Authority.

It has been well said that the human mind recognizes differences more easily than it perceives similarities. The four devices just listed are a case in point. Their relative merits are disputed so strongly that the fact that they are all expressions of a common principle is forgotten. What is really important is to get this principle recognized and utilized to the fullest extent; which device is used makes little difference.

The characteristics thus far mentioned are, so to speak, permanent differences which earmark the Power Industry as distinct from most others. Even more significant, however, is the dynamic phenomenon of rapid change, both in technique and utilization. In an industry where technical development is rapid, and utilization both by industry and the consumer is developing with equal rapidity, economic forces making for a normal equilibrium work too slowly. Costs are falling over the long period, but only with the aid of large new investments of capital. Demand is growing, but it grows faster where the desire for secure profit is tempered by willingness to postpone returns, so as to attract further business. It will be argued, however, that many other industries are in a similar condition of rapid change. Why should the Power Industry be singled out as being affected to an extraordinary degree by accelerated technical progress and by changes in the response of consumers?

The answer seems to lie in the presence of public regulation. The concepts that make up the accepted scheme of regulation in the United States are by their very nature static. The legal theory of an adequate return on the fair value of the investment is valid and sound for an industry in a stable condition. It provides, in effect, that "every tub shall stand on its own bottom"; but the moving equilibrium of a runner in rapid motion is a different phenomenon from that of a man standing still. The accepted theory laid down by the Courts finds support in the instinctive logic and sense of justice of any one who takes the trouble to reason out the problem. However, on every hand, it is seen how the technical and economic changes that keep the industry in constant ferment have already set these theories at naught.

The explanation is to be found in what has been called "the fallacy of misplaced concreteness." The Courts have put their trust in tangible equipment—that is, in property in the physical sense. It is a natural error, seeing how large, impressive, and indispensable is the physical plant necessary in this industry; but the error consists in not recognizing that the physical plant is only the passing embodiment of a technique that is undergoing rapid development. The concept of value envisaged by the Courts is of a property inherent in tangible objects; hence, the elaborate engineering appraisals that support "physical" valuations.

If there is one contribution that the economist should make above all others to the public utility problem, it is to demonstrate that value is not an inherent property dependent upon the dimensions or specifications of a tangible object, but an economic, hence partly psychological, phenomenon dependent upon conditions of use and demand. Obsolescence is not a physical but an economic process. It follows that in a situation where productive methods and productive technique are rapidly changing and improving, and where the nature of the demand is shifting, any attempt to set a value on a piece of property solely as a function of the physical nature of that property is doomed to failure. It is the misfortune of the utility industry that something as transitory and changing as its current technical methods must be embodied in fixed plant which is so permanent and expensive. This plant soon becomes a lifeless shell, when changes in method require a shift in the type of equipment.

The legal theories of regulation have never succeeded in grappling with this problem. The short-run result of such a situation has been to benefit the utility companies. Once rates are set, it has usually been possible to cut costs sufficiently that a considerable addition is made to the rate of return nominally earned on a given investment. Were this not true, all incentive would long since have been removed from the industry. The engineer can be thanked for thwarting the regulator and allowing the industry to adjust itself to the facts of social and economic change.

It seems fair to conclude that this situation is largely responsible for the contemporary emphasis on Federal power projects. It is shallow to regard the Federal program as merely punitive, or as evoked by the misdeeds and abuses of the power companies. It is true that there has been widespread exploitation and abuse, and, in this respect, the Power Industry has accurately followed the pattern set by the railroads a generation earlier. A wide margin of waste, misapplication of funds, and general inefficiency seem to be the normal price of a rapid expansion process. The phenomenal speed and effectiveness with which the railroad and power networks have overspread the continent have been offset by a heavy penalty of frenzied finance and exploitation. Similarly, the war effort of 1917 and the more recent effort to apply Federal funds to large-scale projects both led to colossal economic waste, although some net advantage appears in both cases. All this, although it seems important in the light of the political situation of 1937, will be regarded with a calm and tolerant eye by the economic historian.

It is more accurate, therefore, to regard the Federal power program, whatever its merits or defects, as a response to a long-standing maladjustment, not as a punishment meted out to malefactors. The machinery of State regulation is often said to have "broken down." More strictly speaking, it has always lagged behind the situation with which it strove to grapple. This was true even in the days of local power units, because of the rapid rate of technical change. It is doubly true now that regional inter-connections extend over areas beyond the jurisdiction of regulative authorities.

Are the people then to be content with the imposition of "yard-stick" projects that are manifestly unfair? Is the ineptness of public regulation in the past to be an excuse for unfair competition in the future? It is doubtful whether the problem can ever be solved merely by territorial division and rate adjustments between public and private enterprise. Such a solution, although desirable, is too much to hope for. In practice, some working basis will be arrived at because the demand is still expanding so rapidly as to permit, within a relatively short time, utilization of both private and public facilities without seriously overcrowding the field. As always before in the economic history of the United States, progressive expansion should provide a correction for miscalculations and errors—both those of financial exploitation by private interests, and the equally glaring ones of public policy.

A time will come, however, when economic maturity will overtake the industry. Mistakes will then be more costly because growth will be slower. When that day comes, a lesson may be learned from countries like Switzerland and Sweden. One may hope it will by then be apparent that a public temper which inclines toward moderation and co-operation is indispensable for the full development of a technical and social complex like the Power Industry. The engineer holds the key to continued progress, but unless such a temper prevails, his labors will be in vain.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
DISCUSSIONS

GRAPHICAL DISTRIBUTION OF VERTICAL PRESSURE BENEATH FOUNDATIONS

Discussion

By Donald M. Burmister, Assoc. M. Am. Soc. C. E.

Donald M. Burmister,[40] Assoc. M. Am. Soc. C. E. (by letter).[40a]—The constructive nature of the discussions is gratifying, and the writer feels that the value of the paper has been materially increased thereby. The comments have revealed interesting practical uses, and have suggested valuable "short-cut" procedures.

Although it was not the intent of the writer to go into the limitations of the Boussinesq theory, it is of the greatest importance to have a clear understanding of the situation in any given case. Mr. Cummings has given an excellent discussion of how well the foundation problem conforms with the assumptions on which the Boussinesq theory is based, and has outlined the important influence of discontinuities in the foundation soil. Mr. Paaswell shows that the exact determination of the distribution of pressure is not necessary, since present knowledge as to the nature of foundation material does not yet permit making precise determination. For example, little is known about the actual stress conditions in the soil under various conditions of loading, or the influence of stratification and discontinuities upon the way the stress is transmitted into the soil. Therefore, any method that facilitates computation and reduces the labor and time involved will be useful to the soil engineer in obtaining a good idea of the distribution of stress in the soil beneath foundations.

The fact that others have been thinking about the problem of simplifying the application of the Boussinesq theory indicates the interest in the subject. Mr. Newmark describes an analytical and graphical method, which he has found to be very useful. He has made a valuable suggestion in that one chart may be used for all depths by simply changing the scale

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Note.—The paper by Donald M. Burmister, Assoc. M. Am. Soc. C. E., was published in January, 1937, *Proceedings*. Discussion on this paper has been published in *Proceedings*, as follows: May, 1937, by Messrs. William B. Kimball, I. M. Nelidov, George Paaswell, and Jacob Feld; June, 1937, by Messrs. Nathan M. Newmark, A. E. Cummings, and D. P. Krynine; and September, 1937, by A. A. Eremin, Assoc. M. Am. Soc. C. E.

[40] Asst. Prof. of Civ. Eng., Columbia Univ., New York, N. Y.

[40a] Received by the Secretary October 13, 1937.

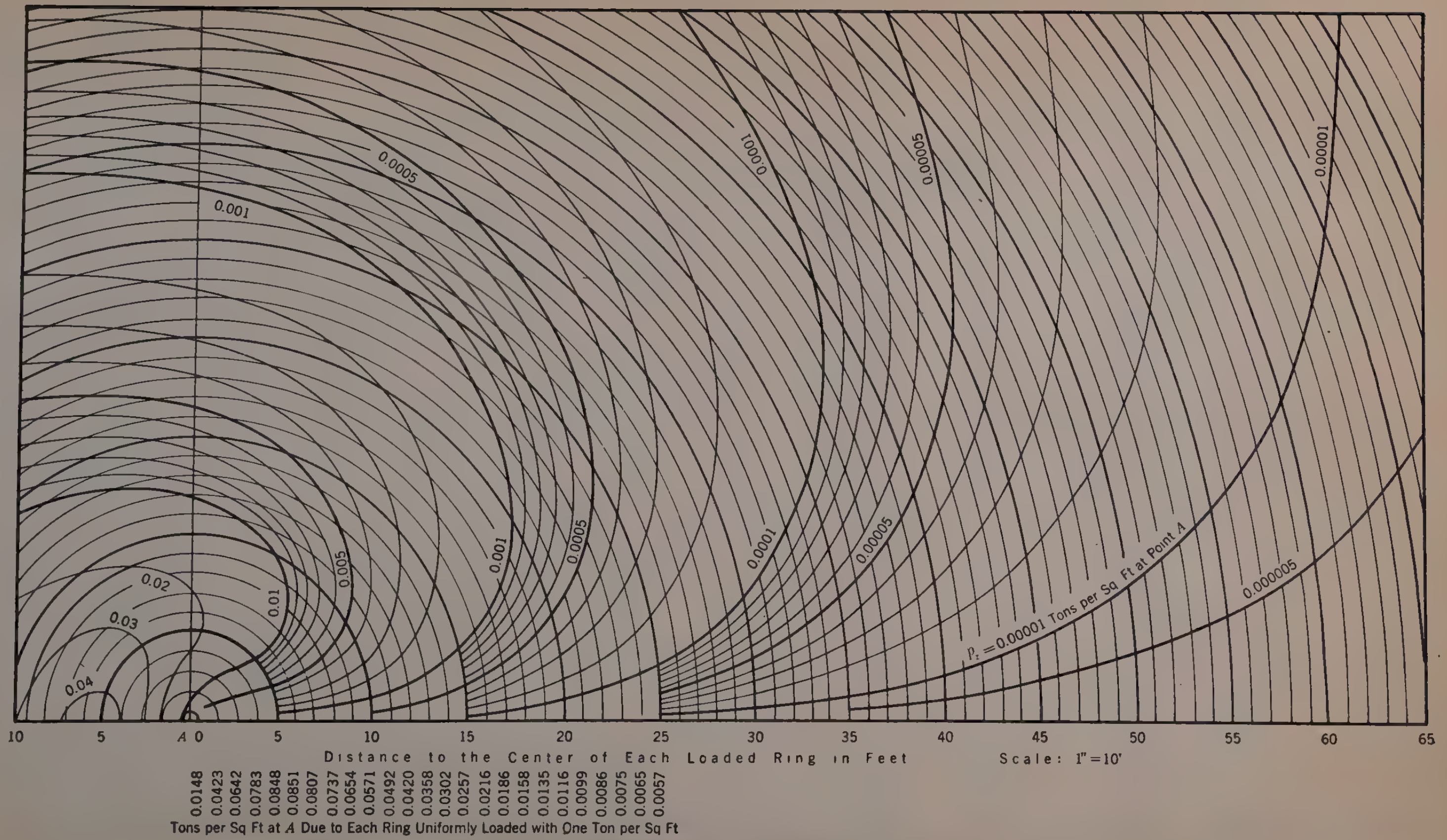

FIG. 17.—PRESSURE CHART FOR $z = 10$ FEET.

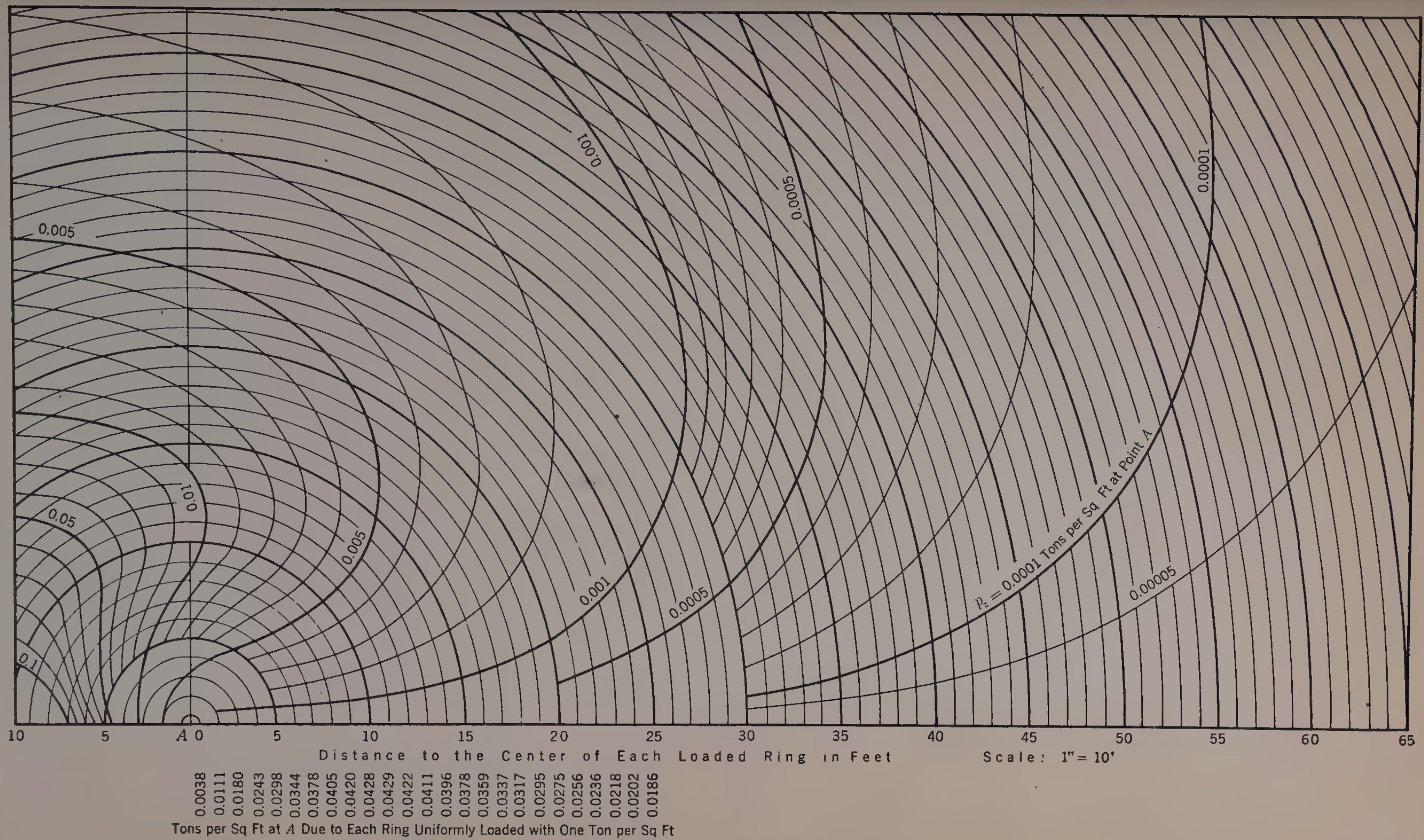

FIG. 18.—PRESSURE CHART FOR $z = 20$ FEET.

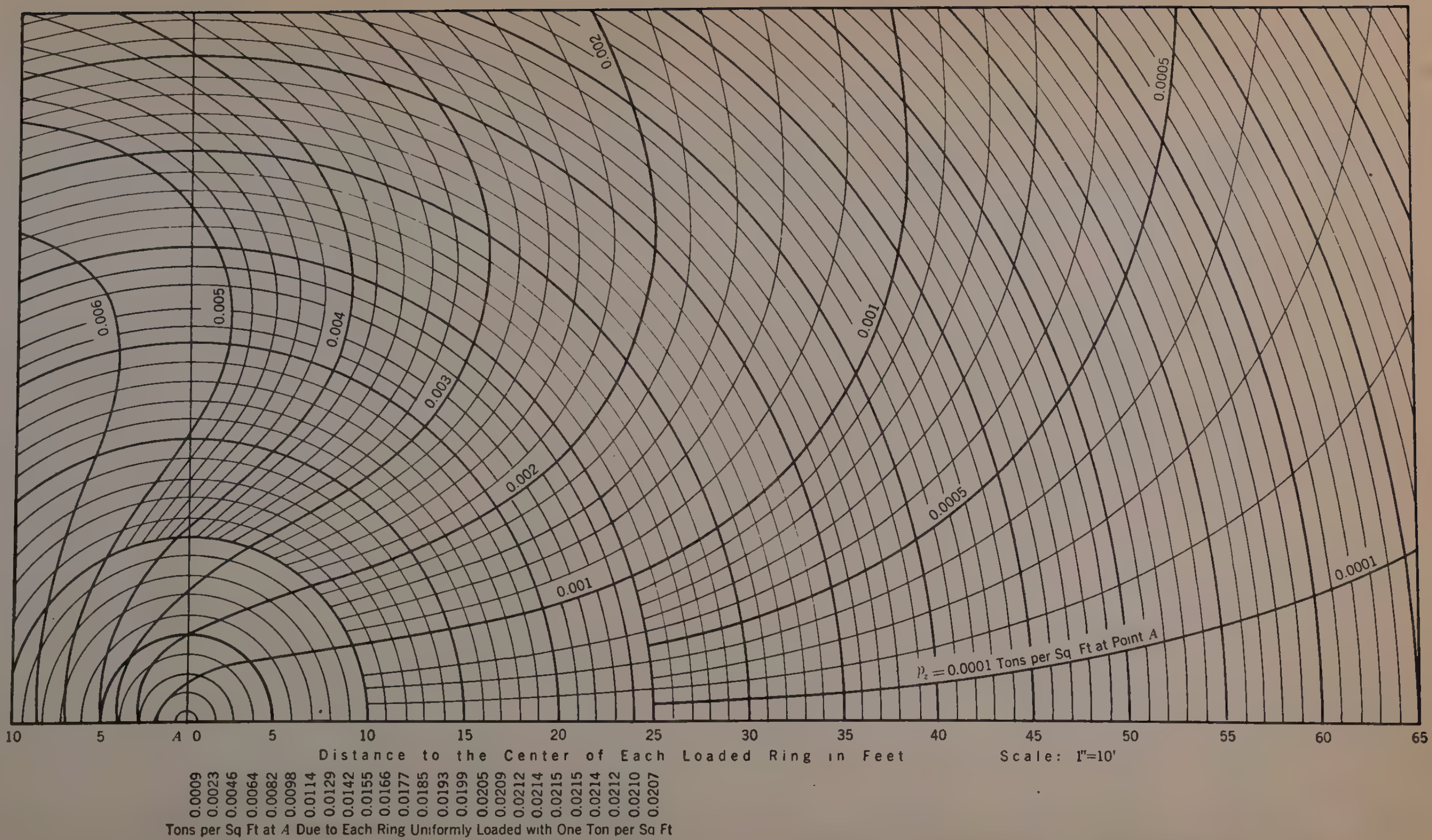

FIG. 19.—PRESSURE CHART FOR $z = 40$ FEET.

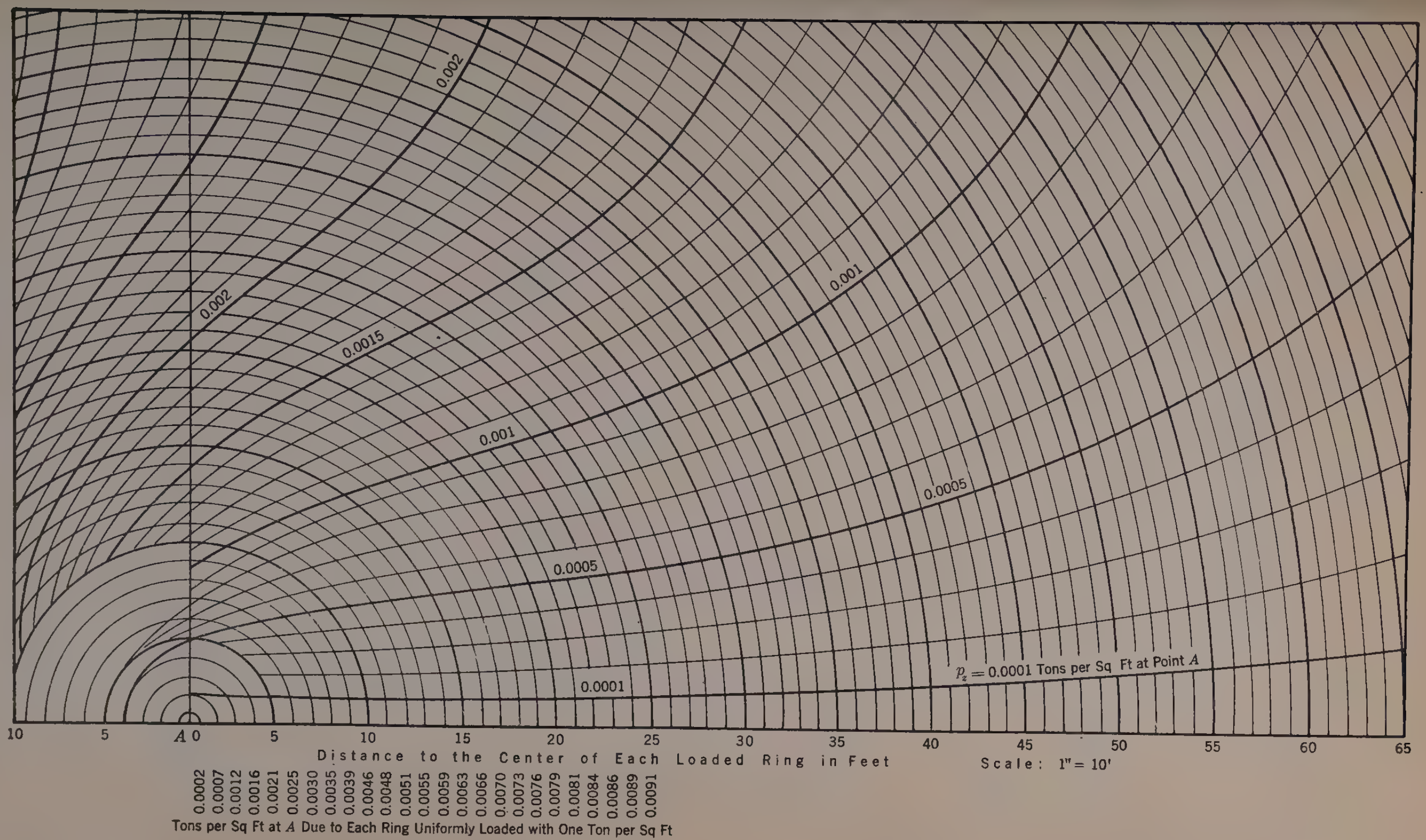

FIG. 20.—PRESSURE CHART FOR $z = 80$ FEET.

of the foundation plan in the ratio of the chart depth to the depth of the plane at which the pressure is desired. However, if the chart for a depth of 10 ft were used for a depth of 100 ft, the individual footings would be too small for an accurate determination of the pressure. This objection can be overcome if the chart nearest to the desired depth is used, provided that the foundation plan can be reduced easily to the odd scale sometimes required. Where the stresses are to be investigated in a relatively thick layer of compressible material, such as clay, it is necessary to know the variation or distribution of stress within the layer, in which case the method outlined in Fig. 4 is most satisfactory. The distribution of pressure can be determined at three levels, obtaining the complete stress picture by interpolation. In this case the charts are used with the foundation plan drawn to the same scale.

Professor Kimball has indicated that the method is equally applicable to a single large area, such as the base of a bridge pier. The settlement of such a rigid foundation is determined by the average pressure acting in the soil beneath the pier. It is interesting to note that the method of taking an equivalent circular area yields practically the same maximum pressure as the chart method.

Mr. Nelidov has extended the usefulness of the charts by developing a practical method for determining the pressure beneath a footing, where the load is not uniformly distributed but varies linearly. Mr. Eremin has suggested a practical and simple method of solving this problem by transforming the area of each footing in accordance with the non-uniform load distribution. These methods may be readily applied to the case of eccentrically loaded structures, such as eccentrically loaded footings, bridge abutments subjected to lateral earth pressure of the fill, etc.

Professor Krynine has suggested a practical improvement and simplification in the procedure, which results in a considerable saving of time. Instead of rotating the foundation plan about Point *A*, the areas of all footings of the foundation are referred to the base line, *AB*, on the foundation plan itself, thereby obtaining an equivalent area, as shown in Fig. 14. The base line, *AB*, of the footing plan is made to coincide with that of each chart in turn, and the readings to the boundary curve of the equivalent area are then added to obtain the total pressure of all footings at the point under consideration. Fig. 3, for $z = 10$ ft, was submitted as a specimen to demonstrate the type of chart recommended. A set, complete enough to serve the needs of practical application in design, is presented in the Appendix.

Appendix.—Charts for Values of z at Various Depths.—Figs. 17 to 20 are pressure charts corresponding to Fig. 3, designed for use in actual practice. The scale reduction has been adjusted purposely to 1 in. = 10 ft.

Corrections for *Transactions:* In Equation (1) change "$\frac{(z)^n}{(R)}$" to "$\left(\frac{z}{R}\right)^n$"; in the denominator of Equation (4) change the first z_2 to z_1; in the caption for Fig. 4, add "Horizontal Linear Scale, 1 Inch = 50 Feet; and, Pressure Scale, 1 Inch = 1.0 Tons per Square Foot"; and, in the over-all heading for Table 3 change "Footings Nos. 1, 2, and 24 to . . . ." to "Footing No. 1 Due to . . . . .";

In closing, the writer wishes to express his thanks to all the discussers for their valuable contributions.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
DISCUSSIONS

FLOOD PROTECTION DATA

PROGRESS REPORT OF THE COMMITTEE

Discussion

By C. S. Jarvis, M. Am. Soc. C. E.

C. S. Jarvis,[18] M. Am. Soc. C. E. (by letter).[18a]—One of the encouraging features of the progress report is the Committee's recognition and definition of some of the main obstacles that have hindered advancement in this field. Other obstacles in addition to those of insufficient data and the human tendency toward unwarranted generalizations enumerated by the Committee, that seem worthy of mention, include the following items:

(1) Lack of most suitable system for summarizing hydrologic data and bringing the most significant items into prominence.

(2) Due to the great expenditure of time and energy required for each user of the basic data to sort, re-arrange, and summarize the data in accordance with his special needs, even a slight step forward entails a strain on the resources of individuals, and properly calls for major group, departmental, or other co-operative action.

One simple device in vogue, for instance, among some published hydrologic records is to use upper case or other distinctive type for recording maximum and minimum quantities, thus facilitating the identification of the extreme values by brief inspection, and doing away with the tedious process of searching long columns, item by item.

There are two widely different approaches to analysis of technical data, which may be compared in many respects with "telescopic" or general definition of limits, trends, and frequencies, and "microscopic" or detailed examination of restricted parts of the field leading up to synthetic and integrating processes. Obviously, there is need for both approaches; they

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Note.—The Progress Report of the Committee on Flood-Protection Data was presented at the Annual Meeting, New York, N. Y., January 20, 1937, and published in March, 1937, *Proceedings*. Discussion on this report has appeared in *Proceedings*, as follows: September, 1937, by Messrs. John C. Hoyt, and H. K. Barrows; and October, 1937, by Robert F. Ewald, M. Am. Soc. C. E.

[18] Hydr. S. C. S., Engr., Washington, D. C.

[18a] Received by the Secretary November 1, 1937.

supplement and reinforce the findings of each other. The investigation of limits, involving the "telescopic" approach, seems to yield satisfactory results regarding extreme phases after a reasonable amount of work. There seems to be no limit for the "microscopic" approach short of the total available hydrologic records; therefore, mature judgment is required for directing research in this field most wisely and most profitably.

Notable advances in facilities for reduction of data to usable form are exemplified in the hydrographic assemblies for the river systems, involving the Neversink,[19] Lehigh,[20] and Mongaup[21] Rivers, have been cited in the publications of the 71st and 72d Congresses. These are tributaries of the Delaware River, and illustrate the use of interpolations and extrapolations for filling out intermittent records of stream flow. Furthermore, a recently tested mechanical integrator for use in flood routing, developed by the Corps of Engineers, U. S. Army, which takes account of both reservoir and valley storage, overflow-spillway discharge, and release through manipulated outlets, seems to hold much promise in that special field.

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[19] H. R. Doc. No. 147, Fig. 13, 72d Cong., First Session.

[20] H. R. Doc. No. 245, Fig. 7, 72d Cong., First Session.

[21] H. R. Doc. No. 660, Fig. 11, 71st Cong., Third Session.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
DISCUSSIONS

NATIONAL ASPECTS OF FLOOD CONTROL A SYMPOSIUM

Discussion

BY MESSRS. W. M. DAWLEY, AND HOWARD M. TURNER

W. M. DAWLEY,[64] M. AM. SOC. C. E. (by letter).[64a]—Under the heading, "Projects in the Ohio Basin," Colonel Covell refers to flood-stages in the Ohio River that attained a gage height of 70 ft (prior to January, 1937), with consequent damage due to the flooding of basements and the lower floors of buildings. There is an Indian tradition of a much higher flood at Cincinnati which, if repeated, would inundate the entire business section of the city, with consequent damages far in excess of those due to the 1937 flood.

In the fall of 1789, Gen. Josiah Harmar sent Maj. John Doughty with a body of troops, and discretionary power, to locate a fort in the Miami country. As a result Fort Washington was built between Third and Fourth Streets produced east of Eastern Row (later Broadway) in Cincinnati, Ohio. Some friendly native Shawnee Indians advised against the location chosen for the site for the fort, stating that it had been flooded by the Ohio River, and designated a point on the hillside to which the waters had risen, which point was at an elevation of 105 ft above the low-water mark.

Since the Indians then were a migratory people, it is not probable that a knowledge of the location of this specific high-water mark would be handed down for more than one or two generations, so this flood must have occurred in the latter part of the Eighteenth Century, possibly coinciding with the high water referred to in the press comments on the 1936 flood in the Allegheny River as being the highest of record, except that of 1763. It oc-

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NOTE.—This Symposium was presented at the Fall Meeting of the Society and at the meeting of the Waterways Division, Pittsburgh, Pa., October 13 and 14, 1936, and published in March, 1937, *Proceedings*. Discussion on this Symposium has appeared in *Proceedings*, as follows: June, 1937, by Messrs. F. C. Scobey, Howard T. Critchlow, T. T. Knappen, M. C. Tyler, Gordon R. Williams, Arthur T. Safford, W. G. Hoyt, J. D. Arthur, Jr., John H. Meursinge, H. K. Barrows, E. D. Hendricks, and Edward W. Bush; September, 1937, by Messrs. H. K. Barrows, Ivan E. Houk, and John E. Field; and October, 1937, by Messrs. C. S. Jarvis and Joseph Jacobs.

[64] Engr., Land and Tax Dept., Erie R. R., Cleveland, Ohio.

[64a] Received by the Secretary October 6, 1937.

curred at a time when the entire drainage area was wooded and uncultivated.

This information was gleaned before 1900 from the papers and personal correspondence of Col. Richard C. Anderson, Principal Surveyor of the Virginia Military District, in Ohio, which were then in the possession of the Rev. R. G. Lewis, of Chillicothe, Ohio. The documentary evidence is not at the moment available, but if it can be verified, it would seem that regulatory measures now proposed would have to be revised.

Howard M. Turner,[65] M. Am. Soc. C. E. (by letter).[65a]—A comprehensive description and general discussion of the 1936 flood is presented in Mr. Uhl's admirable paper. The immediate cause of this great flood in most of the New England States was the storm of March 17 to 19, the run-off from which was superimposed on a very large underlying flow due to melting snow and the run-off from the previous storm of March 12 and 13. A consideration of this particular storm and run-off is thus of interest in analyzing the flood.

The writer questions whether the high rainfall in the second storm (March 17 to 19) covered as large an area as is shown in Fig. 5. The rainfall on the summit of Mt. Washington for these three days was 5 in. and not 8 to 9 in. Any such rainfall as that shown in Fig. 5 would have produced very high flows in the streams draining Mt. Washington to the west. The flow of the Ammonoosuc River, however, was only 56% of that in the 1927 flood. The pattern of isohyetal lines was probably more like that shown for the previous storm and that of 1927. It is even quite possible that the 10-in. rainfall may have covered only a very small area, and the area covered by 8 in., given as 300 sq miles, may have been only a fraction of this, as indicated by the lower rainfall at Randolph, N. H. It would be of great value if a rain-gage at an altitude similar to that at Pinkham Notch could be established on the west side of the Mt. Washington range.

In Fig. 2, showing the rainfall-area relation for several great flood-producing storms in New England, the total rainfall for 13 days in the 1936 storm is compared to other storms of much shorter duration. Considering the storm of March 17 to 19 by itself, it is interesting to note that a plotting of this 3-day storm on the rainfall-area relation chart shows it to be much less than that of the other great storms.

Figures of the total run-off for the entire two-week period of high water from March 12 through March 25 show the enormous quantity of water that flowed down the rivers. The run-off of the concentrated flow during the second peak is also of interest. The writer has made an analysis of this concentrated flood on certain rivers according to the method advanced in the report of the Committee on Floods of the Boston Society of Civil Engineers.[66] This report presented the general theory that the length of the base of the flood hydrograph at a given station for the run-off of a storm within the "concentration period" for that station was constant regardless of the size of the flood, the peak flow thus being proportional to the total flood run-off. Thus, each point on a river has its own flood hydrograph curve which is a measure of its flood characteristics. By this method of analysis the total run-off during a flood is divided into two parts, the "base flow" which is not dependent on the storm rainfall producing the flood, and the "flood run-off" due to the storm rainfall within the concentration period of the basin. From this flood run-off "flood characteristic curves" were developed. These curves were determined from the hydrograph of the flood run-off above the base flow set on a unit basis by dividing the time and quantity of the flow by the square root of the drainage area, and the quantity of flow, in addition, by the number of inches of flood run-off, thus giving a flood hydrograph reduced to a 1-in. flood run-off from 1 sq mile of drainage area. This "flood characteristic curve" is somewhat similar to the "distribution graphs" of the unit hydrograph method of analysis developed by L. K. Sherman,[67] M. Am. Soc. C. E., which express the shape of the flood hydrograph above the base flow in terms of percentage of peak flood run-off, instead of in terms of flow for a 1-in. run-off, and of the actual time instead of time reduced to a unit basis. The base flow is also computed differently, but in cases that the writer has analyzed by both methods the difference is small, and the two curves are very similar.

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[65] Cons. Engr., Boston, Mass.

[65a] Received by the Secretary October 26, 1937.

[66] *Journal*, Boston Soc. of Civ. Engrs., Vol. XVII, No. 7, September, 1930.

The writer has analyzed the 1936 flood on the Connecticut River, at Sunderland, Mass., and on the Merrimack, at Lowell, Mass., comparing it with other floods. Fig. 22(*a*), shows the flood hydrographs of 1913, 1927, and 1933, on the Connecticut River, at Sunderland (8 000 sq miles), and that of 1936 at Montague City, Mass. (7 940 sq miles). Fig. 22(*b*) shows the same flood hydrographs after deducting the base flow in each case. The similarity of the length of the base of the hydrograph for these different floods is clearly shown.

There is some variation possible in the value taken for the base flow. It has been assumed as constant throughout the flood period and, in most cases, equal to the flow existing just before the flood rise began. Usually, this is slightly less than the flow remaining at the end of the flood period, as the flood flow tends to diminish after the peak at a slower rate than it increases to the peak. In some cases, where the run-off before the flood was larger than afterward, it has been taken at somewhat less so as to include the entire flood run-off. A small difference in the quantity of base run-off does not greatly affect the characteristic curve. It may be argued that for the 1936 flood this was not a base flow, strictly speaking, but was a part of the first flood that had not yet run off. This is undoubtedly true, although an examination of the hydrographs and of the rates of diminution of snow on the ground during the period between the two storms seems to indicate that even if the storm of March 17 to 19 had not occurred, the flow prior to that storm would probably have continued high. In this case it would seem to be immaterial just what made up this base flow, as it is about the same before and after the total flood period.

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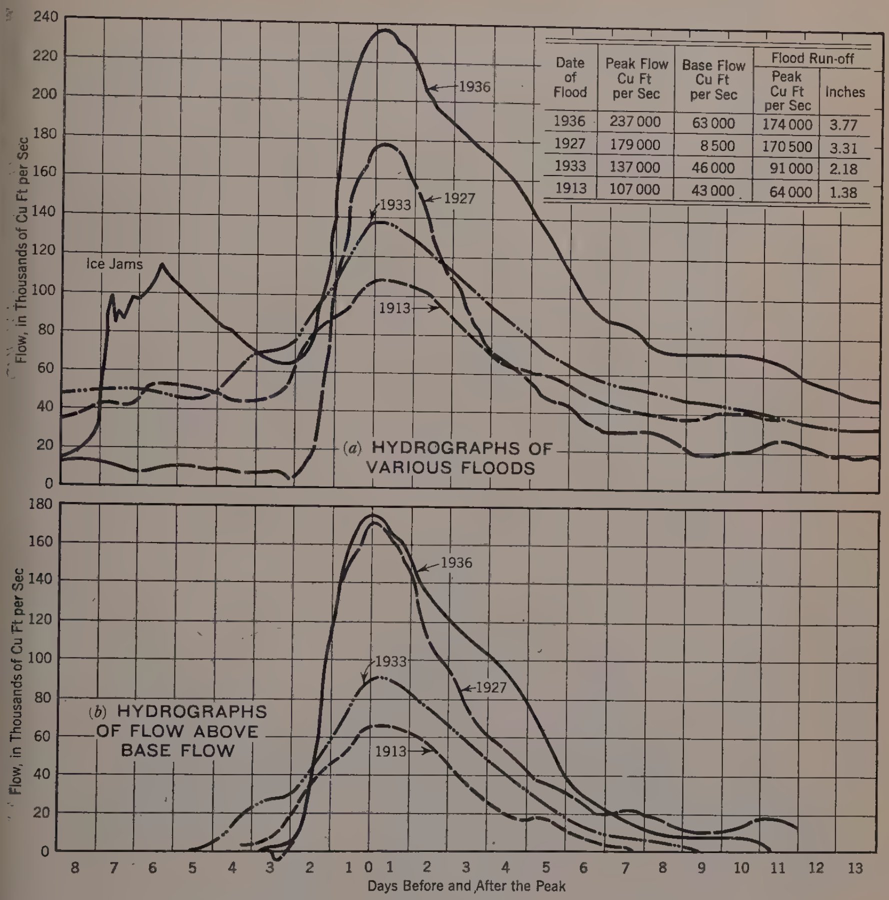
[67] "Stream Flow by the Unit Graph Method," *Engineering News-Record*, Vol. 108, 1932, p. 501; and "Studies of Relations of Rainfall and Run-Off in the United States," *Water Supply Paper 772*, U. S. Geological Survey.

From these "flood run-off" curves the flood characteristic curves shown in Fig. 23(*a*) have been computed. The similarity of these curves for the four floods, with flood run-offs varying from 1.38 in. to 3.77 in., is apparent. That for 1936 is somewhat affected by the additional run-off due to the rainfall of March 21, which shows clearly on the flood hydrograph as a bulge on the curve after the peak. If allowance was made for this condition, the curve would show about the same peak as the curve for 1927.



Date of Flood	Peak Flow Cu Ft per Sec	Base Flow Cu Ft per Sec	Flood Run-off Peak Cu Ft per Sec	Flood Run-off Inches
1936	237 000	63 000	174 000	3.77
1927	179 000	8 500	170 500	3.31
1933	137 000	46 000	91 000	2.18
1913	107 000	43 000	64 000	1.38

FIG. 22.—FLOOD HYDROGRAPHS, CONNECTICUT RIVER AT SUNDERLAND, MASS. (1936, MONTAGUE CITY).

The 1936 flood on the Connecticut River appears, therefore, as a strictly characteristic flood run-off due to the storm of March 17 to 19, including the consequent melting snow caused by it, superimposed on a very large base flow. The latter was probably largely melting snow although in part it was the remainder of the run-off from the previous storm. The flood run-off, 3.77 in., above the base flow was greater than the previous record run-off of 1927 (3.31 in.), and this was added to a base run-off of 8.0 cu ft per sec per sq mile (equivalent to 0.3 in. of run-off per day). The question of what

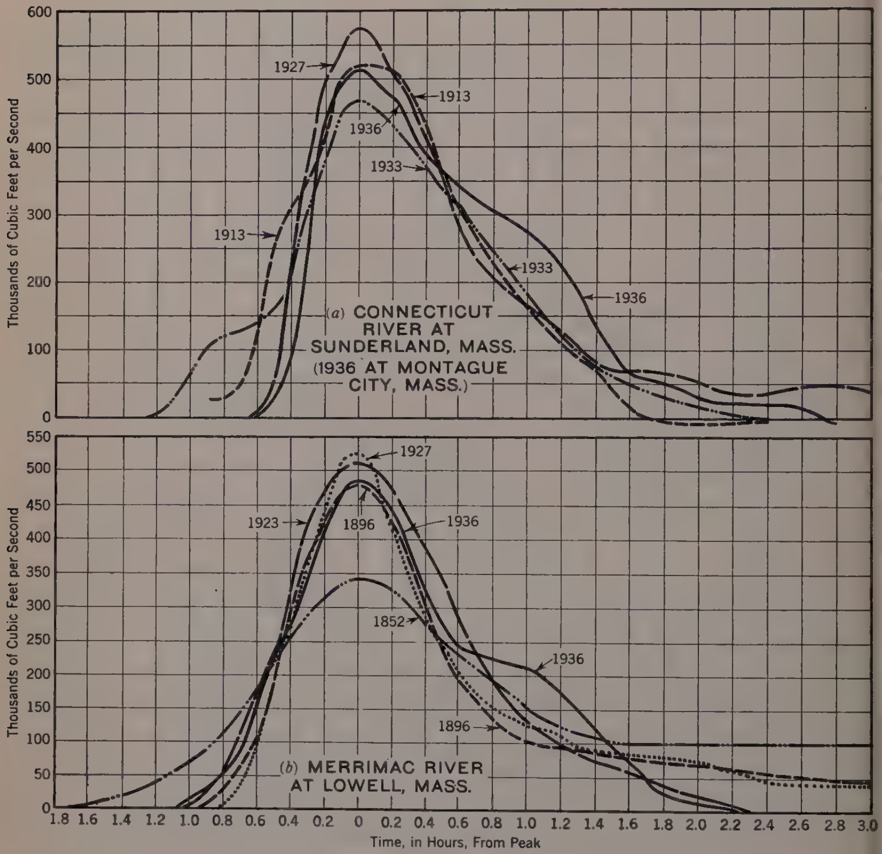


FIG. 23.—CHARACTERISTIC FLOOD CURVES.

is the total run-off including the base flow will depend on the period taken. It seems preferable to express the flood run-off in inches for the total, but the base flow as a rate, continuing as it does before and after the flood period.

Fig. 24(*a*) shows the same hydrographs of various floods at Lowell. These hydrographs are taken from those given by the Committee on Floods of the Boston Society of Civil Engineers,[68] to which has been added the hydrograph of the 1936 flood. Fig. 24(*b*) shows these floods after deducting the base flows. On the Merrimack River the flood run-off above the base flow was greater than any previously recorded flood (1852) by 17%, and 40% greater than any previous recorded flood from a storm within the concentration period (1896). Fig. 23(*b*) shows the flood characteristic curves also taken from the report[69] of the Committee on Floods of the Boston Society of Civil Engineers, re-arranged so that the peaks come at the same time and with the 1936 flood added.

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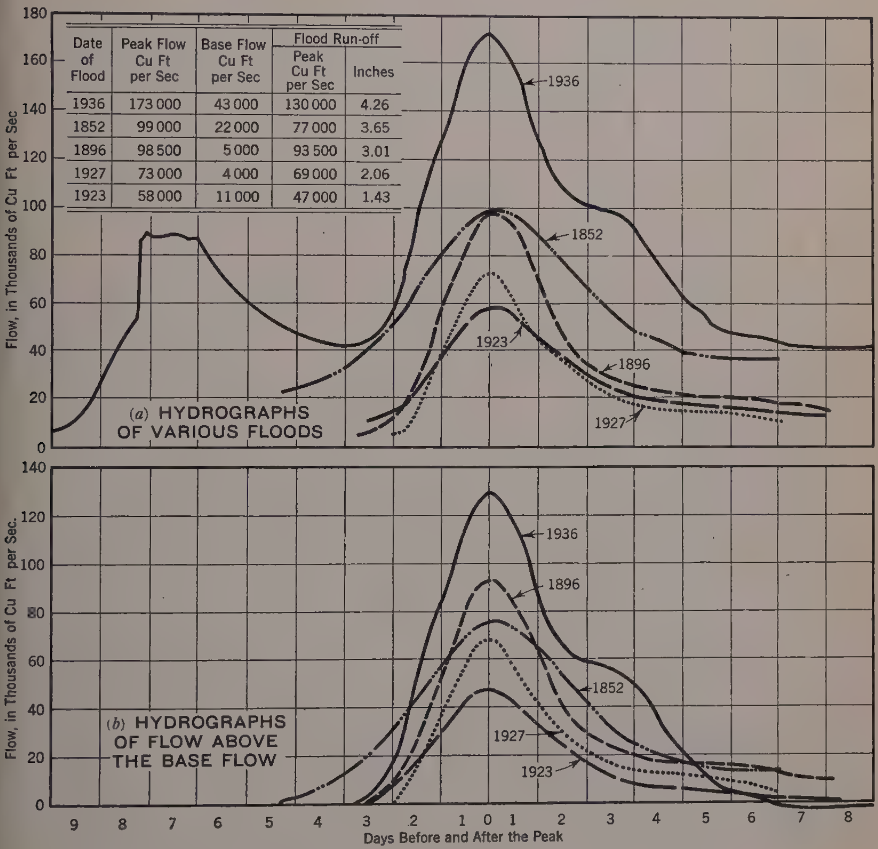
[68] *Journal,* Boston Soc. of Civ. Engrs., Vol. XVII, No. 7, September, 1930, Fig. 16, p. 384.



Date of Flood	Peak Flow Cu Ft per Sec	Base Flow Cu Ft per Sec	Flood Run-off	
			Peak Cu Ft per Sec	Inches
1936	173 000	43 000	130 000	4.26
1852	99 000	22 000	77 000	3.65
1896	98 500	5 000	93 500	3.01
1927	73 000	4 000	69 000	2.06
1923	58 000	11 000	47 000	1.43

Fig. 24.—Flood Hydrographs of Merrimack River, at Lowell, Mass.

Except for the flood characteristic curve of 1852, which apparently shows the effect of a storm considerably longer than the concentration period, the same similarity exists and the flood of 1936 presents the same characteristic. In this case a flood run-off of 4.26 in., in itself much greater than any recorded previous flood, was superimposed on the high base flow of 10.8 cu ft per sec per sq mile, equivalent to 0.4 in. per day of run-off.

Mr. Uhl notes the fact that the 1936 flood generally was relatively less severe on the small drainage areas than that of 1927. Excluding differences in rainfall this was generally due to two factors: First, the storm of three days (March 17 to 19) was longer than the concentration period of many of the smaller streams; and, second, the base flow was a small part of what may be expected for a flood on a small stream; 8 to 10 cu ft per sec per sq mile is not a large flow for a small stream, but on the larger rivers it is in itself of flood magnitude. The base flow of the Connecticut River, at Sunderland (63 000 cu ft per sec) was larger than 27% of the annual 24-hr flood flows in the last 55 yr.

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[69] *Journal*, Boston Soc. of Civ. Engrs., Vol. XVII, No. 7, September, 1930, Fig. 18, p. 394.

With a base flow of flood magnitude and a superimposed flood run-off greater than any on record, it is no wonder that the 1936 flood exceeded previous records by large quantities on many New England rivers.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
DISCUSSIONS

PRESSURES BENEATH A SPREAD FOUNDATION

Discussion

By M. M. Buisson, Esq.

M. M. Buisson,[38] Esq. (by letter).[38a]—The two quite distinct parts into which this paper is divided, are: (I) The description of a graphical method for computing the stresses in soils; and (II) a study of stresses that develop under a rigid foundation.

Part I.—The Method.—In some cases, it has been possible to derive analytical formulas giving the values for horizontal and vertical normal pressures and for principal stresses at any point in a half space. By dividing the loaded area into rectangular, or square, elements, the vertical pressure can be computed easily, as, for instance, the method developed by Dr. Wilhelm Steinbrenner.[39] In some cases, Professor Krynine's method will be an easier one for the solution of similar problems, taking the value of the concentration factor, n, into account, with sufficient accuracy. This is an application of a graphic integration method, which has been well known for many years. An error was made by the author in computing p_h in the case of a two-dimensional problem. In fact, under Michell's hypothesis[5] the ellipsoid of elementary stresses is reduced to a straight line. It is not so when there are many elementary stresses because then they are not all acting in the same direction. The simplified method expressed in terms of $p_h = s - p_z$, may rightly be used when individual elementary stresses are concerned; but it does not enable the resultant stresses to be determined. This computation can only be done by integration, which, incidentally, is not more complicated than the integration that involves p_z.

NOTE.—The paper by D. P. Krynine, M. Am. Soc. C. E., was published in April, 1937, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: September, 1937, by Messrs. O. K. Fröhlich, Donald W. Taylor, and Jacob Feld; October, 1937, by Messrs. G. P. Tschebotareff, and A. Hrennikoff; and November, 1937, by Messrs. Robert G. Hennes, T. A. Middlebrooks, and A. A. Eremin.

[38] Chef du Service CCI, Bureau Veritas, Paris, France.

[38a] Received by the Secretary October 26, 1937.

[39] *Die Strasse*, October, 1934; also a special publication by that Journal entitled "Bodenmechanik und neuzeitlicher Strassenbau," p. 75 (1936).

[5] *Proceedings*, London Math. Soc., Vol. 32, p. 35 (1900).

The foregoing considerations apply also to the computation of the shear stresses, s_s. As the values are:

$$p_h = \frac{p\, n_1}{z\, \pi} \sum \left[dA \cos^{n-1} \alpha \sin^2 \alpha\right] \dots\dots\dots\dots\dots (40)$$

and,

$$s_s = \frac{p\, n_1}{\pi\, z} \sum \left[dA \cos^n \alpha \sin \alpha\right] \dots\dots\dots\dots\dots (41)$$

the forms are quite similar and the integration may be done for s_s in the same manner as for p_z. From Equations (40) and (41) one may compute the principal stresses, s_1 and s_2, from the well-known relations:

$$s_1 = \tfrac{1}{2} \left(p_h + p_z + \sqrt{(p_z - p_h)^2 + 4\, s_s^2}\right) \dots\dots\dots\dots (42a)$$

and,

$$s_2 = \tfrac{1}{2} \left(p_h + p_z - \sqrt{(p_z - p_h)^2 + 4\, s_s^2}\right) \dots\dots\dots\dots (42b)$$

When the values for p_z, s_s, and s_1 are known, Mohr's construction enables one to determine the angle that the principal stress, s_1, makes with a vertical. Thus, using this method, it is easy to solve for the principal stress trajectories.

Moreover, since failure trajectories are isoclinic lines that meet the principal stress trajectories at an angle, $\frac{\pi}{4} - \frac{\phi}{2}$, one may draw easily such failure lines, and thus determine the failure conditions in cases where a purely analytical solution of the problem would not be possible.

In two-dimensional cases, if a semi-infinite plane is loaded and if $n = 3$, the failure lines are parabolas with their foci at the half-plane limit. The axis of the parabola is a straight line, inclined to the vertical by an angle equal to the friction angle. In the case of an infinite loaded strip, it is known that the principal stress trajectories are hyperbolas with their foci at the edges of the loading strip. From this knowledge, failure trajectories can be drawn easily.

Failure would occur if the equilibrium of a certain part of the loaded body should become unstable. This condition will decide the position of extreme failure trajectories and also the loading conditions corresponding to an actual failure along these lines. When it is possible to determine the form of the plastic zones analytically—and, above all, to locate the point of intersection of the marginal plastic zone with the loaded surface—(the abscissa of this point is a function of the applied load)—one may locate the extreme failure trajectory also, in an analytical manner, since this line must necessarily meet the aforementioned point of intersection.

In a particular case, a uniformly loaded strip would show, contrary to expectation, that the extreme plastic-zone lines are not in themselves failure lines; this indicates, incidentally, that the bases of Prandtl's computation are not exact.

Furthermore, if the stress distribution can be studied analytically, it is always possible, to choose from a set of possible failure trajectories, one along which actual failure conditions will be attained. This can be done graphically using a method similar to that applied in determining the slopes of earth dams. Without any difficulty, one may compute the load necessary to produce such a critical instability, whether the loading is rigid or semi-rigid.

Following the same general trend of thought, the writer would attach great importance to the systematic determination of failure conditions in most of the common loading cases. This can be done easily by means of Professor Krynine's graphical method.

On the other hand, actual tests have shown[40] that if an excessive load is applied to an earth mass, and both the earth mass and the superstructure fail, the failure trajectories in both bodies in contact are always prolongations of each other. They do not possess a common tangent at the boundary, however, but will meet at an angle, unless the angles of internal friction are the same in the soil and in the superstructure. It is worthy of note that, in cases where it was possible to trace lines for these failure trajectories, with sufficient accuracy, they were almost the same as the theoretical lines, corresponding to $n = 3$.

Part II.—The Stresses.—Concerning Part II of the paper, which relates to the study of the stresses that would develop under a rigid foundation, the writer would insist, urgently, on the great necessity of making actual tests and measurements on the foundations themselves.

Numerous important factors which are not yet completely understood are then in operation. First, the basic hypothesis must be checked, despite the opinion of several engineering writers to the contrary. Only by actual testing or observation can one obtain reliable results. It is difficult indeed to determine, even approximately, the mutual reactions of both bodies in contact; that is, the superstructure and the supporting soil. Many results seem contradictory only because due attention has not been paid to an analysis of peculiar cases. It was Professor Fröhlich who studied the rare tests existing at the time and compared them, case by case, with the theoretical results obtained by Boussinesq.

In this manner he arrived at the most interesting conception of the "concentration factor," n, thus providing a means of generalizing the formulas for the Boussinesq stress distribution.

Experimental tests and measures, however, are very difficult to carry to a successful conclusion, the principal reason being the presence of important, although secondary, phenomena. In general, pressures are measured by using Goldbeck cells. For small-scale models, it seems better to resort to small manometric capsules, closed in by a rubber diaphragm. It is necessary to conduct the tests in such a manner that the diaphragm cannot become displaced; otherwise, the readings will be affected. For instance, if the movable diaphragm had been pressed into the ground and if, subsequently, one wanted to return it to its original place, the necessary force is not at all in proportion with the actual pressure at the capsule point. Mr. Leo Rendulic[41] made use of a similar method for the measurement of hydrostatic pressures that are developed by various principal stresses in soil cylinder samples.

Actually, this force will be much greater than the pressure caused by any arching effect that may develop in the soil. If sufficient accuracy is to be attained, one must press the diaphragm beyond its definite stationary position and then release the force gradually, noting the pressure at the very moment when the initial position has been reached.

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[40] *Annales des travaux Publics de Belgique.*

[41] "Spannungszustände in der Umgebung eines Hohlraumes," by Leo Rendulic, *Die Wasserwirtschaft*, Vol. 27, June, July, and August, 1934.

Nevertheless, even when such precautions are taken, errors can be made. The best way would be to mark a capillary zone on a tube, and then to make use of a tank filled with air under compression. The pressure could be exerted by water, for instance, as the measured pressures will be always low. If the air compression is kept constant, the meniscus will stay at the mark, and then the measured pressure will be the actual pressure in the soil.

With such apparatus in operation, one could detect such stress variations as those indicated by Professor Krynine. It is normal that such variations should occur because after loading the soil, its properties will be altered locally. If, previous to tests, the soil behaved as an isotropic and homogeneous body, it will no longer be so after the tests due, among other causes, to unequal settlements. It is obvious that factors for isotropic properties always express extreme conditions that are never absolutely obtained in actual soils.

Tests will show changes of two different types: (*a*) Rapid stress variations; and (*b*), slow changes in stress alterations.

The rapid variations will transform the curves (which appear continuous in Fig. 13 of the paper) into saw-toothed lines with maxima and minima. With the passage of time the difference between these maxima and minima will decrease. The curves in question reflect the process of re-grouping of soil particles. The slow changes are a result of the adjustment of the soil structure to the external load.

In testing some models of wharf retaining walls the writer used apparatus such as the aforementioned to extend the research to other tests more especially intended to study the form of failure curves, and to determine the conditions under which failure occurred. A certain number of capsules were placed in the soil, more or less at random and only at the end of the tests could a satisfactory experimental procedure be devised. As new tests concerning this question are being conducted, the writer is not yet in a position to furnish definite results.

The original tests were performed in a large basin, 3.93 ft wide, 3.28 ft deep, and 11.5 ft long. Such a basin may be flooded or emptied, at will. A glass panel on one side permitted photographs to be taken which indicate, with the profile of the quay model, the particles in the soil.

By fixing the photographic apparatus in a definite steady position many pictures could be taken at various loading stages. By superposing such pictures taken at different times, one could easily trace (very approximately) the trajectories of movement in the soil particles as well as displacements of the wall itself; thus, the actual failure curves were derived in quite a satisfactory manner. Capsules were placed in the soil at different levels, as the soil was placed in the basin. Datum marks were fixed to measure vertical movements at the soil surface, as well as movements of the loading sheets, and of the quay model. These movements were measured by dials with an accuracy that could easily reveal 0.005 mm (= 0.002 in.).

Some test results were recorded during a loading test on a relatively long strip of homogeneous clay, 10 cm (3.94 in.) wide. The load was applied through rigid plates placed end to end along the entire width of the basin. A layer of fine sand, about $\frac{3}{8}$ in. thick, was laid between the plates and the clay, which provided for drainage through the soil immediately beneath the plates. Many difficulties had to be overcome:

(1) In some places, the pressure differences that were recorded during the application of the load were negative instead of positive as one would have expected. Such phenomena seem to be due to suction that occurs near the capsule, from the flowing clay. The capsules were practically fixed by seating them in the soil mass. It follows, therefore, that such readings cannot yield precise information as to stress distribution.

(2) To date (1937) test measurements have been made only at, or near, the critical loads, when the p_h-value (as recorded by a vertical capsule set 10 cm deep and 10 cm from the plates) is practically the compressive resistance of the same clay as that measured on cylinder samples. This value would be greater only if the material could consolidate.

(3) One may overload the soil beyond the critical loads obtained by rapid application, if the sample is permitted to consolidate after each partial loading. In a particular test, clay possessing a moisture content responding to the Atterberg liquid limit, offered a shearing resistance of 10 gram per sq cm (20.4 lb per sq ft). Failure under a rapid rate of loading took place under the action of a weight of 60 grams per sq cm (122.6 lb per sq ft). In a slow test process a load amounting to more than 120 grams per sq cm (245.3 lb per sq ft) would be required for the same purpose.

(4) Although the following statement bears only a distant relation to stress redistribution under rigid plates, it is useful to emphasize that, as the load approaches its critical value, one detects a considerable increase in the permeability. This is accompanied by a large and almost immediate settlement under the plate and, at the same time, a settlement occurs to the right and left of the plate. Thus, the extent of consolidation settlement beyond the plate, is much greater than the swelling that should have occurred as computed from the instantaneous settlement under the plate and from the constant-volume soil deformation.

Long before failure, however, one can note that instantaneous consolidation is occurring to a large extent, as the uplifted volume is practically nil, although the settlement under the plate is rather large; this is absolutely contrary to results derived from measurements of the permeability coefficient determined in the usual manner.

Observations on the movements of soil surfaces show that the flow influence varies with the distance to the plate; for instance, at a distance of 3 cm (1.18 in.) the maximum settlement was noted about 150 hr after the loading was completed, although at 20 cm and 30 cm (7.87 in. and 11.81 in.) such a maximum was recorded 70 hr after loading; the next swelling to occur was stabilized 200 hr after loading. One may conclude that the length of time necessary for flow stabilization in clay is almost equal to, or longer than, the time necessary for consolidation itself. Indeed, during the tests, the consolidation was not yet complete as the plate continued to settle.

The form of the settlement curve was readily adapted to logarithmic plotting, settlements being in proportion to logarithms of time. Thus, in this testing time, the settlements were due at once both to flowing and to consolidation processes, the effects being equal but opposite beyond the plate. After a certain time had elapsed the importance of the continuous flowing process eclipsed the consolidation noticeably.

The volume of soil raised on both sides of the plate was then equal to the product of the depth of settlement by the area of the plate. In this case, again, the settlement-to-time relationship was adaptable to logarithmic plotting. From the foregoing considerations, it seems that the logarithmic law, as given by Professor Buisman[42] for long-time settlements in eudiometric tests, would apply also to settlements from plastic flow.

Conclusions.—Research in the United States should be directed toward an extension of the ideas herein described, in order to obtain true information as to the behavior of the soil under foundations. This would not minimize, in the least, the merits of Professor Krynine's paper; but it is necessary in this connection to emphasize the urgent need of an experimental verification of theoretical results. At present, experimental data are much less available than theoretical research. It seems timely to minimize the importance of the latter and to strive for a rational balance with the importance of confirmatory tests.

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[42] "Results of Long Duration Settlement Tests," by A. S. Keverling Buisman, *Proceedings,* International Conference on Soil Mechanics and Foundation Engineering, Vol. I (1936), p. 103 (Paper F–7).

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
DISCUSSIONS

EARTHQUAKE RESISTANCE OF ELEVATED WATER-TANKS

Discussion

BY FRANKLIN P. ULRICH, M. AM. SOC. C. E., AND
DEAN S. CARDER, ESQ.

FRANKLIN P. ULRICH,[15] ASSOC. M. AM. SOC. C. E., AND DEAN S. CARDER,[16] Esq. (by letter).[16a]—In mentioning structural characteristics and the superficial nature of past earthquake design, Professor Ruge has touched upon a subject which is of considerable interest and for which a large amount of observational data has recently been obtained.

The strong-motion records obtained by the U. S. Coast and Geodetic Survey show the complexity of earthquake motions. The Los Angeles record of the Long Beach shock (used by Professor Ruge) shows periods ranging from 0.12 to 4.8 sec with only a few periods greater than 2.5 sec.[17] Only a few periods were of the same order as the fundamental period of this particular tank which was later measured and found to be about 1.5 sec.

The Coast and Geodetic Survey has made vibration observations on nineteen selected steel water tanks having capacities of 50 000, 75 000, and 100 000 gal, and an elevation of 100 ft above the ground. Vibration observations on eighteen other tanks, including the prototype of Professor Ruge's models, have also been made.[18] For the most part, however, this discussion will be confined to the nineteen tanks selected for comparison at the time the tests were made. All the tanks were filled or nearly filled with water.

NOTE.—The paper by Arthur C. Ruge, Assoc. M. Am. Soc. C. E., was published in May, 1937, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: October, 1937, by Messrs. N. H. Heck, Mason A. Stone, and H. C. Boardman.

[15] Chf., Seismological Field Survey, U. S. Coast and Geodetic Survey, San Francisco, Calif.

[16] Asst. Observer, U. S. Coast and Geodetic Survey, San Francisco, Calif.

[16a] Received by the Secretary October 20, 1937.

[17] "Destructive Earthquake Motions Measured for First Time," by N. H. Heck and Frank Neuman, *Engineering News-Record*, June 22, 1933.

[18] "Earthquake Investigations in California, 1934–1935," by U. S. Coast and Geodetic Survey, *Special Publication No. 201*, p. 75; and "Observed Vibrations of Steel Water Towers," by D. S. Carder, *Bulletin*, Seismological Soc. of America, Vol. 26, No. 1, January, 1936, p. 69.

Three orders of periods of vibration have been recognized:

(1) The most important type of motion is probably translational vibration which has periods ranging from 0.95 to 1.75 sec depending on the effective water load, the soil conditions, and the elastic properties of the tower. Other things being equal, towers reinforced from 0.08 to 0.15 *g* for side loads have periods 10% to 30% less than standard towers designed to withstand a 100-mile wind. Towers on piling in yielding soil have periods as high as 20% greater than similar towers on firm ground.

The range of periods of translatory vibration lies within the frequency range of some of the possibly destructive earth waves as observed on accelerograms of the Long Beach earthquake. It is to be understood that observed periods of these tanks apply to low-amplitude vibrations. On the tanks where some of the tie-rods went out of action or failed during the earthquake, the periods undoubtedly became variable and much longer, according to Harry A. Williams,[19] Assoc. M. Am. Soc. C. E., perhaps in sufficient amounts to throw the structure out of resonance with the earth. Broken or stretched tie-rods would further serve temporarily to damp the structure. On the other hand, if the rigidity of a tank tower is to be increased, the purpose should be to strengthen the structure rather than to decrease its period, because destructive earth waves have been known to have periods less than 1 or 1.5 sec. For instance, the highest acceleration registered during the October 31, 1935, earthquake at Helena, Mont., was associated with periods of about 0.15 sec, even though a wave with a 1-sec period was also in evidence. At Long Beach, waves having a 0.3-sec period contained the highest acceleration.[17]

(2) Rotary or torsional vibration has been recorded on all tanks of this group. Observed periods of this type of motion usually range from 0.30 to 0.45 sec on standard structures, and from 0.20 to 0.30 sec on towers with an 0.08 *g* to 0.15 *g* side-load design. The magnitude of this period is dependent on the elastic properties and the steel load of the tower and perhaps also on the type of soil. It is independent of the water load because the water within the tank does not partake in this type of vibration.

Torsional vibrations of water tanks should not be overlooked in experimental procedure because it is quite possible that large torsional motions may be set up in resonance with destructive earth motions having the same frequency range.

(3) A period of the order 2.7 sec, assigned by Professor Ruge to swinging motion of water within the tank, has been recognized on several tank-tower vibrograms. Observed periods are 2.63 sec on one 50 000-gal tank, 3.0 sec on two 75 000-gal tanks, and 2.85 to 3.0 sec on five 100 000-gal tanks. One of the 100 000-gal tanks which had bracing to withstand an 0.08 *g* side load, had a period of this order, the same as similar standard tanks, supporting the belief that this period is due to water motion in the tank. However, the observed periods on tanks having different capacities cannot

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[19] "Dynamic Distortions in Structures Subjected to Sudden Earth Shocks," by Harry A. Williams, *Transactions*, Am. Soc. C. E., Vol. 102 (1937), p. 838.

be explained satisfactorily by simple gravitative oscillations of water within the container. Furthermore, a supposition that water in the tank re-acts upon the tower in such a way as to produce a period approximating the translational period of the tower has considerable experimental support.

Strengthening a tank, primarily, to reduce its period of vibration is not the entire solution to making the tank more resistant to earthquakes. It is quite possible to have the reduced periods of the tank equally as destructive, or more so, than the original periods, especially if the reduced period happens to be close to one of the destructive ground periods.

It might be possible to avoid resonance between ground and tank, if the tank had a definite fixed period and if the destructive ground periods for that particular location are known. To date (1937) information on destructive ground periods is too meager to form any definite theory. Observations made by the Coast and Geodetic Survey show that tank periods fluctuate with different types of ground and with changing tank conditions, such as depth of water in the tank or looseness of tie-rods. Tests on one tank showed that the period varied from about 0.36 sec for an empty tank to about 1.0 sec for a filled tank. In another case the period was 1.44 sec with tight tie-rods and 1.66 sec, or more, with loose tie-rods.

The question of merits or objections of spring elements, which Professor Ruge proposes to introduce in the tie-rods, is a structural problem and is beyond the scope of this discussion, but there is little room for doubt that a provision for some kind of damping will reduce, materially, the earthquake hazard of any structure. That standard elevated water tanks are poorly damped has been demonstrated on several of these structures on which a side force of 2 000 to 4 000 lb was suddenly released and the resulting vibrations recorded at the platform of the tank. The ratio of two successive amplitudes (damping ratio) of towers with taut tie-rods is never more than 1.02. One tank tested in this manner had loose tie-rods and the damping ratio was higher—about 1.1. The translatory period of the latter tank at first was 1.74 sec which decreased to 1.65 sec as the amplitudes became smaller. A minute after the release of the side force, vibrations due to elastic properties of the tower became subordinate, and a motion with a period of 2.85 sec was dominant for several minutes until the end of the record. If the idea is correct that this longer period is contributed by swinging action of water in the tank, it appears that this type of motion continued long after structural vibrations had diminished to normal amplitudes. Later, the tie-rods of the tower were tightened and the tests were repeated. The period was then invariable at 1.45 sec and structural vibrations disappeared much more slowly, and the longer 2.85-sec period was not recognized on the record.

The statements in the preceding paragraph should not be interpreted as an argument in favor of loose tie-rods in existing structures, but it does seem that some method of breaking up the high resilience of these structures, without destroying their resistance to dynamic side loads, is desirable.

It is possible that Professor Ruge's idea of spring elements, with damping, is a solution.

It is unfortunate that more earthquake data are not available and that Professor Ruge was unable to continue his tests under a wider range of conditions. However, additional tests would probably show results comparable to this series. The errors of small models cannot be avoided and no doubt they are within the limits of error of present-day structural science. It is hoped that several full-sized tanks, embodying the spring design, can be constructed so that comparisons can be made with the present design, and their relative capabilities to withstand earthquakes can be determined.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
DISCUSSIONS

EFFECT OF DOWEL-BAR MISALIGNMENT ACROSS CONCRETE PAVEMENT JOINTS

Discussion

By Messrs. W. O. Fremont, and George A. Smith

W. O. Fremont,[12] M. Am. Soc. C. E. (by letter).[12a]—The type of failures described in this paper depends to a great extent on the direct action of the bar on the concrete in the vicinity of the dowel. This action depends on the bending moment, shear force, and longitudinal force in the dowel. Similar functions of the outer forces are produced in the dowels also by vertical loads as well as by the curling and warping of the slabs due to changes of moisture and temperature. Therefore, the effects of the opening or closing of the joint must be studied in conjunction with the effects of all the other factors. If the concrete fails at a certain amount of opening or closing of the joint, it does not mean that it will not fail at a much smaller opening or closing when the vertical load and curling, with warping, are acting in addition.

Conclusions drawn on the basis of joint opening and closing tests alone, therefore, without at least the vertical forces, seem insufficient, if not useless, for practical purposes. It is difficult to believe that the small specimens, cut from the entire pavement construction, truly represent or simulate the actual service conditions. The rotation of the slabs mentioned under the heading, "Permissible Error Determined by Experiment," will be quite different under actual service conditions. The distribution and magnitude of pressure on the sub-grade from the joint-opening forces, or by the closing forces alone, will also be different in actual large pavement slabs. Therefore, the action of the dowels will also be quite different.

The authors state that the dowel bars were coated with asphalt paint and oil. No information is given about the thickness of the coats applied and other details. From theoretical considerations it seems that errors

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Note.—The paper by Arthur R. Smith, M. Am. Soc. C. E., and Sanford W. Benham, Assoc. M. Am. Soc. C. E., was published in June, 1937, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: October, 1937, by Messrs. L. W. Teller, David J. Peery, and L. J. Mensch.

[12] Engr. in Chg. of Pavement Joint Research, State Highway Dept., Ann Arbor, Mich.: formerly Prof. of Eng. Mechanics, Russian Polytechnic Inst., Harbin, China.

[12a] Received by the Secretary October 2, 1937.

in the alignment of the dowels of the magnitude used in the tests are permissible only because of the clearances between the dowels and the concrete formed by the coating applied. It seems that no exact definition is given as to what is understood in Table 1 under "Dowel-Bar Movements."

In the last paragraph of the paper the authors recommend that "common 24-in. dowel bars be placed with a degree of accuracy such that the error of no one bar in an entire installation shall exceed 1 in." This conflicts with the statement a few paragraphs preceding it, that "errors of 1 in. caused the slabs to crack and spall." According to the authors it required a tension force of 3 000 lb per dowel to open a contraction joint to a width of 0.5 in., which means a tensile stress of 50 lb per sq in. in the concrete slab 5 in. thick. Considering the high tensile stresses in the concrete slab from vertical loads, an addition of 50 lb per sq in. is appreciable inasmuch as this addition might become larger in the actual pavement slabs farther away from the joint, because of the friction between slabs and sub-grade.

George A. Smith,[13] Esq. (by letter).[13a]—Those interested in the design and construction of concrete highways should appreciate the information contained in the paper by Messrs. Smith and Benham and should be grateful to learn that relatively large errors in the alignment of dowels in joints in concrete pavements may not contribute materially to failures at contraction and expansion joints. The design and construction of joints in concrete highways have probably been given as much consideration as any other single factor relating to such work and, although the particular tests reported relate to a single phase of the subject, their findings certainly answer a moot question.

A review of this paper cannot fail to leave the impression that the work involved was ably conducted and that the results, although general, were presented in a clear, concise, and accurate manner. The paper offers little, if any, grounds for critical comment, but does prompt the asking of several questions which, should the studies be continued and the investigation extended, may be worthy of some consideration.

The authors indicate that, when the misalignment of dowel bars is not in excess of 1 in., no particularly serious results need be anticipated. This was based on conditions obtaining after ten cycles of opening and closing, or closing and opening, of the joints. It would be of interest to know whether there was any progressive destructive action noted during the tests, either when the error was small or when large errors existed. There is also the question of what would have happened had the number of cycles been such as would compare with the number that would occur during the life of the pavement.

From an analysis of conditions that exist when the error in alignment is large, it appears that two types of failure may occur. For a vertical misalignment the failure appears to be in tension, whereas a shear failure, the result of a compressive stress, is indicated when a horizontal error exists. Since the resistance to failure is less in the case of vertical errors, one wonders what the maximum permissible errors in the two directions might be, and if there is actually more likelihood of a failure in the case of a vertical error than for a horizontal error of the same magnitude. From the fact that the tendency to spall was decreased as the thickness of the slab was increased, it would appear that such might be the condition.

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[13] With National Park Service, U. S. Dept. of the Interior, Washington, D. C.

[13a] Received by the Secretary November 20, 1937.

It is of interest to note that the effect of errors in alignment is greater at expansion joints than it is at contraction joints. Except for slight differences in the location of the dowels relative to the top and the bottom of a slab and for the fact that only one end of the dowel is free to move in the case of an expansion joint, the conditions are much the same. However, a 3 000-lb pull per bar in the case of a contraction joint, as opposed to a 4 000-lb compression for an expansion joint, would be expected to accentuate the tendency to spall in the case of the contraction joint, as is evidenced by the fact that some spalling is observed occasionally in pull-out tests where bars are axially stressed in tension and the concrete is stressed because of the bond. In the tests reported, an effort was made to reduce the bond. However, the pull required is evidence of the fact that some direct tension was imparted to the concrete when opening a contraction joint, and there is little doubt but that some of the tension was resisted by the concrete immediately adjacent to the joint. One wonders also if confining the movement of the dowel to one part of the slab might not have been in some degree responsible for the greater tendency to spall. Although it is difficult to understand how the condition could produce the effect observed, the increased load required to close an expansion joint, as compared with that for opening a contraction joint, is more readily explainable when reference is made to load-slip data obtained in pull-out tests.

On the basis of their observations, the authors have recommended that .24-in. dowels be placed so that the maximum error will not exceed 1 in. Any deviation from perfect alignment is undesirable, despite the fact that small errors appear to cause no serious results. Consequently, it is the writer's opinion that engineers should require and insist on the making of every reasonable effort to insure all dowels being installed as nearly as possible parallel to the longitudinal elements of the pavement. Particularly should this be the case since, as stated by the authors, there is always some likelihood of there being some movement of the dowels during the placing of the concrete.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

SOIL REACTIONS IN RELATION TO FOUNDATIONS ON PILES

Discussion

By Messrs. Harry E. Sawtell, and J. Stuart Crandall

Harry E. Sawtell,[10] M. Am. Soc. C. E., (by letter).[10a]—In the Synopsis the author states his purpose "to use the construction field as a source of information, and to correlate the collected data on pile foundations under varying conditions so that the reason for success and failure become evident."

The success of his attempt, when citing different examples of construction, depends largely on the accuracy with which the conditions that surround each piece of construction are reported. This accuracy is not always present. The writer has elsewhere called attention to a case in which inaccurate assumptions led the investigators into some curious errors.[11]

Mr. Miller discusses soil reactions due to driving piles into clay, but does not cite examples of the remoulding effect on clay. The writer does not think that the results of actual construction or any published experiments can justify any assumption that driving piles into soft or medium clay will completely or largely remould it. In and around Boston, Mass., it has been the practice to support foundations by driving piles into clay, and the comparatively few cases of bad building settlements no doubt are caused by the slow but sure compacting of underlying clay beds due to the additional loads placed upon them.

Driving piles into clay cannot be compared to remoulding a clay sample in the laboratory. In fact, the usual close spacing of piles gives an area of soil around each pile of about 5.5 sq ft, and assuming a pile well above the point to be 7 in. in diameter, the pile is found to occupy less than 5% of this area.

To add an example to Mr. Miller's citations showing "Soil Reactions to Foundations on Piles," the writer wishes to cite the soil conditions, assumptions

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Note.—The paper by R. M. Miller, M. Am. Soc. C. E., was published in June, 1937, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: September, 1937, by Messrs. Hibbert M. Hill, and George P. Stowitts; and October, 1937, by Messrs. George A. McKay and Chandler Davis.

[10] Structural Engr., Care, Charles T. Main, Inc., Engrs., Boston, Mass.

[10a] Received by the Secretary October 26, 1937.

[11] *Proceedings*, Am. Soc. C. E., November, 1933, p. 1467. *et seq.*

for design, and some results of the foundation design for the Eastman Laboratory, a Chemical Research Building for the Massachusetts Institute of Technology, built in 1931.

Soil.—This building is supported on piles which were driven through about 20-ft of low-grade fill (silt) and which stopped in a good sand bed under the northerly half of the structure; but for the southerly part the piles were driven through a loose sand bed, an average of 3 ft thick, and about 23 ft into a stratum of medium blue clay. Under the tips of the piles the clay was about 47 ft thick where boulder clay is reached. The clay has some small quantity of sand in it, in the form of layers or lenses, but the layers are seldom continuous.

Design of Pile Supports.—The basic idea of the design was to spread the load reaching the plastic (medium) clay over as great an area as possible, in order to impose upon the clay as small an additional loading per square foot as possible. The piles were spaced so that the pressure bulbs (having a small diameter in clay) did not overlap. The piles were designed to carry small loads and were deeply embedded, in order to transfer the load to the clay with a surface frictional value of about 300 lb per sq ft.

Load Tests.—To develop the value of skin friction certain piles were loaded covering a period of 200 hr. One pile was driven through a stratum of loose sand and into 22.7 ft of clay; and after being allowed to set, it was given a test load of 40 000 lb applied slowly, without causing failure. This pile (one of a group) settled less than $\frac{5}{16}$ in. and recovered to a total settlement of $\frac{3}{16}$ in. when the load was removed.

The original condition of the clay, evidently, was not changed by driving to such an extent as to destroy the elasticity and grip on the piles. The recovery at unloading was somewhat more than one-third the maximum settlement under the load of 40 000 lb. The working load is about 14 000 lb per pile in this foundation.

Surface Friction.—At the foregoing loadings, the pile showed a surface friction resistance equal to about 900 lb per sq ft for the total test load, and about 300 lb under the proposed working load.

Tests to determine the probable frictional value of a pile in soil under load show whether or not the actual working load upon a pile can be transferred to the surrounding soil safely. This is correct whether or not the pile is a part of a group. The effect of group action under certain loads is largely a question of what settlement occurs under the total loading of the group over a long period of time in which the moisture content of the clay may be pressed out into the surrounding soil. The question of how much settlement is permissible must be answered by other considerations, and must be suited to the surroundings, connecting buildings, or other structures.

To demonstrate, in another manner, the effect of driving piles into this kind of clay and the possible deformation and remoulding of the clay, a series of re-driving tests were made upon piles driven for the support of this Laboratory.

Re-Driving Pile Values.—Re-driving values, after hours of rest varying from 0.5 hr to 32 hr, have been reported in detail elsewhere.[12] The same formula and a single-acting steam hammer were used in both driving and re-driving and, therefore, the results were comparable. The clay was a so-called "medium" grade, but was inclined toward the soft-medium. No evidence of remoulding of clay was shown; nor was the elasticity of the soil destroyed as would be the case if the clay was remoulded much when the piles were driven.

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[12] *Journal,* Boston Soc. of Civ. Engrs., October, 1933, Table 1, p. 166.

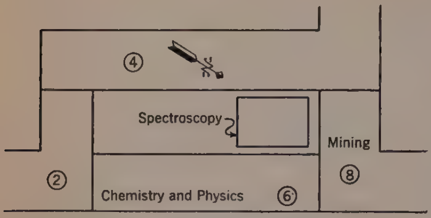

Fig. 7.—Key Plan, New Eastman Laboratory.

Action of Clay During Driving of Piles.—Another demonstration of the action of clay soil, at this same building, when driving piles, was observed, namely, one end of this Eastman Laboratory (Building 6, Fig. 7) joined one of the other buildings (Building 2) of the group, also supported on piles driven into the soft-medium clay. The piles under Building 2 were 45 ft long, whereas those under the new Laboratory are 50 ft long, with the tips of the new piles 5 ft deeper than the old piles.

After driving the new piles adjacent to the old ones, it was found that the wall and near-by corridor of Building 2 had been raised $\frac{3}{16}$ in. After two years had elapsed Building 2 settled back to its original level and has remained there.

The ground surface heaved around the new piles during the driving, but equalled less than one-half the displacement of the piles.

The lifting action and the displacement of soil around the piles indicate that the clay around the new piles was not appreciably remoulded but resisted compression to such an extent as to exert a positive pressure on the clay around the new, and especially around the old, piles under Building 2. It was evident that the clay had sufficient stability and density to transmit the upward component of the compression force to the old piles and to raise the building slightly.

This favorable action cannot take place except when piles are properly spaced, deeply embedded, and have a large reserve value of surface friction.

Pile Formulas.—There appears to be no one pile formula suited to all, or even to any two types or combinations of, soils. The engineer will use many times, therefore, the formula best known and recorded for its deviations and limitations when applied to the largest combination or types of soil.

No formula is used by the writer as a rule, but occasionally, the *Engineering-News* formula is favored as a yardstick or gage only, and seldom to obtain correct pile values directly. No other formula is so well known in regard to its faults and limitations, and, therefore, so safe when used as a gage.

Elastic Properties.—All the writer's research in soils seems to confirm the repeated statement that soils are somewhat similar to "weak solids" having elastic properties, and must be considered as such in foundation design.

Economic pile design is one in which just enough piles are used to distribute the load uniformly and properly to the highest stratum capable of carrying the total load without producing an unwarranted settlement.

Each pile corresponds to a small isolated footing or unit and has its own bulb of distributed pressure. Therefore, the most economical spacing of piles is one in which the pressure bulbs are far enough apart to prevent overlapping of the substantial pressure lines, thus using the entire area to good advantage and producing a fairly uniform pressure on the soil under such piles. If piles are spaced too far apart the subsoil pressure is uneven and permits upward movements of moisture and clay between piles where there is no pressure.

When piles are used in medium or soft clay, it is imperative that small loads be given them, and that they be given deep embedment to keep the surface friction very low.

The total load per square foot under the entire pile group must be kept small so as to obtain as little deformation or future settlement as possible, and it is this fact which must control the design of foundations on compressible soil rather than any result of a single pile-load test.

Examples of Settlement.—The material beneath the Laboratory has been described herein under the heading, "Soil." The design of the superstructure was based upon the idea that the unit loading on the deep bed of clay would be about the same (about 1 500 lb per sq ft additional pressure) whether piles were embedded in the clay or in the sand bed above it. This basic idea was used in the design of the original buildings as it was known at that time (1914) that the actual settlements would result from the consolidation of the clay bed below all fill or sand which might be found above it.

The settlements of this building have been recorded occasionally since its erection in 1931 and now (1937) amount to about 1.25 in. at the southerly end and about 2.25 in. at the other end, but with a somewhat greater settlement at the center of about 2.5 in. A greater settlement was anticipated at the center of the building and was allowed for by cambering the building.

The ends of the building were butted up to, but had no physical connection with, older buildings beneath which it was assumed that the subsoil was consolidated to a greater extent than would be the case in the center away from the old buildings (see Fig. 7).

The older buildings which surround this new Laboratory were built from 1914 to 1916 and the soil and foundations were similar to those described herein for the new Laboratory. These older buildings have settled almost exactly 6 in., one having shown no further settlement since 1927, and the others have had no appreciable settlement since 1932. It is anticipated that some time in the future, these buildings, which appear to have ceased settling, will show some slight additional settlement due to the small but continued consolidation of the clay.

The interesting point regarding these examples is that, from the very first, settlement was anticipated, and the foundations were designed so that the settlement would occur in the clay by consolidation and not at the piles or as a result of driving piles.

The proof of the soundness of this design is that under a number of the buildings of this group where a part of the piles were stopped in a sand bed above the clay, and a part were driven deeply into the clay (the sand did not extend under all the full length of each building), the settlements under both ends of the buildings were found to be almost exactly the same.

The author's "Conclusion" is quite proper and justified, as it clearly states the case as developed from actual work done. The writer agrees with most of the conclusions, but wishes to contend that, when designing foundations, engineers should insist on having authority to make more thorough soil investigations than is customary. The comparatively small extra cost of making more explorations, including the taking of "undisturbed" samples by boring machines or open pits, should not stand in the way of obtaining all the information possible, especially if the soil is stratified or includes compressible material. Such thoroughness not only results in a safer foundation for a structure, but frequently saves more than the extra cost of complete exploration.

A case in point was the foundation for the Baltimore (Md.) Plant of the American Sugar Refining Company. A sufficient number of borings were taken to permit a closely spaced contour map being made, showing the surface of the sub-stratum which was capable of supporting all the piles under this group of large heavily loaded buildings. This map was used by the contractors when ordering piles and when driving them, and they estimated their savings at many thousands of dollars in less waste of piles and in smaller cost for the routine handling of the piles on the job.

The writer wishes to emphasize the positive danger of using prepared tables either for driving values of piles or of following pile-driving formulas, or of using values showing safe friction per square foot of surfaces of piles for different kinds of soils. These values should be determined by investigation and study for each job.

Such a study should include certain developments brought forward by the science of soil mechanics, but caution should be exercised to use only those which have been substantiated and proved to be applicable.

J. Stuart Crandall,[13] M. Am. Soc. C. E. (by letter).[13a]—Many sources of data are cited by the author and these should be helpful in permitting more logical design of foundations, and particularly pile foundations. The accumulation and careful interpretation of such information are vital to a proper checking of the assumptions now made or to be made in theoretical analysis. Many of the apparently antagonistic results can be interpreted readily with present knowledge of soil mechanics. Some require further research.

The elapse of time under the dynamic load, during the driving of piles, is very short as compared with the period of actual static load. Therefore, during driving the only data of value are those which give some measure of the behavior of the foundation or any part of it under static load. In a pile foundation:

(1) Each pile must have adequate strength to resist the stress developed in driving (usually several times the static load);

(2) Each pile must have adequate resistance against movement with relation to the soil; and,

(3) The load on the soil, created by the entire group of piles, must not cause dangerous or excessive differential settlement.

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[13] Pres. and Chf. Engr., Crandall Dry Dock Engrs., Inc., Cambridge, Mass.

[13a] Received by the Secretary November 9, 1937.

Requirement (1) depends on the size and the materials comprising the pile and on the actual stress caused by driving. Much over-driving of wooden piles has been due to the use of drop hammers and the application of the *Engineering-News* formula in determining so-called safe loads, which may have a factor of safety of 12, or more, thus stressing the piles beyond the ultimate strength of the material. Requirement (2) is measured by a dynamic pile formula, or by a load test; and Requirement (3) depends on the net weight of the structure and the nature of the underlying supporting soil. As a maximum over the entire area, this imposed load is the weight of the structure per square unit, and this unit load can be reduced only if piles permit spreading the load over a larger area and provided that no loads are imposed by adjacent structures, present or future. As the author intimates, there is no doubt that many foundation difficulties have been due to blind reliance on Requirement (2) and complete neglect of the others. Too frequently this second requirement has been determined by the *Engineering-News* formula, the primary merit of which is that the uninitiated can work out a result, however far it may be from reality.

One of the most logical driving formulas is that of Alfred Hiley[14] which is in current use in England:

$$R_d = \frac{e\,W\,h}{s + k} \times \frac{W + n^2\,W_p}{W + W_p} \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

in which R_d = dynamic resistance to driving; e = efficiency of the hammer; W = weight of the striking parts of a hammer; h = the drop, or length of stroke; s = penetration of pile per blow; k = one-half the elastic rebound of the pile-head after blow; W_p = weight of pile; and n = Newton's coefficient, or coefficient of restitution. Inasmuch as for the materials in contact the values of n are usually small, the writer proposed[15] eliminating the term, $n^2\,W_p$, and has suggested the formula:

$$R_d = \frac{e\,W\,h}{s + k} \times \frac{W}{W + W_p} \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

If the actual resistance may be determined accurately, a factor of safety of 3 should be adequate. Applying such a factor with the usual efficiency of hammer (namely, 0.75 for a drop hammer overhauling the fall and drum; and 0.90 for a single-acting steam hammer), measuring drop, in feet, and penetration, in inches, the allowable load is:

For a drop hammer,

$$R_a = \frac{3\,W\,h'}{s + k} \times \frac{W}{W + W_p} \ldots\ldots\ldots\ldots\ldots\ldots (3a)$$

and for a single-acting steam hammer,

$$R_a = \frac{3.6\,W\,h}{s + k} \times \frac{W}{W + W_p} \ldots\ldots\ldots\ldots\ldots\ldots (3b)$$

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[14] "Pile-Driving Calculations," by Alfred Hiley, *The Structural Engineer*, London, England, Vol. VIII, July and August, 1930.

[15] *Journal*, Boston Soc. of Civ. Engrs., May, 1931, p. 176.

These formulas appear in the proposed new Boston (Mass.) Building Code.

In Equations (3) the term, k, is a measure of the energy loss due to elastic compression of pile and soil. Evidently, for any given pile, soil condition, and hammer, there is a certain height of fall below which the pile does not move, h_0, which is also a measure of this energy loss due to elastic compression. Based on this fact the writer has suggested the following formula[16]:

$$R_d = \frac{W\,(h - h_0)}{s} \times \frac{W}{W + W_p} \quad \text{.................} (4a)$$

Making the same assumptions as for Equation (3*a*):

$$R_a = \frac{3\,W\,(h - h_0)}{s} \times \frac{W}{W + W_p} \quad \text{.................} (4b)$$

in which h_0 is the maximum drop of the hammer below which there is no penetration. The value of h_0 for any particular case may be determined readily in the field. Measuring the penetration accurately for, say, four different heights of fall, plotting the penetration as abscissas and the drop as ordinates, these points should fall on a straight line whose intercept with the Y-axis gives the value of h_0. If the points do not fall in a straight line the test should be repeated. As h_0 varies as the square of the dynamic resistance, therefore, these determinations should be made when at the probable desired resistance.

Furthermore, the determination of h_0 permits a verification of the theoretical calculations of the values of k, because,

$$k = \frac{h_0\, s}{h - h_0} \quad \text{.........................} (5)$$

The value of h_0 was determined by the writer, at Boulogne-sur-Mer, France, in 1933,[17] for 35-ft greenheart piles driven into sand with a 3 080-lb hammer for a safe load of 25 metric tons (factor of safety = 6), giving a value of 1 ft. Applying these data in Equation (5) gave a value of k equal to 0.06 in. as compared with a theoretical value of 0.12 in. Recently, a similar determination was made of an oak pile driven at the Massachusetts Institute of Technology.[18] The pile was 26 ft long, driven into sand with a 3 000-lb hammer for a safe load of 22.5 short tons (factor of safety = 3). The value of h_0 was determined as 2.2 ft, giving, by Equation (5), a value of k equal to 0.35 in. as compared with a theoretical value of 0.30 in.

A sound driving formula can give only the resistance to rapid penetration or, in other words, the load on the pile during driving. This is of value in determining the driving stresses in a pile. Furthermore, in soils where the resistance to rapid penetration is the same as that to slow penetration, such formula can give the resistance of a pile to movement in the soil under static load. This is true of sands, gravels, and such permeable incohesive soils, but is not true for such soils as clays and silts.

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[16] *Journal*, Boston Soc. of Civ. Engrs., May, 1931, Equation (14), p. 181.

[17] Four piles in railway dry dock designed and constructed for Service des Ponts et Chaussées. Not published.

[18] By Stone & Webster Eng. Corp., October, 1937, Pile Z 240, New Architectural Bldg., Cambridge, Mass. Not published.

In clays and silts the remolding and disturbance of the soil during driving tends to reduce the frictional resistance which in most cases again sets up after an elapse of time at even a greater value, although on rare occasions the opposite occurs. In these soils there is no known relation between dynamic and static resistance. Therefore, it would appear that the logical procedure in such cases is to base the resistance to movement in the soil on the frictional resistance. An equation for this condition would take the form:

$$R_a = \frac{A_s f}{3} \qquad \text{.........................(6)}$$

in which A_s = the area of contact surface; and f = "skin" friction per unit area. In Table 3 the author gives certain values[19] for friction which apparently are taken from the writer's paper. It should be pointed out that the soil at Hull, England (Item No. 1) was not soft clay, but actually stiff blue clay which makes the tabulated value of 1 850 lb per sq ft appear more reasonable.[19a] Furthermore, the writer verified certain data of pile-pulling tests at Boulogne-sur-Mer, France, where a value equal to, or greater than, 1 800 lb per sq ft was found for stiff clay. In some tests the friction per unit area decreased with depth; in others, it increased, and then became constant. Therefore, friction tests should be made for each locality and for the lengths of pile used unless other tests are available for the same soil and locality. Even then, corroboration by test is advisable for any important foundation.

To this point the writer's remarks have been concerned with the stress in individual piles and the resistance of individual piles to movement in the soil. The results of driving formulas and of load tests can give no indications of the action of a group of piles under static load. The load from a single pile stresses the soil over a fairly wide area. With a group of piles in close proximity these zones of stress overlap, causing more important stress concentrations at greater depths. The writer cannot agree with Mr. Miller regarding the value of load tests on a group of piles. Admitting that such tests on single piles can give no measure of the settlement of an entire pile foundation, particularly where underlaid by compressible soils (because the load caused by a single pile is distributed over considerable area so that the time-settlement effect is infinitesimal), the same is true of load tests of small groups of piles forming part of a larger foundation, although to a different degree. The settlement for the group would be greater than for a single pile, but it would not be comparable to that of the entire foundation. Assuming a structure of infinite extent, the additional load on the soil, whether or not piles are used, would be equal to the weight of the structure per square unit, less the weight of any excavation. If the weight of material excavated is equal to the weight of the structure there would be no additional load and, hence, no comparative settlement. On the other hand, if there is a net load increase and the structure is underlaid by such compressive soils as clay and silt, then there will be settlement. The magnitude of this settlement can be predicted only after an accurate exploration of the soil and a settlement analysis based on undisturbed samples.

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[19] *Journal,* Boston Soc. C. E., May, 1931, Table 4, p. 185.

[19a] Corrections for *Transactions:* In Table 3, Item No. 1, should read "Stiff blue clay"; in Item No. 9, the friction value should read "95."

The author has cited cases in which the soil has "heaved," others of subsidence, and, again, cases of no evident movement. This action is a result of the soil structure. The voids of clays and silts are usually filled with water and, being relatively impermeable, cannot be compressed during the short period of pile-driving, so that the soil must be displaced equivalent to the volume of the embedded piles, resulting in "heaving." In compact sands and gravels, piles cannot be driven without displacing the soil, causing "heaving" either by the driving process alone or with the aid of jetting. In loose sands, gravels, and fills, the action of driving tends to re-arrange the soil particles into a more compact structure. This may result in some subsidence of the surface, which would depend on the degree of volume change by reason of the reduction in voids. Where layers of several kinds of soil are penetrated, a combination of these effects may take place.

In cases where piles are driven through incompletely consolidated soil to a bearing in a more resistant one, there are certain special conditions to be considered. The imposed load on such a pile would be transferred through friction in the upper layer, and largely through end bearing in the lower one. However, as the upper layer continued to consolidate under its own weight, eventually it would cause an additional load on the pile through friction, so that the load carried by the point would amount to the superimposed load on the pile plus the total frictional load imposed by this consolidating layer, and thus cause settlement. This has been observed in Holland and is a factor in design.[20]

The profession has given much attention to refinement in the design of structures, all based on certain assumptions as to rigidity of foundations, while all too frequently the foundations are only superficially considered and empirically designed; yet the settlement of these foundations may completely vitiate the assumptions in the super-structure design. It seems to be particularly this point which Mr. Miller desires to emphasize and which merits serious thought. The mere driving of piles to some resistance does not, by itself, constitute a solution. The question of foundation design and its limitations should be the first consideration, after which definite conditions affecting super-structure design can be logically determined.

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[20] *Proceedings*, Conference on Soil Mechanics and Foundations, 1936.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
DISCUSSIONS

WATER TRANSPORTATION VERSUS RAIL TRANSPORTATION
A SYMPOSIUM
Discussion

By W. D. Faucette and J. E. Willoughby,
Members, Am. Soc. C. E.

W. D. Faucette,[10] and J. E. Willoughby,[11] Members, Am. Soc. C. E. (by letter).[11a]—The region included in Mr. Wonson's paper is a land area encompassing the Mississippi River and its tributaries. The truth, as disclosed by Mr. Wonson, is sound for general application to all inland river and canal transportation in the United States. That truth applies whether the relative land area to length of river and canal is great (as for the great interior land area drained by the Mississippi), or of less extent, as for the peninsula of Florida This peninsula has a form such that no producing area is more than 75 miles from the coast. The annual rainfall is large, and there are many rivers from the interior to the sea. The rivers are drainage channels now flowing in submerged valleys, the filling of which was the result of the recent subsidence of the peninsula. Florida has a long shore line with many harbor indentations, and much Federal money has been spent on recommendation of the Chief of Engineers, U. S. Army, to develop a river traffic for Florida products to the cities along the Atlantic and Gulf Coasts, the river to be used to the seashore, and then the Atlantic Ocean or the Gulf of Mexico. This peculiar condition makes for minimum river haul and maximum ocean haul, as compared with the area discussed by Mr. Wonson. The inland waterways in Florida that have been created into a system at Federal cost aggregate: Thirty projects, including 1 869 miles, and with a 1935 traffic of 57 233 000 ton-miles. Federal expenditure, including cost of construction equipment, to June 30, 1936, has been $31 836 000. The average annual maintenance for the 5-yr period ending in

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Note.—The Symposium on Water Transportation Versus Rail Transportation was presented at the meeting of the Waterways Division, Little Rock, Ark., April 25, 1936, and published in September, 1937, *Proceedings*. Discussion on this Symposium has appeared in *Proceedings* as follows: October, 1937, by George Hartley, Esq.

[10] Chf. Engr., S. A. L. Ry. System, Norfolk, Va.

[11] Chf. Engr., A. C. L. R. R., Wilmington, N. C.

[11a] Received by the Secretary November 9, 1937.

1936 was $425 000. Service at 3½% on the total expenditure of $31 836 000 is $1 111 250 per yr. The total annual maintenance and service charges are $1 536 250. The cost to the Federal Government in 1936 was 27 mills per ton-mile, not including the cost of vessel operation, or service on capital invested in vessels, wharves, etc. This ton-mile cost of 27 mills will be increased as compared with railway and highway transportation, due to the circuity of the water route. The four principal railways in the peninsula of Florida in 1935 handled a freight traffic of 4 337 243 000 ton-miles at a cost of 10 mills per ton-mile, which includes the total cost to the user—service on capital, maintenance of railroad and equipment, and cost of transportation.

The high cost of inland water transportation in the Florida Peninsula is demonstrated as follows:

Description	Cost, in mills per ton-mile
Inland Water Transportation:	
Service on total of permanent-way expenditures to 1936	20
Maintenance of permanent way (5-yr average, ending 1936)	7
Total paid by taxpayers	27
Estimated cost of transportation, paid by user	4
Aggregate cost for inland transportation	31

as compared with inland rail transportation:

Service on total expenditures for permanent way and equipment, cost of maintenance of permanent way and equipment in 1936, and cost of transportation	10

It is of interest to note that the service charge on the cost of permanent way of a water carrier for freight service is 20 mills per ton-mile. The service charge on the cost of permanent way for rail transportation, when allocated as between freight and passenger traffic, develops only 2 mills per ton-mile for the Florida Peninsula.

In addition to the low cost per ton-mile of rail transportation as indicated in the foregoing list the four principal railways in Peninsular Florida paid taxes for the year, 1936, to the State, and other civic units in Florida, equal to $2 698 401, which is equal to 0.6 mill per ton-mile on freight traffic carried, leaving a net cost of inland rail transportation, as compared with inland water transportation, of 9.4 mills for rail against 31 mills for water.

Mr. Wonson touches upon the question of waterways having rigidity of location, which prevents universal and flexible service; and, in his "Summary," he shows that improvement when made, tends to accrue not to the nation as a whole, but to special and localized industries and localities.

This prompts the suggestion that, when it is being considered, inland water transportation should also comprehend consideration of the service features in connection with water-borne traffic. In the final analysis the question of service of transportation to the nation, is inextricably interwoven with considerations of economics. Probably at no time during the past 10 or 20 yr has the matter of transportation service entered so largely into problems pertaining to commerce and industry. This transportation, of course, means moving one's goods from where they are to where they are to be sold; and delivering them, with reasonable and suitable promptness, in a condition such that they can be sold. In order to serve the greatest number and maximum requirements for all classes of commerce and industry, of course, the complete transportation instrumentality should not be limited and rigid in its location and should not be circumscribed by being adaptable to only certain classes of industrial products, which narrows the field of usefulness. On the contrary, railroad facilities as they stand, offer and are able, in substance, to carry all classes and character of goods and commercial products with speed, safety, protection, and responsibility, and (when all factors of cost are properly considered and taken into account) economically, as compared to water transportation. The railroads permeate and serve the entire nation. They are not rigidly held to any one geographical location. They are performing transportation to a nation, highly developed and still developing. Therefore, in the public interest and in the expenditure of public funds, consideration must be given to the fact that rigid location for water transportation narrows it to certain commodities and is not a broad instrumentality that finds itself easily justified. The facts are that when goods and commerce are to be moved, it is service and quality and expedition of that service that is sought—a fact which cannot be ignored.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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DISCUSSIONS

PRE-STRESSED REINFORCED CONCRETE AND ITS POSSIBILITIES FOR BRIDGE CONSTRUCTION

Discussion

BY MESSRS. DEAN F. PETERSON, JR., AND M. HIRSCHTHAL

DEAN F. PETERSON, JR.,[25] JUN. AM. SOC. C. E. (by letter).[25a]—The method of analysis to be used in beams containing pre-stressed steel has been presented clearly in this paper, and Mr. Rosov has opened for discussion a subject, which, pending further mathematical and experimental investigation, may lead to some appreciable economy in reinforced concrete design.

The writer does not agree with Mr. Rosov's analysis of the principal stresses. Applying Equation (31) with $v = 150$ and $f_l = -300$, he determines f_d as equal to $-\frac{1}{2}$. However, if these values are substituted in Equation (30), from which the author derives Equation (31), $f_d = -150 \pm \sqrt{150^2 + 150^2}$; or -362 and $+61.5$ in which $+$ denotes tension.

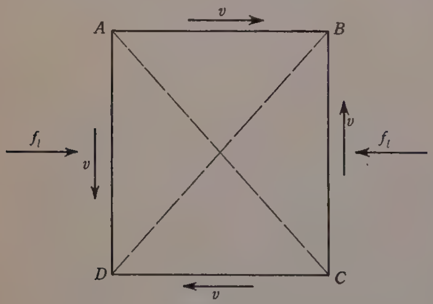

FIG. 8.

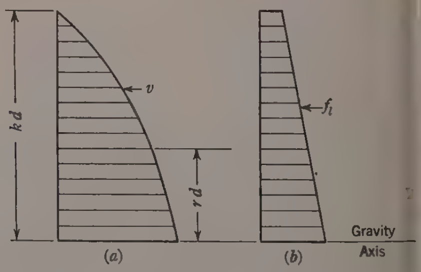

FIG. 9.

Analysis of Diagonal Tension.—Consider a longitudinal stress that is always compressive for pre-stressed beams. The stress on Plane *BD* (see Fig. 8) is minimum tension or maximum compression, and that on Plane *AC*

NOTE.—The paper by Ivan A. Rosov, M. Am. Soc. C. E., was published in September, 1937, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: November, 1937, by Messrs. Charles S. Whitney and Karl W. Lemcke.

[25] Troy, N. Y.

[25a] Received by the Secretary November 10, 1937.

is maximum tension or minimum compression. The former case is not important since, as Mr. Rosov states, the maximum value of f_l does not occur at a point of maximum unit shear stress, v, and the diagonal compression will never be much in excess of the allowable. In the case in which the principal stress occurs on Plane AC, f_d will always have a tensile value if shear exists. Writing,

$$f_d = -\frac{1}{2} f_l + \frac{1}{2} f_l \left(1 + \frac{v^2}{\frac{1}{4} f_l^2}\right)^{\frac{1}{2}} \quad \text{..........} (46)$$

which may be expanded to,

$$f_d = -\frac{1}{2} f_l + \frac{1}{2} f_l \left(1 + \frac{2\,v^2}{f_l^2} - \frac{2\,v^4}{f_l^4} + \cdots\right) \quad \text{..........} (47)$$

The series in Equation (47) is convergent only if $\frac{4\,v^2}{f_l^2} < 1$, or if $v < \frac{1}{2} f_l$. For values of v greater than, or equal to, $\frac{1}{2} f_l$, the series does not define the radical. For the first approximation,

$$f_d = \frac{v^2}{f_l} \quad \text{..........} (48)$$

instead of $\frac{v}{f_l}$ as in Equation (31). As an example choose $v = 100$ and $f_l = 400$; then, $f_d = \frac{100 \times 100}{400} = 25$ lb per sq in. If the series is carried to the second approximation, $f_d = 23.44$. The value of f_d is between the first and second approximations, since the series is alternating. Equation (30) gives $f_d = 23.6$. The purpose of the foregoing comment is merely to show that the assumption expressed by Equation (31) is not valid. There is no gain in using Equation (48) for purposes of computation. Appreciable diagonal tension may occur near the end of the pre-stressed beam, although not to the extent that it would occur on ordinary beams of the same depth.

Assuming the section to be entirely in compression, the unit shear, v, may be determined at any point by using the transformed section and the formula,

$$v = \frac{V\,Q}{I\,b} \quad \text{..........} (49)$$

in which Q = the first moment about the gravity axis of that portion of the transformed cross-section lying above the point for which v is to be determined. Since the end moments will be small the stress diagram will resemble Fig. 9(*b*), and it is obvious that a value of r will occur in which f_d will be a maximum. However, if,

$$\frac{d\left(\frac{v^2}{f_l}\right)}{dr} = 0 \quad \text{..........} (50)$$

is taken, assuming a parabolic variation of v and a straight-line variation of f_l, a fifth-degree equation in r results, and it is probable that it would be easier to investigate three or four points on the section for v than to attempt to find r for each section.

Effect of Shrinkage and Plastic Flow.—It is also to be emphasized that the design pre-stress is not the initial tension imparted to the bar. The effect of shrinkage will be to reduce the pre-stress directly, with no effect on the concrete, except to close the cracks and release the tension. The reduction in f_{ps} due to shrinkage is $m\,E_s$ in which m is the coefficient of shrinkage. The elastic deformation of the beam will further reduce pre-stress. The change in length of the concrete fibers, ΔL_c, must equal the change in length of the steel, ΔL_s, in any unit length of beam; but

$$\Delta L_c = \delta_1 + \delta_2 \qquad (51)$$

in which δ_1 and δ_2 = changes in length due to rotation (moment), and, shortening (direct stress), respectively. Furthermore,

$$M_{ps} = T_1(q_b - k_1)\,d \qquad (52a)$$

and,

$$\theta = \frac{1}{R} = \frac{M}{E\,I} = \frac{T_1\,(q_b - k_1)\,d}{E_c\,C_1\,b\,d^3} \qquad (52b)$$

in which, T_1 is the pre-stressing force remaining on the section; but,

$$\theta = \frac{\delta_1}{(q_b - k_1)\,d} \qquad (53)$$

from which,

$$\delta_1 = \frac{T_1\,(q_b - k_1)^2}{E_c\,C_1\,b\,d} = \frac{A_s\,f_{ps1}\,(q_b - k_1)^2}{E_c\,C_1\,b\,d} \qquad (54a)$$

and,

$$\delta_2 = \frac{T_1}{E_c\,K_1\,b\,d} = \frac{A_s\,f_{ps1}}{K_1\,E_c\,b\,d} \qquad (54b)$$

in which f_{ps1} is the unit pre-stress acting on the section. Finally,

$$\Delta L_s = \frac{f_{ps0} - m\,E_s - f_{ps1}}{E_s} \qquad (55)$$

in which f_{ps0} is the unit pre-stress imparted to the reinforcement before placing the concrete. Equating $(\delta_1 + \delta_2) = \Delta L_c = \Delta L_s$, and reducing,

$$f_{ps1} = \frac{f_{ps0} - m\,E_s}{1 + p\,n\left(\dfrac{(q_b - k_1)^2}{C_1} + \dfrac{1}{K_1}\right)} \qquad (56)$$

is the value of f_{ps1} before the effect of plastic flow has occurred. Since the member is under compression from the effect of the pre-stressing force the fibers adjacent to the steel will tend to shorten due to plastic flow. This effect will occur progressively with time. The ultimate plastic flow modulus is approximately 1 000 000, which, when combined with E_c, will give a secant modulus, N, representing the combined effect of flow and elasticity of $N = 750\,000$, and a ratio of $\frac{E_s}{N}$, n', of 40. The dead loads will also have an effect on the designed pre-stress due to flow. The longitudinal shortening of the member will tend to release the designed pre-stress, and the sagging of the member will tend to increase it. The deformation of the unit length of fiber at the steel due to the dead load will be,

$$\delta = \frac{M_D\,(q_b - k_p)}{N_c\,C_p\,b\,d^2} \qquad \text{.......................}(57)$$

in which C_p is based on n'. Equating the deformation of the concrete to the deformation of the steel,

$$\frac{f_{ps0} - E_s\,m - f_{ps2}}{E_s} = \frac{A_s\,f_{ps2}}{N_c\,b\,d}\left(\frac{(q_b - k_{1p})^2}{C_{1p}} + \frac{1}{K_{1p}}\right) - \frac{(q_b - k_p)\,M_D}{N_c\,C_p\,b\,d^2} \quad \text{..}(58a)$$

and,

$$f_{ps2} = \frac{f_{ps0} - m\,E_s + \dfrac{M_D\,(q_b - k_p)\,n'}{C_p\,b\,d^2}}{1 + p\,n'\left(\dfrac{(q_b - k_{1p})^2}{C_{1p}} + \dfrac{1}{K_{1p}}\right)} \qquad \text{.............}(58b)$$

in which f_{ps2} is the residual stress remaining after the effect of shrinkage, elastic, and ultimate plastic deformations have occurred, and the beam is carrying the total dead load moment. The beam at this stage still behaves elastically as far as the external loads are concerned, the effect of the plastic deformation being entirely represented in the pre-stress. The tension in the steel due to dead loads, the beam behaving elastically, is $\dfrac{M_D\,(q_b - k)\,n}{C\,b\,d^2}$. The pre-stress is then:

$$f_{ps1} = \frac{f_{psc} - m\,E_s + \dfrac{M_D\,(q_b - k_p)\,n'}{C_p\,b\,d^2}}{1 + p\,n'\left(\dfrac{(q_b - k_{1p})^2}{C_{1p}} + \dfrac{1}{K_{1p}}\right)} - \frac{M_D\,(q_b - k)\,n}{C\,b\,d^2} \qquad \text{......}(59)$$

As an example of the possible effects of plastic flow Example 1 of the paper is investigated (see Fig. 5). The values of the section functions are given by Mr. Rosov. The plastic functions are $k_p = 0.586$, and $C_p = 0.113$. K_{1p}, k_{1p}, and C_{1p} are equal to K_1, k_1, and C_1, respectively. A designed pre-stress of − 36 500 lb per sq in. is to be used. Substituting the proper values in Equation (56), assuming $m = 0.0004$, and making the necessary algebraic transpositions, $f_{ps0} = 60\,000$ lb per sq in. After ultimate plastic deformation has occurred, the effective tensile pre-stress in the steel is found by Equation (59):

$$f_{ps1} = \frac{60\,000 - 12\,000 + \dfrac{(0.844 - 0.586)\,(22\,900)\,(12)\,(40)}{(0.113)\,(12)\,(16)\,(16)}}{1 + 0.0086\,(40)\left(\dfrac{0.347^2}{0.0843} + \dfrac{1}{0.9914}\right)}$$

$$- \frac{22\,900\,(0.307)\,(12)\,(15)}{0.096\,(16)\,(16)\,(12)} = 26\,300 \text{ lb per sq in.}$$

Then, $f_{tp} = -0.00885\ (26\,300) = -230$ lb per sq in.; and, $f_{bp} = 0.0265\ (26\,300) = +700$ lb per sq in. For dead loads: $F_t' = -230 + 500 = 270$ lb per sq in.; $F_b' = 700 - 430 = 270$ lb per sq in.; and, for live loads: $F_t'' = 270 + 478 = 748$ lb per sq in.; and, $F_b'' = 270 - 408 = -162$ lb per sq in. At this condition, cracks would occur, and it is evident that the compression area would no longer extend over the face of the section and the foregoing analysis would be invalid. The condition tends to approach that of an ordinary beam, somewhat "under-designed." It is to be noted that the effect of plastic deformations due to dead load is opposite to the effect of plastic deformations due to pre-stressing. The case investigated is designed for a dead load greater than 50% of the total load. In cases where the dead load is less significant, the effect of reducing the pre-stress due to flow will be even greater.

Spirally Reinforced Columns.—Another application of the possibility of using a designed pre-stress to advantage might occur in the case of spirally reinforced columns. Shrinkage of the columns will induce compressive stresses of from 12 000 to 18 000 lb per sq in. in the longitudinal steel, with zero stress in the concrete. This compressive stress in the steel will be further increased by plastic flow of the concrete under dead load, with the result that when full-load conditions for the column are reached, the steel may be stressed from 20 000 to 30 000 lb per sq in.[26] This is borne out by the fact that test columns usually fail when the longitudinal reinforcement reaches its yield point and bulges. By pre-stressing the steel in tension, more of the stress would be transferred to the concrete, or, at least, the stresses in the steel due to shrinkage and the flow could be neutralized. It would be desirable, of course, to introduce a greater percentage of spiral reinforcement to restrain the additional deformations of the concrete. This is a matter for thorough investigation by experiment.

In conclusion, it seems to the writer that, although stress control due to induced pre-stress in beams may be a means of saving in design, the effect of deterioration of the pre-stress by plastic flow should be considered and an attempt made to evaluate it. Further investigation of the specific effect of flow on pre-stressed beams is needed. Although the particular beam investigated by this discussion apparently develops a rather unfavorable condition due to plastic flow it may be possible, using high-strength steel, to design beams of greater economy by using the principle of pre-stress, the limiting condition being that the stresses before and after plastic flow has occurred will be within the allowable limit. It is also concluded that appreciable diagonal tension may develop in spite of pre-stress, and that its occurrence should be fully investigated, especially since the unit shear will increase as the depth of the beam decreases.

M. Hirschthal,[27] M. Am. Soc. C. E. (by letter).[27a]—The pre-stressing of reinforced concrete in bridge construction, as suggested by Mr. Rosov, is very interesting, but it is to be feared that it is purely academic, particularly for members that are cast in place. To the writer, the most interesting angle is that referring to the elimination of shrinkage stresses in slabs and beams by means of pre-stressing.

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[26] "Principles of Reinforced Concrete Construction," by F. E. Turneaure, Hon. M. Am. Soc. C. E., and E. R. Maurer, Chapter VII, 1932.

[27] Concrete Engr., D. L. & W. R. R., Hoboken, N. J.

[27a] Received by the Secretary November 22, 1937.

In this connection, it may be interesting to note that the writer has requested the personnel of several university experiment stations to make tests on a form of pre-stressing, with special application to pavement slabs. The shrinkage (or temperature) reinforcement of a concrete slab (similar to a concrete pavement slab) is to project beyond the slab ends and be restrained from responding to normal shrinkage action which puts this steel in compression and the concrete in tension, thus causing the shrinkage cracks. The action resulting from the foregoing restraint would tend to put the steel reinforcement in tension and the concrete in compression—the normal action of reinforced concrete. In practice, two adjoining slabs would have the projecting reinforcement bars interlock.

However, in bridge work that is cast in place, the assumption that the reinforcement is free at the construction joints is a false assumption, in the main, because practically all construction joints are specified to be made at sections of minimum shear. This is invariably the point of maximum moment where the bars are certainly continuous through the joint because even the necessary laps in bars are excluded at such points or sections in careful design.

In the author's method of operation at Stage (2) when the tensile force is removed, there is a great probability of high concentration of bond stresses at the ends of the beams or slabs, and although Mr. Rosov notes that Freyssinet reported no difficulty in this connection, the 150-lb tensile strength for the very high-strength concrete indicates otherwise.

Moreover, pre-stressing is applicable only to a material assumed to be homogeneous and elastic. Although every designer specifies that concrete shall be of a composition and grading such as to result in a "homogeneous mass" when thoroughly mixed, the writer feels that the results, even with the best of care, fall far short of homogeneity. Any material of this character cannot be perfectly elastic even within the limits of working stresses. When the stresses approach the ultimate, the material may begin to fail from strains characteristic of a composite material such as concrete. The failure may result from fracture of the coarse aggregate, from fracture of the matrix that binds it, or by loss of adhesion or bond between the aggregate and the matrix. This type of material is not an ideal one for the application of pre-stressing. The material (concrete) is very different from mortar, on the action of which, under tests by Freyssinet, the author bases his assumption. Mr. Rosov cites Freyssinet's tests of treated concrete, in which a strength of 14 000 lb per sq in. in compression was developed, but the tensile strength was as low as 150 lb per sq in. Such concrete would not be very effective in bond and, when used with 45 000-lb per sq in. working stresses in steel reinforcement, would require an inordinate amount of embedment of the steel, or provision against the heavy bond stress developed on release of the "pre-stress."

The author's analysis is confusing at times; assuming the concrete section as taking tension at one time, then assuming the entire section to be in compression;

taking into consideration the reinforcement, and then dropping it. If the final assumptions in the paper are made at the beginning, Equations (20) and (21) could be derived directly instead of by means of the intervening steps. Mr. Rosov does not apply his test (*D*) for stresses at the bottom of the beam after having completed that of Condition (*C*). Incidentally, at the beginning of the derivation of the formulas the author assumes the entire concrete section to be in compression; and yet he indicates the neutral axis in the proximity of the center, implying it to be the gravity axis and so computing his moments of inertia of the section.

On the subject of economy, the author seems to be too optimistic as to the cost of special steels. He would find that a specification calling for steel reinforcement suitable for a working stress of 45 000 lb per sq in. would result in quotations that could scarcely be considered "only slightly" higher than, say, standard intermediate grade reinforcement which is usually specified.

In the author's comparison of slabs, he selects one, citing the writer's series of articles on pre-cast bridge slabs, published in 1926.[13] Some of these structures were designed as early as 1909, and the structure selected by Mr. Rosov is purely hypothetical since the overhead highway bridges there described consist of pre-cast T-beam units with clear spans varying from 30 ft $8\frac{1}{2}$ in. to 35 ft 6 in.; and corresponding depths of 2 ft 6 in. and 2 ft 8 in. These bridges were designed in 1913, when a concrete having a 6-bag cement content (the 1 : 2 : 4 variety) was computed as having a working stress of 650 lb per sq in. Had the author made a comparison with an actual design in the articles referred to and selected a railroad bridge at Lackawanna Place, Millburn, N. J., with pre-cast slabs continuous over a center support and having a maximum length of 60 ft $3\frac{1}{2}$ in., with maximum spans of 28 ft 0 in., from the center support to the inner edge of the abutment, such comparison would not have been so favorable to the "pre-stressed design." (In effect, a pre-stressed design is based on the assumption that pre-stressing leaves the entire concrete section above the tensile reinforcement in compression much after the assumption made by Charles S. Whitney, M. Am. Soc. C. E., in 1936,[28] without the requirement of pre-stressing.) The slabs have a depth at the center of 2 ft 4 in., and 2 ft 2 in. at the edges (thus providing drainage). They are subjected to a maximum bending moment of 104 400 ft-lb per ft of width for live load and impact as against the 21 700 ft-lb selected by the author for his slab—a moment more than $4\frac{1}{2}$ times as great.

In recent years, many methods have been followed to reduce the slab depths. Among them are the use of compressive reinforcement, continuity, end restraint (practically the same effect as pre-stressing), the use of higher-strength concrete, two-way, or multi-way, reinforcement combined with continuity, and end restraint. A girderless flat-slab bridge, carrying railroad loadings for spans about 30 ft, center to center of columns, required a slab thickness of only 2 ft.

Another comparison that would not have been so favorable to pre-stressing, would be the use for the moment in the author's slab, of a working stress of 45 000 lb per sq in. in the reinforcement in tension and the $0.4 f_c'$ allowable for a concrete with a 6-bag cement content—a concrete with a water-cement ratio of 6 gal per bag. Under these conditions, the ultimate strength is 3 500 lb, or f_c = 1 400 lb per sq in., and f_s = 45 000. Then:

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[13] "What the D. L. & W. Is Doing in Concrete Design," by M. Hirschthal, M. Am. Soc. C. E., *Railway Review*, October 9 and 16, 1926.

[28] *Journal*, Am. Concrete Inst., 1936.

$$k = \frac{12 \times 1\,400}{45\,000 + 12 \times 1\,400} = 0.272;$$

$$j = 0.909;$$

$$K = \frac{0.272 \times 0.909 \times 1\,400}{2} = 172;$$

$$d = \sqrt{\frac{44\,600}{172}} = 16 + 2.5 = 18.5 \text{ in.}$$

(which gives a somewhat higher dead load moment);

$$A_s = \frac{44\,600}{0.909 \times 16 \times 45\,000} = 0.82;$$

and, $p = 0.00425$. If 4 500-lb concrete were used, the value of f_c would be 1 800, and,

$$k = \frac{12 \times 800}{45\,000 + 21\,600} = 0.324;$$

$$j = 0.892;$$

$$K = \frac{0.324 \times 0.892 \times 1\,800}{2} = 260;$$

$$d = \sqrt{\frac{44\,600}{260}} = \pm 13 \text{ in.},$$

or 16 in. total; and $p = 0.006$. (For a value of $n = 1$, d becomes 13.5 in., giving a total depth of 16 in., which is the same as in the author's example.)

Aside from these considerations, the practical difficulties encountered on a structure in connection with pre-stressing would be almost insuperable, and probably only in a casting yard could such installation be considered.

Although the writer has cited his objection to the use of pre-stressing in this particular case (bridge construction), he realizes the value of such treatment for structures such as reservoirs, tanks, and poles. This paper will have accomplished a valuable service if it leads to research activities, so that possibly shrinkage stresses may be counteracted in concrete—an undoubted incipient cause of deterioration and final failure of reinforced concrete.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
DISCUSSIONS

THE DESIGN OF ROCK-FILL DAMS

Discussion

BY MESSRS. CECIL E. PEARCE, AND H. B. MUCKLESTON

CECIL E. PEARCE,[3] ASSOC. M. AM. SOC. C. E. (by letter).[3a]—There has been no good compendium on the subject of rock-fill dams and, therefore, the paper by Mr. Galloway is a timely one. The writer wishes to make several comments on it.

The sliding factors mentioned in the paragraph following Table 2 have been checked, and, at the same time, it is found that the cases which have sliding factors of 4.50, 5.14, and 6.45 have up-stream slopes of (ratios of base to height) 0.775 : 1, 1.02 : 1, and 1.54 : 1, respectively. Although it may be true that these values represent safe sliding factors in the sense usually applied to gravity dams, it may also be true that the stability of the up-stream slope is a more important criterion. If dumped rock assumes a natural slope, then does not a natural phenomenon offer a perfect index of what should be done? Carrying the case to an extreme illustrates the point. Assume that the up-stream face is vertical for a high dam (say, 300 ft high). Obviously, it would not stand; the entire vertical face would tumble, slide, and roll until equilibrium was reached at its natural angle of repose. Any slope steeper than such natural slope would mean that there were still some forces at work trying to arrive at such a slope. Any slope as flat as the angle of repose, or flatter, would mean that the material had reached a state satisfactory to the forces of Nature, and that there was no longer a condition that would cause rocks to roll down the slope.

Although it is true that Man may place these rocks somewhat carefully, and gain certain advantages in friction and interlocking, which will permit the face to stand steeper, it is also true that it is difficult to appraise the degree of success of such operations. On the other hand, one cannot assume that an up-stream face, placed as flat as the angle of repose, or flatter, has much tendency, if any, to become still flatter.

NOTE.—The paper by J. D. Galloway, M. Am. Soc. C. E., was published in October, 1937, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion of the paper.

[3] Acting Project Design Engr., Hiwassee Project, Tennessee Valley Authority, Knoxville, Tenn.

[3a] Received by the Secretary November 5, 1937.

The author has stated (see heading, "Definition and Principles") that "although the design of rock-fill dams is mainly based upon experience certain definite theoretical principles apply to such structures which should always be considered." It is the writer's contention that the slope of the up-stream face is one of these points which should be based upon the rational. Considering the angles of repose which have been observed for dumped rock, the writer would recommend that the up-stream slope of important rock-fill dams be not less than 1 on 1.3, as recommended by the author in his "Conclusions."

In discussing construction and expansion joints, Mr. Galloway states that (see heading: "Design: Construction and Expansion Joints"): "In practically all cases, all horizontal joints become construction joints as such joints, being always in compression, never tend to open although they may be subjected to shear." The statement that the horizontal joints will always be in compression is quite debatable. In fact, in discussing the concrete facing, the author states (see heading, "Design: Concrete Facing"): "The concrete of the slab is poured directly upon the rubble wall that supports it. The concrete enters into the interstices of the rock, fills the inequalities, and adheres to the rock. The concrete is thus held in position, and expansion or contraction is prevented." It is agreed that the slab should be poured in this manner. Such being the case, if the job has been properly done the slab is truely supported by the fill, so that the horizontal joints will or will not be in compression, depending on whether or not the settlement shortenings overtake the original contraction of the concrete on cooling. In fact, one of the most important reasons for pouring the concrete facing as recommended is to prevent the inclined thrust down the slab and the consequent possible shearing across the corner of the joint.

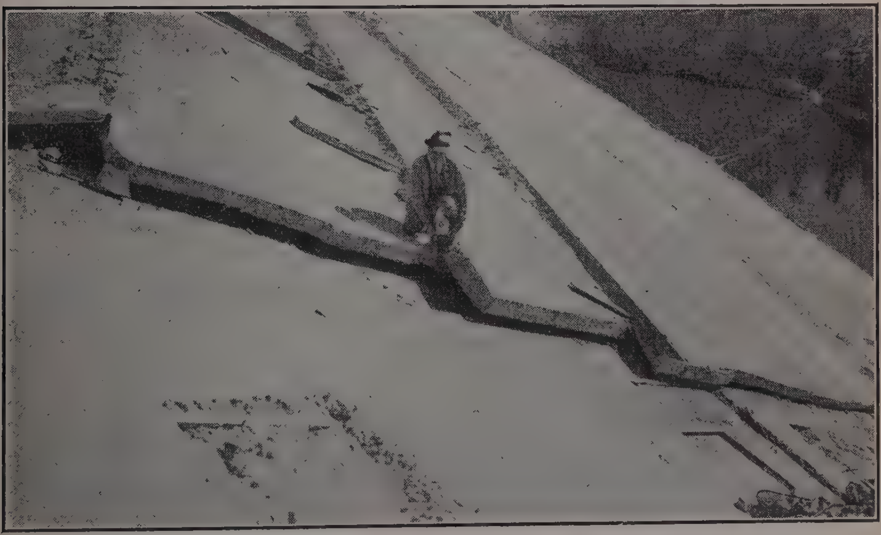

FIG. 13.—FAILURE OF CONCRETE FACING, SAN GABRIEL DAM NO. 2, IN CALIFORNIA.

Another question that arises in connection with this paper is the lamination of face slabs. From time to time various engineers have either recommended or built the facing on rock-fill dams laminated instead of in a single slab. San Gabriel Dam No. 2, built by the Los Angeles County Flood Control District on the San Gabriel River in 1930, was constructed with laminated slabs. Fig. 13 shows what happened. The settlement of the dam was irregular, and the upper slabs were offset enough at the joints so that they did not bear across the joint against the slab below. Hence, the slabs slipped over or under each other, and the corners sheared and spalled off.

If the facing had been a single slab instead of several slabs the same offset at the joints would still have left a considerable proportion of the joint area for transmitting thrust down the slope to the adjacent slab below. However, the idea of the laminated slab runs into still other undesirable conditions. It becomes difficult to anchor such a slab to the slab beneath it, even with reinforcement steel, as was done in the case shown in Fig. 13, so that there is a decided tendency for the top slab to slip on the plane top surface of the slab below. If the facing is poured as a single concrete slab on the rubble cushion, as recommended by Mr. Galloway, the slab is fixed to the rubble by bond, and by the shear in the concrete which fills the interstices in the rock so that the tendency to slide down hill is minimized. The possibility of the slab sliding down the slope has led to a general practice of establishing horizontal benches at intervals on the up-stream face of the rubble cushion, which benches are filled with concrete and act as anchors or thrust-blocks for the concrete slab.

The author's contention that the rock-fill should be sluiced while placing is a very important one, because if the fill is placed otherwise, the later rains wash the fines downward, settlement is excessive, and trouble is experienced with the concrete facing slab.

The author states that at Salt Springs the sheet-copper water-seals were brazed together. Experience with gravity dams has shown that the heat requirements of brazing seem to be such as to cause a burning of the copper, and a piling of braze upon braze, because of the rapid cooling and consequent poor job, until a kind of stiffener effect is formed.

The brazing metal, on cooling, becomes brittle, and repeated bending tests caused the braze to crack and break away from the copper. A better method has been to rivet and solder. Tinners' copper rivets, about $\frac{1}{8}$ in. in diameter, are spaced on $\frac{3}{4}$-in. centers in each of two rows, being staggered, say, $\frac{3}{4}$ in. and $1\frac{1}{2}$ in. from the end of each sheet of copper. The sheets of copper, as shipped, are tinned on the ends for about 2 in., and can be soldered easily and thoroughly. A thin film of solder results, by sweating, over the 2 in. of contact, using a soldering iron, and also a blow torch to heat the entire contact area and sweat it together. The resultant joint is as thin as possible and, consequently, stands repeated deflections without cracking.

H. B. MUCKLESTON,[4] M. AM. SOC. C. E. (by letter).[4a]—Apparently the only recorded failures of the rock-filled type of dam were not due to defective design, but to inadequate spillway capacity. The Walnut Grove Dam, cited by the author in Table 1, Item No. 4, was designed with an up-stream slope of 20 : 47. The rear slope was 70 : 108, with a talus of loose rock on 1 : 1 slope for about one-half the height. It stood successfully for two years and would doubtless be still standing had the spillway been sufficient. The Escondido Dam (Item No. 7, Table 1) nearly as frail, is still (1937) doing its work after forty years. In both dams the slope of the water-face was much steeper than the angle of repose for loose rock, and dry rubble retaining walls were necessary to hold them. Both dams are excellent illustrations of the "great Hydraulic Principle" enunciated in 1912 by Mr. George L. Dillman[5]: "Construct one impervious surface, and build the rest of the structure to support that surface."

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[4] Cons. Engr., Vancouver, B. C., Canada.

[4a] Received by the Secretary November 6, 1937.

In solution of that principle, the author advocates reinforced concrete for the impervious surface and a backing or cushion of dry rubble masonry to minimize the effects of unequal settlement within the rock mass which is the supporting structure. Experience seems to show that this manner of construction meets the requirements successfully, but it also seems to show that it is not the only manner of construction that will do so. In every case, the designer must find the answer to the question: What type of construction will "make the owner's dollar earn the most interest?"

The object of the dry rubble backing is to take up and distribute the inequalities of settlement within the rock mass so that the impervious surface will not be ruptured, either directly by water pressure, or indirectly by unequal settlement. If this end can be achieved by cheaper means than dry rubble, the end will justify its adoption. Dry rubble work is expensive and, moreover, the type of skilled labor needed for its construction seems to be nearly extinct in many localities.

The cause of settlement in a loose rock mass is mainly crushing of the individual rocks at points of contact; and inequality of settlement is mainly due to localized arching. Both can be greatly minimized if the voids in the rock mass are filled with a fairly well graded but coarse material, such as a mixture of coarse sand and quarry waste. Such a material could be sluiced into the mass from the up-stream face and allowed to take its own slope through the fill. The escaping water would find its way out through the down-stream face taking the undesirable fines with it. The resulting mass should be stable; it would be sufficiently pervious; and settlement should be relatively small and much more nearly uniform. Thus constructed, there would be no need for the rubble backing, and the thickness of the impervious surface could be fixed by consideration of impermeability alone.

For the impervious surface, many materials are available. If the dam is for head only and if storage is not a factor, a well-built timber surface should be almost everlasting. If storage is a great factor, the life of a timber face may be short but, even so, it may be the cheapest in the end.

If an adequate supply of material were available, a thick blanket of puddled clay, loaded with rock for protection, and also to confine it, would meet all requirements for impermeability. It would be sufficiently plastic to stand considerable deformation without rupture, and it should be much cheaper than concrete. If clay is not available, a blanket of well-graded sand, impregnated with a heavy asphaltic road oil, would serve as well. It probably is not possible to consolidate clay by artificial means as thoroughly as Nature has sometimes done it. The writer knows of one clay deposit (underlying the Bassano Dam, in Alberta, Canada) which was so dense that a disk 6 in. in diameter by 1 in. thick, under a head of 50 ft scarcely passed sufficient water in 24 hr to fill a tea-cup. Asphaltic concrete would be impervious—more so than cement concrete—and it would be flexible to a high degree. It would have to be loaded, however, to hold it on a slope of $1\frac{1}{2}:1$. On one small dam, 30 ft high in a V-shaped gorge, the writer used old brick coated with, and laid up in, asphalt. It has been in service for twenty years and is apparently good for a hundred years more.

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[5] *Transactions*, Am. Soc. C. E., Vol. LXXV (1912), p. 52.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
DISCUSSIONS

DESIGN OF REINFORCED CONCRETE IN TORSION

Discussion

By Messrs. C. W. Deans, and L. E. Grinter

C. W. Deans,[6] Esq. (by letter).[6a]—The interesting method presented by Professor Andersen is rational, and the resulting designs appear to be safe. The material in Table 2 could be presented in such a manner that the final results are found more readily by inspection. Table 3 demonstrates the writer's suggestion. The lines underscoring the balancing moments indicate that the joint moments are balanced at this stage. If the individual carry-over moments (see "C. M." in Table 3) are recorded as shown, the total un-

TABLE 3.—Suggested Simplification of Table 2

Description*	Joint *x*†				Joint *y*†					Joint *z*†				
	xa	*xb*	*xc*	*xy*	*yx*	*ya*	*yb*	*yc*	*yz*	*zy*	*za*	*zb*	*zc*	*zu*
S. R.	0.08	0.18	0.08	0.33	0.33	0.08	0.18	0.08	0.33	0.33	0.08	0.18	0.08	0.33
C. F.	0.5	0.5	0.5	1.0	1.0	0.5	0.5	0.5	1.0	1.0	0.5	0.5	0.5	1.0
F. M.	..	..	..	..	..	..	..	..	..	..	..	..	..	+540
D.	..	..	..	..	..	..	..	..	..	−180	−40	−100	−40	−180
C. M.	..	..	..	..	..	..	..	..	−180	..	..	..	..	..
D.	..	..	..	..	+60	+14	+32	+14	+ 60	..	..	..	..	..
C. M.	..	..	..	+60	..	..	..	..	..	+ 60	..	..	..	+ 60
D.	−5	−10	−5	−20	..	..	..	..	..	− 40	− 9	− 22	− 9	− 40
C. M.	..	..	..	..	−20	..	..	..	− 40	..	..	..	..	..
D.	..	..	..	..	+20	+ 5	+10	+ 5	+ 20	..	..	..	..	..
C. M.	..	..	..	+20	..	..	..	..	..	+ 20	..	..	..	+ 20
Total	−5	−10	−5	+60	+60	+19	+42	+19	−140	−140	−49	−122	−49	+400

* S. R. = stiffness ratio; C. F. = carry-over factor; F. M. = fixed-end moment; D. = distribution; and, C. M. = carry-over moment. † See author's Fig. 3(*c*).

Note.—The paper by Paul Andersen, Assoc. M. Am. Soc. C. E., was published in October, 1937, *Proceedings*. This discussion is published in *Proceedings*, in order that the views expressed may be brought before all members for further discussion of the paper.

[6] Estimator, Western Bridge Co., Ltd., Vancouver, B. C., Canada.

[6a] Received by the Secretary October 27, 1937.

balanced moments that are ready for distribution at each step are apparent. In the example solved, the moments in members meeting at Joints u and v are identical to corresponding members meeting at Joints x and y, respectively, and can be omitted.

Attention should be called to Equation (12), in which p is defined, simply, as the pitch. It should be made clear that this is the value of the pitch at right angles to the spiral bars and not the distance between bars measured parallel to the axis of the spiral. The writer had some difficulty in checking the formula until he recognized this fact.

Corrections for *Transactions:* In the first diagram inset in Table 1, change 150 kips to 37.5 kips and change the lever arm from 2 ft to 8 ft; and in Diagram (a), Table 1, insert M beneath 150 kips.

L. E. GRINTER,[7] ASSOC. M. AM. SOC. C. E. (by letter).[7a]—The thought of studying a continuous framed structure as a space problem may seem a great refinement that strains all practical conceptions of design. However, the writer takes the position that no refinement of analysis is wasted that clarifies the picture of the action of the structure. Certainly it is a serious crudity to neglect entirely the torsional resistances of large beams and girders in the design of continuous frames. Whether a full space-frame study can be justified will depend largely upon the relative flexural and torsional resistances of the various members.

Distribution Factors.—The stiffness factor is proportional to $\frac{E\,I}{L}$ for a beam, whereas this factor is $\frac{E_s\,J}{L}$ for a circular torsional section, J, of course, being the polar moment of inertia. In this form the comparison is evident although the latter expression can be determined readily from an elementary study of virtual work. Since $\theta = \int \frac{T\,t'\,ds}{E_s\,J}$ and since both T and t', the real and virtual torque moments, will be constant and equal to unity, $\theta = \frac{L}{E_s\,J}$; and $\frac{1}{\theta}$, the stiffness factor, becomes $\frac{E_s\,J}{L}$. Obviously, the carry-over factor is unity since a torsional moment passes from one end to the other of a member without reduction.

Signs of Moments.—Several sign conventions would be possible in the study of continuity in space frames. However, there seems little justification in this case for any convention other than the one introduced by the writer for balancing moments in planar frames.[8] Although a sign convention based on fiber stresses (cantilever moment negative) may serve adequately for continuous beams and even passably for planar frames, it is fundamentally unsound for the study of space structures because of its inapplicability to torsional problems.

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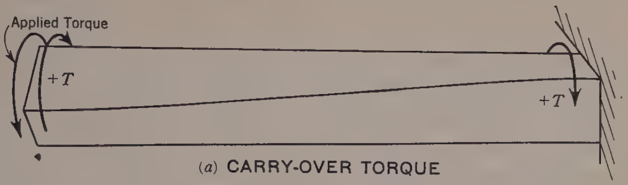
[7] Dean, Graduate Div., and Director of Civ. Eng., Armour Inst. of Technology, Chicago, Ill.

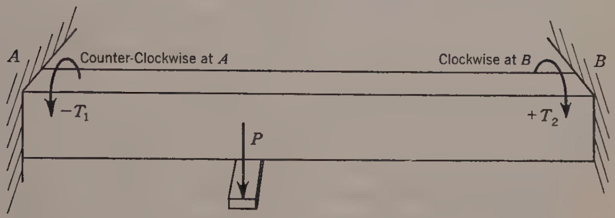
[7a] Received by the Secretary November 10, 1937.

[8] *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 14.

The basic sign convention should be that a positive moment (either flexural moment or torque moment) is one which tends to rotate the adjacent joint clockwise. Fig. 4 illustrates this sign convention applied to torque moments.



(a) CARRY-OVER TORQUE



(b) FIXED-END TORQUES

FIG. 4.—SIGN CONVENTION FOR TORQUE MOMENTS.

The sign of the carry-over factor based on the foregoing sign convention has the usual positive value for moment distribution. Furthermore, a positive torque moment must also be carried over with the same positive sign (see Fig. 4(*a*)). Hence, the common carry-over factor for bending moment, which is + 0.5, becomes + 1.0 for torque moment. The entire sign convention for planar frames, therefore, remains unchanged for solving space frames. No confusion is possible if one always considers only the action of the member on the joint, which is the fundamental conception used in classical truss analysis by the graphical method.

Fixed-End Torque Moments.—As the author mentions, when an internal torque moment is applied, the fixed-end torque moments are supposed to be inversely proportional to the relative distances to the two ends of the member. For instance, if a torque moment, T, is applied to the member, AB, as shown in Fig. 5, the larger resisting moment occurs at the near end, A, and is equal to,

$$M = T\left(\frac{b}{a + b}\right) \dots\dots\dots\dots\dots\dots\dots\dots (15)$$

This will be evident from the fact that the rotation over the length, a, must be equal to the rotation over the length, b. Since the rotation is proportional to the acting torque moment and to the length, the relation expressed in Equation (15) follows for a homogeneous beam. For a haunched beam or one in which the torsional resistance is influenced by the addition of much steel for shear resistance, this simple relationship does not hold. Approximate fixed-end torque

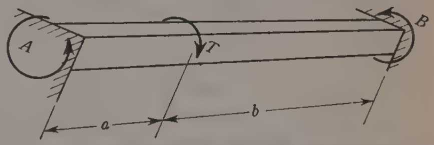


FIG. 5.—FIXED-END TORQUE MOMENTS.

moments for a haunched beam might still be found by an integration. The writer calls attention to the fact that such fixed-end torque moments must be accepted as approximate since the action of vertical or diagonal shear reinforcement for torsional resistance is not predictable with mathematical accuracy. It is worth mention that torsional resistance or stiffness for reinforced concrete depends very largely upon the resistance of stirrups, the most unpredictable quantity in the analysis of reinforced concrete. In contrast, the flexural resistance or stiffness of a beam is dependent principally upon the action of tension steel, the most predictable factor in the analysis of reinforced concrete.

Arrangement of Computations.—The author appears to favor the use of tabulated coefficients for his analysis. More than a decade of experience with the method of balancing moments and with numerous similar balancing procedures seems to have convinced the originator of the method[9] and others who have used it widely, that such tabulation leads to no benefit. In fact, it eliminates the major reason for the choice of the method of balancing moments; that is, the close connection between the action of the structure and the arithmetic procedure of the computations. The use of a table of moments leads to a purely routine procedure and divorces the computations from the possibility of their interpretation as successive causes of the structural action. Fig. 6 was prepared to show the simplicity of arrangement of the computations at their proper places on a line perspective of the structure.

These computations also illustrate the desirability of combining torque and flexural moments into one analysis although they must be entered in the separate groups designated in Fig. 6 as T and F. The writer's preference for this attack is simply that it is not possible to divorce these separate actions anyway because a flexural moment can turn into a torque moment in passing around a corner. Since torque cannot be studied separately from flexure in space frames, there seems little reason to separate the terms in the manner suggested by the author. The final point to notice in Fig. 6 is the separation of the flexural moments in the columns under the heading of F and F'. This designation is used to indicate that these flexural moments act in different planes and, hence, must be applied individually in design.

Accuracy in Balancing Moments.—It will be noticed that the final moments computed in Fig. 6 do not check exactly with those shown by the author in Table 1. It is of no moment which solution is more nearly exact since either would be more accurate than the standard tools of internal stress analysis for reinforced concrete sections. For instance, one flexural moment is computed as 75 kip-ft by the author and as 77 kip-ft by the writer. The variation of 3% is about the standard of accuracy that the writer considers desirable for the analysis of concrete framed structures. Naturally, it is understood that this value does not represent the probable inaccuracy inherent in any attempt to predict the true stresses in the actual structure.

Resistance of Reinforced Concrete in Torsion.—The author has clarified the necessary constants for investigating the stresses in a rectangular reinforced concrete section under torsion. This analysis is subject at least to the same questioning that in the past has been directed to the analysis of reinforced concrete beams. Actually, one should feel some question about the use of Equations (6) to (14) until tests have justified the author's procedure in the same manner that they have justified the ordinary theory of reinforced concrete beams. However, it is natural and proper that theory should precede full confirmation by tests.

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[9] "Analysis of Continuous Frames by Distributing Fixed-End Moments," by Hardy Cross, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 5.

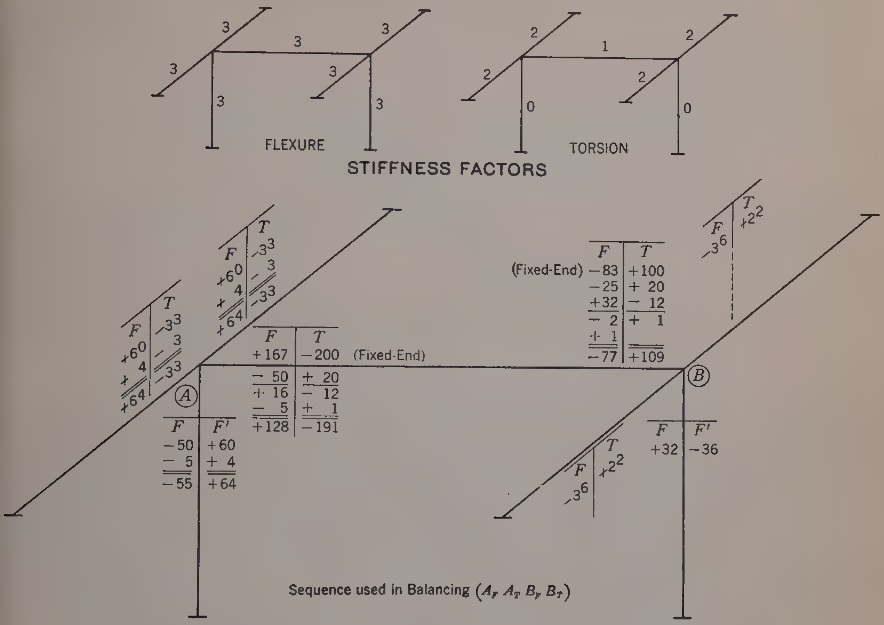

FIG. 6.—BALANCING MOMENTS FOR A SPACE FRAME.

One interesting feature of the design of a reinforced concrete crane girder for torsion, as indicated by the author, is the use of continuous diagonal stirrups. Since vertical stirrups and horizontal rods have been found quite adequate for resisting diagonal tension resulting from shear and flexure, there would seem to be no necessity for changing this proved and simple device when torsion is present. Failure is still by diagonal tension. The design criterion should still be to space hoops of any type in such a manner that every crack must cross at least one shear bar which itself is anchored adequately. The slope of the bar is less important than its area and anchorage.

Conclusion.—The author has served the profession by collecting in one paper many data on the analysis and design of reinforced concrete space frames, in which torsion is almost inevitably present. Perhaps this excellent paper may help convice designers that it is not too difficult, or at least not impossible, to design a building frame as a space structure.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
DISCUSSIONS

ECONOMICS OF THE OHIO RIVER IMPROVEMENT

Discussion

By Eugene L. Grant, Assoc. M. Am. Soc. C. E.

Eugene L. Grant,[24] Assoc. M. Am. Soc. C. E. (by letter).[24a]—The author is to be congratulated on his attempt to take the question of the economic justification for a high investment in the improvement of inland waterways out of the realm of conjecture and into the realm of measured facts, and for his careful statement of the assumptions of his study. Likewise, the Corps of Engineers should be commended for its records of river traffic which made possible such a factual study. However, a critical examination of the stated assumptions and of the calculations of Table 5 casts considerable doubt on the expressed conclusion that the Ohio River improvement has been an economic success.

The most important of the assumptions, of course, is Assumption (1) to the effect that all savings are passed on to the public in the form of lower prices. Even under the assumption of free competition of classical economic theory this is not necessarily the case. A saving in transportation costs to all the competing producers in a given industry would be passed on, but a saving to a limited number of producers might or might not be passed on, depending on their competitive positions—that is, on whether or not they are "marginal" producers in the classical economist's sense. Thus, even under free competition it would be likely that a considerable part of the saving in transportation costs from the Ohio River improvement would not be passed on. Under the well-known fixed-price structure of the steel industry, which has been in existence throughout the period of this study, it seems probable that, as far as this industry is concerned, none of the savings has been passed on; there has merely been a transfer of funds from the owners of the railroads that would have carried the traffic, had there been no river improvement, into the pockets of the owners of the steel companies.

Note.—The paper by C. L. Hall, M. Am. Soc. C. E., was published in October, 1937, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion of the paper.

[24] Associate Prof. of Economics of Eng., Stanford Univ., Stanford University, Calif.

[24a] Received by the Secretary November 3, 1937.

With regard to Assumption (2), it may be noted that although the difference between the costs of water transportation and rail rates is a fair measure of the economic advantage to the shipper in instances where the commodity would move by rail if water transportation were not available, this is not the case with traffic that would not move at all if only rail transportation were available. Where traffic is created by the availability of cheap water transportation, the economic advantage to the shipper is the difference between a rate which would just serve to move the traffic (that is, "what the traffic will bear") and the cost of water haul. Although this is not measurable, it constitutes a reason for discounting, somewhat, the author's estimates of savings.

In connection with Assumption (3)—that the savings in the cost of ferriage have been neglected—it might be noted that the increased costs of highway and railway bridges due to the requirements of navigation have also been neglected.

Thus, it appears that the author's carefully stated assumptions favor the waterway to a much greater degree than he has indicated.

Table 5 is an example of a somewhat unconventional method of setting up an engineering economy study.[24b] It is in effect a compound amount calculation for all the disbursements and receipts (assuming "annual commercial savings" as receipts), with interest at 4 per cent. Such a calculation with respect to a terminated enterprise would tell whether the invested moneys had been recovered with a 4% return; however, its suitability for judging the economic success of the Ohio River improvement may be questioned.

This is particularly the case because there are apparently two separate questions which should have been asked: (1) Did the moderate level of improvement of the early days (with $13 000 000 invested before 1905), pay for itself in reduced transportation costs? and (2) does it appear as if the present canalization of the river (with its investment of $135 000 000) is likely to pay?

From an inspection of Table 5, the historical picture seems to be somewhat as follows:[24b] In 1905, and for a few years thereafter, the investment in river improvement paid handsomely (not as much, however, as is implied by the 266% annual return shown for 1905 and the high returns for the years immediately succeeding; these high values resulted because the author failed to include the $13 300 000 spent on capital improvements before 1905 as part of the investment on which the return was calculated; he also omitted depreciation on these improvements throughout his study). Then the competition of other forms of transportation apparently resulted in rapid obsolescence of the river improvements, and, by 1919, the commercial savings were barely sufficient to cover operation and maintenance costs, with no allowance for depreciation or return on investment. After this the increasingly high level of river improvement served to stimulate river traffic to the point of increasing the annual commercial savings to more than $12 000 000 in 1934.

[24b] Corrections for *Transactions:* The legend at the head of Column (24), Table 5, should read "+Column (14)" instead of "—Column (14)." All the values of page 1503 are affected by the change, detailed corrections to be made for *Transactions*. Figs. 2 and 3 will then be corrected correspondingly. These modifications do not affect the conclusions.

The author's plan of writing off subsequent investments against previous savings, and of assuming an interest income from calculated "surplus," has the tendency to show last year's savings as a return on next year's improvements. It should be clear that this is misleading—that the high commercial savings of the early years which resulted in the "surplus" in the years 1906 to 1912 have no relation at all to the question of whether the subsequent higher level of improvement will prove profitable.

Another objection to the use of the type of calculation involving the determination of a compound amount is the great importance of the assumed interest rate on the apparent conclusion. This dependence of the conclusion on the interest rate assumed is not always obvious either to the engineer making such calculations or to his audience. If, for instance, the author had used 6% as his assumed interest rate, his conclusion would have been that no profit existed in any year after 1915; this would have implied that the high level of improvement of the Ohio River was not an economic success.

The real question with regard to a proposed investment in capital goods—either in private enterprise or in public works—is whether the investment seems likely to be recovered during the economic life of the capital goods plus a return which is attractive in the light of the apparent risks involved. In private enterprise, it is customary not to make such investments unless the prospective rate of return is considerably greater than the cost of money; thus a concern with an average cost of capital of 7% might be likely to require a prospective return of 15% before undertaking a proposed investment. Where the possibility of obsolescence exists, the required safety factor may be much greater; a common requirement in the manufacturing industries is that a proposed investment in cost-reducing machinery must "pay for itself" in two or three years, or, in some cases, in one year.

Two considerations should be noted in connection with interest rates in economy studies regarding public works projects. The first is that when the Government collects taxes it is taking funds which might otherwise be productively invested by the taxpayers. It follows that public works so financed are not socially profitable unless they show capital recovery with a return at least as great as the return from the private investments which are displaced by the diversion of taxpayers' funds to public uses. This is a sound principle even if the difficulties of placing a money value on the benefits from public works make it difficult to calculate a rate of return in many instances; but where benefits are measured, as in the present instance, a project is not economically successful unless the return is as great as the average return obtainable by the investment of capital in private enterprise.

The second consideration in selecting an interest rate for economy studies for public works projects is that of the necessity of a safety factor. The high safety factors commonly used with regard to proposed cost-saving improvements in private enterprise have been mentioned. These safety factors are used because experience has shown that estimates of savings and of the economic lives of capital goods are likely to "go wrong." As errors in such estimates are not confined to private enterprise, it would seem reasonable that a safety factor should also be used in public works projects. This is particularly true in the present case, in which many arbitrary assumptions necessarily enter into the evaluation of benefits. It may also be noted that, although obsolescence does not operate as rapidly in public works as in many private enterprises, the history of the Ohio River navigation presented by the author shows a rapid obsolescence of the earlier moderate level of improvement.

The use of a 4% interest rate in this economy study is in effect the assumption that 4% is a satisfactory return on the Government's investment in the Ohio River improvement. In the light of the considerations which have just been mentioned it would seem as if the return should be considerably higher than this, before it can be said that the improvement has been economically justified.

For this reason it should be of interest to determine the apparent rate of return in each of the thirty years of record, making calculations which avoid any interest charges or any assumed interest rate. At first glance, it might appear that Column (20) of Table 5, gives the excess above, or deficiency below, a 4% return. This is not the case, however, because of the author's neglect of the investment prior to 1905 and because of the fact that the interest charge or credit has been based on the "cumulated capital account" rather than on the depreciated investment. Therefore, the writer has computed, in Table 7, the apparent profit or loss that would have been shown each year in the income statement of a private enterprise with receipts equal to the commercial savings of Column (16), Table 5, and operation and maintenance costs equal to the

TABLE 7.—ANNUAL PROFIT AND LOSS STATEMENT (IN MILLIONS OF DOLLARS) AND RATE OF RETURN ON DEPRECIATED INVESTMENT FOR OHIO RIVER IMPROVEMENT

Fiscal year	Gross income (1)	Annual expense operation, maintenance, and depreciation (2)	Net income Column (1) − Column (2) (3)	Plant investment less depreciation (4)	Percentage return on depreciated investment, Column (3) ÷ Column (4) (5)	Fiscal year	Gross income (1)	Annual expense operation, maintenance, and depreciation (2)	Net income Column (1) − Column (2) (3)	Plant investment less depreciation (4)	Percentage return on depreciated investment, Column (3) ÷ Column (4) (5)
1905...	3.03	0.56	2.47	10.69	23.1	1920..	2.44	2.27	0.17	50.33	0.3
1906...	3.05	0.49	2.56	11.59	22.1	1921..	2.17	2.57	−0.40	55.84	−0.7
1907...	3.06	0.56	2.50	13.08	19.1	1922..	2.01	3.00	−0.99	58.80	−1.7
1908...	2.39	0.60	1.79	14.06	12.7	1923..	2.75	3.59	−0.84	62.34	−1.3
1909...	2.14	0.73	1.41	15.68	9.0	1924..	3.41	3.67	−0.26	66.70	−0.4
1910...	2.44	0.72	1.72	16.53	10.4	1925..	5.71	4.05	1.66	73.13	2.3
1911...	3.10	0.76	2.34	17.77	13.2	1926..	6.54	4.35	2.19	79.56	2.8
1912...	1.92	0.99	0.93	19.49	4.8	1927..	7.57	4.01	3.56	84.53	4.2
1913...	2.16	1.06	1.10	21.57	5.1	1928..	8.68	4.48	4.20	92.41	4.5
1914...	1.69	1.05	0.64	25.53	2.5	1929..	10.28	5.01	5.27	95.44	5.5
1915...	1.61	1.17	0.44	32.74	1.3	1930..	10.09	5.75	4.34	96.68	4.5
1916...	1.22	1.31	−0.09	37.23	−0.2	1931..	10.03	6.29	3.74	96.15	3.9
1917...	0.93	1.41	−0.48	40.17	−1.2	1932..	8.75	7.20	1.55	93.93	1.7
1918...	1.15	1.81	−0.66	42.40	−1.6	1933..	10.36	7.97	2.39	91.70	2.6
1919...	1.03	2.01	−0.98	46.26	−2.1	1934..	12.36	6.94	5.42	90.34	6.0

values shown in Column (7). In the absence of any information regarding the dates of the $13 000 000 investment prior to 1905, the depreciated investment at the beginning of 1905 has been assumed arbitrarily as $10 000 000 and depreciation has been charged on the investment on the basis of an assumed 40-yr life; the annual depreciation charges shown in Column (14), Table 5, are thus increased by $250 000. All values have been expressed to the nearest $10 000, as it does not seem that more significant places are justified by the character of the available data.

The record as shown from 1912 to 1934 in Column (5), Table 7, is not impressive; the rate of return is never great enough to be considered attractive in the light of the risks involved. It will be noted that this is so despite the fact that Table 7 gives the waterway considerably more than it is entitled to by assuming the revenue as 100% of the author's estimate of commercial savings. If, as suggested at the beginning of this discussion, much of the savings in transportation costs are not passed on to the public in the form of reduced prices, the "picture" becomes even more unfavorable to the waterway. For instance, if it is assumed that only 50% of the savings are passed on, there has been no profit in twenty-two of the last twenty-three years. It would seem, therefore, that even if weight is given to the "Non-Statistical Considerations" noted at the end of the paper, the Ohio River improvement can scarcely be considered an economic success.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
DISCUSSIONS

PRACTICAL APPLICATION OF SOIL MECHANICS
A SYMPOSIUM

Discussion

BY EDWARD ADAMS RICHARDSON, ESQ.

EDWARD ADAMS RICHARDSON,[50] ESQ. (by letter).[50a]—This discussion of the paper by Mr. Hough will touch upon three subjects: (1) The proper stress to choose for the limiting shear in the case of plastic materials; (2) factors affecting the computation of the factor of safety of the foundation; and (3) methods for improving the design of such foundations as suggested by the tests and investigations.

In one of his model tests, the author utilized "a bed of uniform, remolded clay" to represent the foundation material. Such remolded clay exhibits materially lower shear resistance than the undisturbed material. It seems improper to base the design of a foundation on the strength of the fully remolded material rather than on that of the undisturbed material.

Considerable kneading is required to convert the original clay body to the uniform, fully remolded condition. This is true even in the case of very soft clay. This amount of kneading appears to be greatly in excess of that which may be found in the most stressed elements of material beneath the foundation. Most of the material that flows plastically can scarcely be said to be subjected to appreciable kneading action. Such action is measured by the relative movement or strain of adjacent particles of clay; not by the distance moved by the most traveled point of the plastic body as measured relative to a fixed point on the boundary. That being the case, it would appear that the proper value of the limiting shear stress for the plastic material should be closely that of the undisturbed material. In view of the considerable differences in this limiting stress, it becomes a matter of great importance to choose the value most nearly approximating actual conditions if an economical structure is to be secured.

NOTE.—This Symposium was presented at the meeting of the Soils Mechanics and Foundations Division, at Boston, Mass., October 7, 1937, and published in September, 1937, *Proceedings*. Discussion on this Symposium has appeared in *Proceedings*, as follows: September, 1937, by the members of the Committee of the Society on Earths and Foundations; and November, 1937, by Messrs. S. C. Hollister, T. T. Knappen, and L. F. Harza.

[50] Associated with Bethlehem Steel Co., Bethlehem, Pa.

[50a] Received by the Secretary November 9, 1937.

The factor of safety is computed by the Haines analysis through an adaptation of the method presented by K. E. Petterson, a member of the Swedish Geo-Technical Commission. Due references were made to the investigations by Jurgenson[51] and Professor Dr. H. von Krey. In using similar methods, as in the case of sloping banks, it has been customary to apply loads to the surfaces of the rotating segment and then to assume that such added loads increase the friction of the sector on the surface of sliding, or at least contribute a friction force along this surface. In the case of dams, the friction force thus calculated may be considerable. It should be noted that a plastic clay segment does not develop any friction force due to such loads until time for consolidation has elapsed. As this time may be long (perhaps nearly as long as the life of the structure) such friction forces should be omitted from the initial stability calculations. It will be recalled that any load added to clay is initially carried entirely by hydrostatic pressure in the contained water and not through particle contacts; therefore, no friction force can be developed by the added load.

An approximation to the curved part of the surface of failure may be obtained in the following manner: The equations for the shearing stresses involved are those for an elastic body of indefinite extent loaded at a point of the surface, and were derived from the corresponding case given by Mr. John Prescott.[52] Such a derivation yields not only the p_z of the Boussinesq theory, but also p_x, p_y, and the shearing stresses.

For reference purposes, the formulas for all stress components at (x, y, z) are given herein, as Equations (11) and (12). Only Equation (11*c*) for p_z is commonly given. Fig. 57 may be used as a diagram, with OY perpendicular to the X–Z plane. Poisson's ratio is $\frac{1}{m}$. Direct stress components are p_x, p_y, and p_z parallel to OX, OY, and OZ, respectively. Shear stress components are s_1, s_2, and s_3, the axes of torque being likewise parallel to OX, OY, and OZ, respectively. That is, the stresses for s, producing a torque component about an axis parallel to OX, may be considered to lie in a plane, containing (x, y, z), which is parallel to the Y–Z plane. The equations are:

$$r = \sqrt{x^2 + y^2 + z^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (11a)$$

$$p_x = \frac{P}{2\pi}\left[\frac{r^5 - 2r^3y^2 + r^2z^3 - 2r^2x^2z - 6rx^2z^2 - 3x^2z^3}{r^5(r+z)^2} - \frac{2}{m}\,\frac{r^5 - 2r^3y^2 + r^2z^3 + r^2x^2z}{r^5(r+z)^2}\right] \quad \ldots\ldots\ldots\ldots (11b)$$

$$p_y = \frac{P}{2\pi}\left[\frac{r^5 - 2r^3x^2 + r^2z^3 - 2r^2y^2z - 6ry^2z^2 - 3y^2z^3}{r^5(r+z)^2} - \frac{2}{m}\,\frac{r^5 - 2r^3x^2 + r^2z^3 + r^2y^2z}{r^5(r+z)^2}\right] \quad \ldots\ldots\ldots\ldots (11c)$$

---

[51] "The Shearing Resistance of Soils," by Leo Jurgenson, *Journal*, Boston Soc. of Civ. Engrs., Vol. XXI, No. 3, July, 1934, p. 242.

[52] "Applied Elasticity," by John Prescott, Chapter XIX, p. 623, Longmans, Green & Co., London, 1924.

$$p_z = \frac{P}{2\,\pi}\left[-\frac{3\,z^3}{r^5}\right] \qquad (11d)$$

$$s_1 = \frac{P}{2\,\pi}\left[-\frac{3\,y\,z^2}{r^5}\right] \qquad (11e)$$

$$s_2 = \frac{P}{2\,\pi}\left[-\frac{3\,x\,z^2}{r^5}\right] \qquad (11f)$$

and,

$$s_3 = \frac{P}{2\,\pi}\left[\frac{(2\,r^3 - 2\,r^2\,z - 6\,r\,z^2 - 3\,z^3)\,x\,y}{r^5\,(r+z)^2} - \frac{2}{m}\,\frac{(2\,r^3 + r^2\,z)\,x\,y}{r^5\,(r+z)^2}\right] \qquad (11g)$$

When $y = 0$, or OX is so chosen that the X–Z plane contains r, the case of Fig. 57 is obtained. It should be obvious that the symmetry of point loading is such that all points having the co-ordinates, r, α, will have the same stress components on the plane, X–Z, chosen to contain r; hence Fig. 57 is a cross-section upon which the stress system is fully characterized for the entire body. The corresponding equations are:

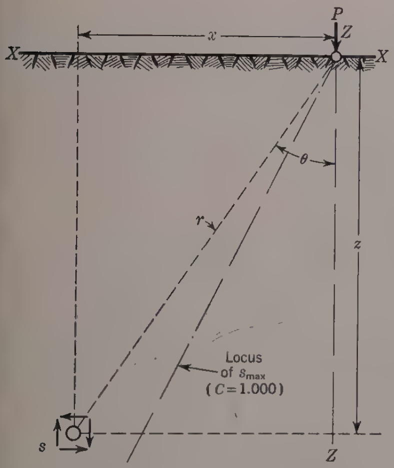

FIG. 57.—LINE OF SEPARATION, ELASTIC AND PLASTIC REGIONS UNDER A POINT LOAD.

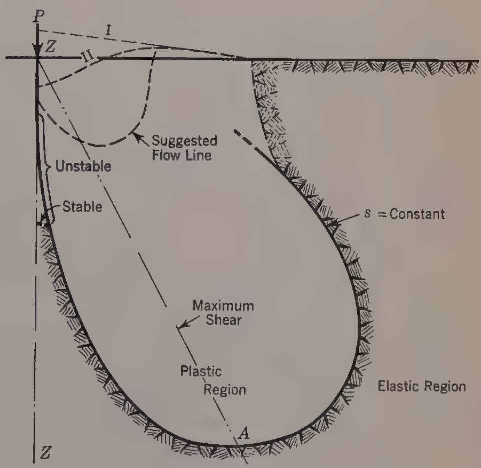

FIG. 58.

$$r = \sqrt{x^2 + z^2} \qquad (12a)$$

$$p_x = \frac{P}{2\,\pi}\left[\frac{r^5 + r^2\,z^3 - 2\,r^2\,x^2\,z - 6\,r\,x^2\,z^2 - 3\,x^2\,z^3}{r^5\,(r+z)^2} - \frac{2}{m}\,\frac{r^5 + r^2\,z^3 + r^2\,x^2\,z}{r^5\,(r+z)^2}\right] \qquad (12b)$$

$$p_y = \frac{P}{2\,\pi}\left[\frac{r^5 - 2\,r^3\,x^2 + r^2\,z^3}{r^5\,(r+z)^2} - \frac{2}{m}\,\frac{r^5 - 2\,r^3\,x^2 + r^2\,z^3}{r^5\,(r+z)^2}\right] \qquad (12c)$$

$$p_z = \frac{P}{2\pi}\left[-\frac{3z^3}{r^5}\right] \quad \text{(12d)}$$

$$s_1 = 0 \quad \text{(12e)}$$

$$s_2 = \frac{P}{2\pi}\left[-\frac{3xz^2}{r^5}\right] \quad \text{(12f)}$$

and

$$s_3 = 0 \quad \text{(12g)}$$

By combining point loads and their stresses, it is known that solutions approximating the present case may be obtained. The stresses in the soil will be closely approximated only when the entire soil mass behaves elastically under the applied load and the earth is of indefinitely great depth. The approximation may be good in those cases in which at least a portion of the earth is subjected to shearing stresses beyond the yield point of the material. In such cases, one may determine a surface of shearing for the limiting shear values from point to point. This surface will set the boundary approximately between the elastically strained material and that which is in the plastic condition. Whether plastic flow does or does not occur will depend upon other factors, such as the restraint opposed to the direct pressures of the plastic portion of the mass. The character of an iso-shear line for a point loading will be seen in Fig. 58. The boundary is determined from a set of these lines for a series of s-values. As drawn, however, the boundary is determined by one s-value.

When the elastic body is a solid of great cohesive strength relative to the force of sliding friction, the boundary between the elastic and the plastic regions will approximate very closely to the s-curve having the value of the cohesive or shearing strength of the material. Treating Fig. 58 as representative of such a material, Curve II suggests the character of surface deformation for elastic behavior, whereas Curve I suggests that for plastic flow when the elastic limit is exceeded. These curves are purely diagrammatic.

If, however, the elastic body is more like a soil body in having a cohesive strength which may be small compared with frictional resistance throughout the stressed region, the procedure is more complicated. Friction increases with soil depth and depends upon the slope of the sliding surface, whereas cohesion remains substantially unchanged, or changes slightly. It is necessary to assign, to the soil, shearing resistance stresses, s_r, appropriate to the z-values investigated. It is also necessary to construct a set of constant (iso-) shear curves for the stresses due to the load, with equal increments in s between adjacent curves, the set to cover a sufficient range of values. The boundary surface is then obtained by choosing a z-value, noting the s_r-value considered appropriate, and plotting this value where the s-curves give its equal. Proceeding in this manner, a number of points may be located and the curve drawn. The s_r-values should now be checked. It is necessary to measure the normal force on the curve, compute the friction force tangential to it, and add the cohesive force. If the s_r-values, so computed for the curve as drawn, check the assumed values, the curve is really the probable surface of failure. If not, a second approximation must be made. It will be observed that this procedure is similar to that of Haines, but differs in that Haines approaches the problem from the standpoint of choosing, by trial, the curve yielding the lowest factor of safety. Whereas the curve to choose is virtually unknown in the Haines method, the stress method enables one to approximate the probable surface of failure.

The procedure used in drawing the iso-shear lines, or lines of constant shearing stress, is based on Equation (12*f*). It may be shown by differentiation of Equation (11*f*), holding z constant, that the maximum shearing stress, s_2, occurs when $x = 0.5\,z$. The value of this maximum stress, s_2, is denoted by s_m and,

$$s_m = \frac{0.1366\,P}{z^2} \qquad \text{(13a)}$$

The value of the shearing stress for any other value of x, z remaining constant, may be found from s_m by using a factor, k;

$$s = k\,s_m \qquad \text{(13b)}$$

By dividing s_2 by s_m, k may be obtained in terms of $\frac{x}{z}$ alone; therefore, k applies to any level, z, provided $\frac{x}{z}$ is the same:

$$k = \frac{25\,\sqrt{5}}{16}\,\frac{x/z}{\{1 + (x/z)^2\}^{5/2}} \qquad \text{(14)}$$

In Fig. 57 the line of maximum shear stresses is shown making the angle, θ, with OZ such that $\frac{x}{z}$, or $\tan \theta_5 = 0.5000$. The right circular cone with such lines for elements is the locus of such maximum shear stresses, s_m, for the point load, P.

In Fig. 59, k has been plotted in terms of $\frac{x}{z}$. Certain other lines numbered n from 0 to 10 cross the chart parallel to the $\frac{x}{z}$-axis. These are located so that,

$$k = 0.01\,n^2 \qquad \text{(15)}$$

Suppose the curve, s, is to be drawn. The first point may be located by substituting s in Equation (13*a*) and solving for z. This point will lie at the z-value thus found and at $x = 0.5\,z$. Divide this z-distance into ten equal parts along OZ, pass horizontal lines through the points of division, and number them, n, from 0 at the surface to 10 at the first line found. Now,

$$s_{mn} = \frac{100\,s^2}{n^2} \qquad \text{(16)}$$

Hence, k at Level n must have the value of $0.01\,n^2$. The lines, n, in Fig. 59 give the $\frac{x}{z}$-values at the n-level where they intersect the k-curve, thus enabling one to determine the two x-values at each n-level. It merely remains to draw the

s-curve through the points so found, and repeat for such other s-curves as may be required for the set. More exact values of k are given in Table 8.

If a distributed load is involved, it becomes necessary to combine stress components at each of a set of points, and thereby build up s_2-values on appropriate planes. In the case of a line loading of indefinite extent, this might be a center plane perpendicular to the line. Integration is easy for s_2. Such planes experience a principal shearing stress and, therefore, are critical.

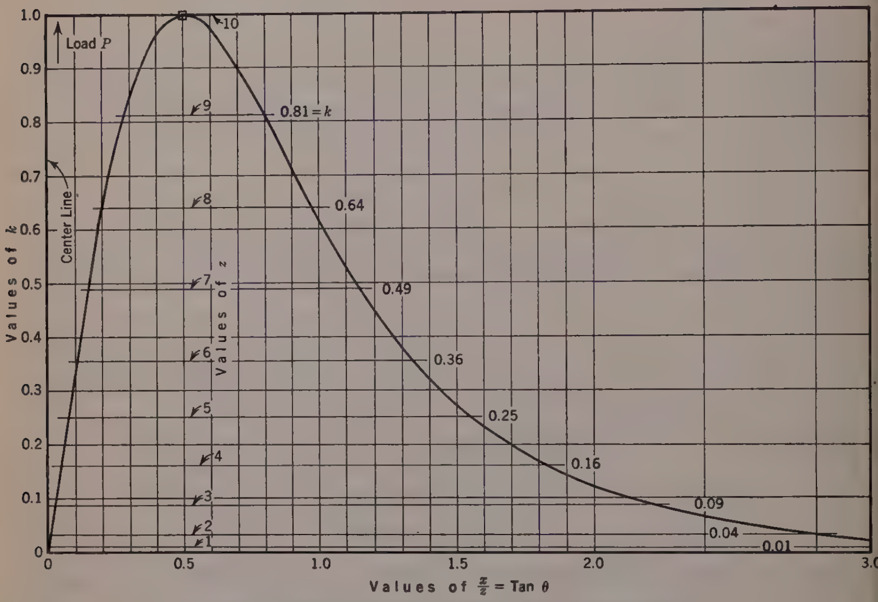

FIG. 59.—RADIAL VARIATION OF SHEAR ON THE X–Z PLANE.

In passing, it should be noted that a flowing plastic body may have higher shear stresses within it than the limiting shear stress for the material. Furthermore, a plastic body, restrained from flowing, cannot support a greater shear stress. That part of the stress system producing the limiting stress in the elastic body remains unchanged in the plastic body. The excess stresses are

TABLE 8.—VALUES OF k, FOR SUBSTITUTION FOR EQUATION (12)

Ratio, $\frac{x}{z}$ (1)	Coefficient, k (2)	Ratio, $\frac{x}{z}$ (1)	Coefficient, k (2)	Ratio, $\frac{x}{z}$ (1)	Coefficient, k (2)	Ratio, $\frac{x}{z}$ (1)	Coefficient, k (2)
0.0	0.0000	0.6	0.9764	1.2	0.4508	1.8	0.1699
0.1	0.3408	0.7	0.9025	1.3	0.3827	1.9	0.1455
0.2	0.6335	0.8	0.8115	1.4	0.3245	2.0	0.1250
0.3	0.8450	0.9	0.7134	1.5	0.2752	2.4	0.0706
0.4	0.9643	1.0	0.6176	1.6	0.2338	2.8	0.0421
0.5	1.0000	1.1	0.5293	1.7	0.1990	3.2	0.0264

purely direct in the case of the non-flowing plastic body. This shift in the character of the direct stresses tends to modify the shape of the surface bounding the plastic region, because the surface computed in Fig. 58 involves the assumption of pure elastic stressing, and direct pressures on the corresponding surface. The shift in the boundary should not be great, however. When the plastic material flows, the stress system in the plastic portion is intermediate between the elastic and the non-flowing plastic cases, and, therefore, should result in even less boundary shift. Plastic flow requires higher shearing stresses than the limiting ones since the rate of flow is proportional to the shearing stress excess.

Methods such as the foregoing are useful in checking the Haines analysis. Since the elastic equations are approximately applicable even if the elastic body consists of two layers of differing properties, it should yield useful results up to the point at which the surface of failure touches the rock boundary for some particular value of $\frac{a}{L}$.

This discussion should tie the problem raised by Mr. Hough more firmly into the entire body of results thus far obtained in earth mechanics.

The limiting shear value will be based on the cohesion of the material and the friction due to the direct pressure, making due allowance for the degree of consolidation, but the designer must disregard the value of the load, P, which, as before stated, can contribute nothing to the friction force. The force of this argument may better be appreciated by reference to Fig. 60(a), which shows the variation of the coefficient of friction, based on the pressure, p, when both friction and cohesion are considered, provided the clay is considered fully consolidated for the actual value of p used. Fig. 60(b) applies if the clay is not fully consolidated. This curve represents the solution between the apparent coefficient of friction (based on the full applied load) and the degree of consolidation under that load. For all practical purposes the plot is a straight line. A sudden application of pressure is equivalent to a sudden reduction in the degree of consolidation. There have been a number of cases in which the

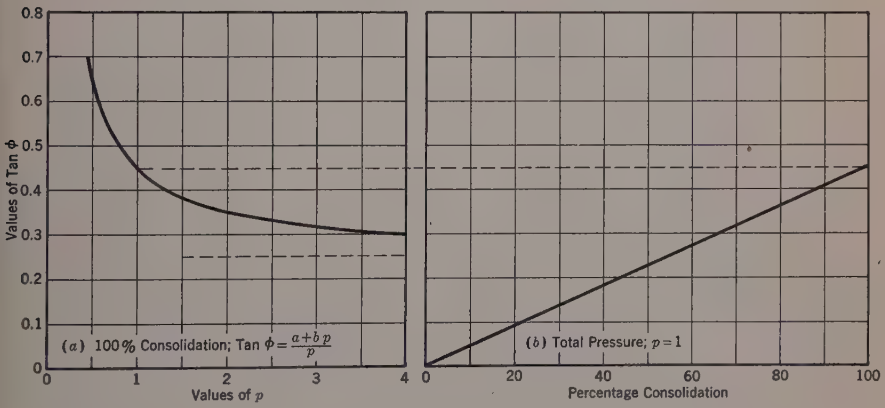

Fig. 60.—Shear in Clay.

disregard of such principles has resulted in failure. The coefficient of friction enables one to calculate the limiting shear for the direct pressure acting.

This Symposium is related primarily to dams and other structures erected on the surface of soft materials with the idea of permitting the structures to sink to equilibrium. Curves such as Figs. 37 and 38, suggest that it may prove much more economical to sluice and dredge soft material into the position where the "mud waves" must form. The cross-section of these waves appears to be much less than that of the dam or other structural material which must be lost by the sinking action. With mud waves formed in place in excess of the sizes shown, the structures erected on the surface of the soft body should remain substantially at that surface without any sinking action. The question of protecting the "waves" from erosion exists in either case.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

HYDRAULIC TESTS ON THE SPILLWAY OF THE MADDEN DAM

Discussion

By Morrough P. O'Brien, Assoc. M. Am. Soc. C. E.

Morrough P. O'Brien,[22] Assoc. M. Am. Soc. C. E.[22a]—The use of the hydraulic jump at the base of a spillway for dissipating energy has been frequently discussed in the technical journals, but several features have not been sufficiently emphasized. The sloping apron of the Madden Dam spillway illustrates one of these points.

In the analysis of the jump, the hydraulic elements are obtained from the summation of pressure force and rate of transportation of momentum. This analysis gives the depth below the jump, but fails to give an adequate explanation of the fact that energy is dissipated in a flow system assumed to be frictionless. The explanation lies in the moments applied which are of such a nature as to produce rotation. At the down-stream section, the same total energy per unit weight is present in mechanical form (potential plus kinetic), but a portion of the kinetic energy is accounted for by the rotation of small masses of fluid and is unavailable. Stated more simply, the adverse slope of the water surface and large bottom velocities combine to bring about a rotation which is the first step in the final conversion of a part of the mechanical energy into heat. The practical significance of this fact is that "drowning" of a jump reduces this moment by eliminating the sloping water surface and results in less effective operation. The sloping apron at Madden Dam prevents drowning and thus increases the energy dissipation over a range of discharges. Only in the event that the back-water curve from the next control down stream produces submergence at each discharge exactly equal to the "jump depth" can a horizontal apron be as effective as a sloping apron which produces a true jump but prevents drowning.

Note.—The paper by Richard R. Randolph, Jr., Esq., was published in May, 1937, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: October, 1937, by Messrs. Ettore Scimemi, and J. C. Stevens; and November, 1937, by F. W. Edwards, Jun. Am. Soc. C. E.

[22] Prof., Mech. Eng., Univ. of California, Berkeley, Calif.

[22a] Received by the Secretary November 17, 1937.

Information on the jump produced at the bottom of a steeply sloping channel is meager. Fig. 27 shows data obtained in a small laboratory channel by Lieut. B. D. Rindlaub, Corps of Engineers, U. S. Army.[23] The dimensionless variables entering the problem are, $\frac{d_2}{d_1}$; $\frac{l}{d_1}$; and, $\lambda = \frac{Q_2}{g\,d^3}$. The curves show constant values of $\frac{l}{d_1}$ at three slopes. The dashed line, $\frac{d_2}{d_1}$, is a function of λ for horizontal channels.

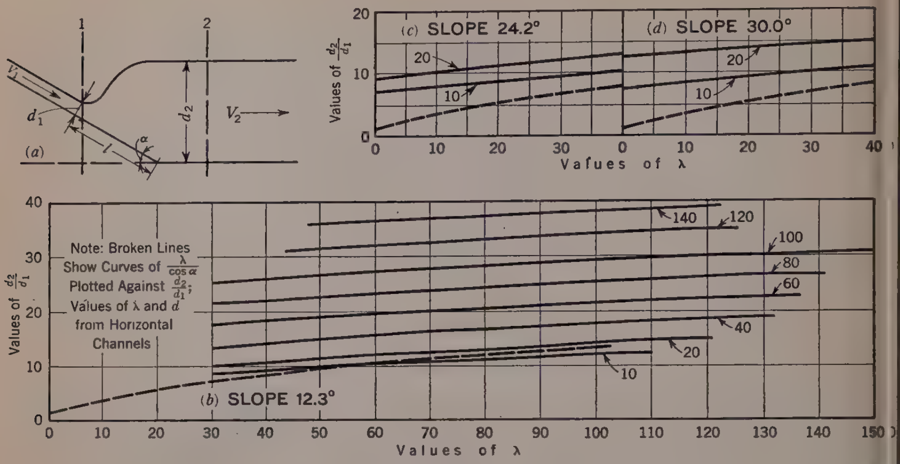

FIG. 27.

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[23] Unpublished Master's Thesis, Univ. of California, May, 1935.

PROCEEDINGS

of the

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Civil Engineers

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ABRIDGED TRANSLATIONS
OF
HYDRAULIC PAPERS

November, 1937

AMERICAN SOCIETY OF CIVIL ENGINEERS
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TRANSLATIONS

ABRIDGED TRANSLATIONS OF HYDRAULIC PAPERS

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NOTE.—These translations are not subject to discussion, and will not be published in *Transactions*. Reprints of this collection of papers, however, are available for limited distribution at a nominal cost of 80 cts. per copy, with 50% discount to members.

FOREWORD

In 1933, an informal organization of recipients of the John R. Freeman Traveling Scholarships was first suggested, for the purpose of stimulating the translation and abstracting of foreign papers on hydraulics that would not otherwise be available to American engineers.

In April, 1935, a detailed proposal for the publication of such translations was presented to the Committee on Publications. In that proposal it was pointed out that although the work was initiated "as an expression of appreciation for John R. Freeman's efforts in behalf of the profession," the identity of the translators was immaterial, and the participation of all interested persons was welcomed.

The Committee on Publications gave its general approval to the proposal, and in April, 1937, it gave specific consideration to the papers that had been collected during the interim. Following endorsement and co-operation by the Committee of the Society on Hydraulic Research, and the customary review by other experts, the papers presented herewith were accepted for publication.

PRESSURE, ENERGY, AND FLOW CONDITIONS IN CHANNELS WITH HIGH GRADIENTS[1]

By Harald Lauffer,[2] Esq.

Translated and Abstracted by Donald P. Barnes,[3]
Assoc. M. Am. Soc. C. E.

At any point in a liquid at rest the hydrostatic pressure, p, is equal to the external pressure on the free surface increased by the product of the distance from the surface and the unit weight. If the fluid is in straight-line horizontal motion (Fig. 1), there is no change in the pressure condition, and, if the external pressure is zero,

$$p = \gamma (t - z) \qquad (1)$$

The total potential energy, h_s, of a particle of water whose unit weight is 1, may be expressed with reference to the bottom of the channel in terms of the energy of position, z, and the pressure energy, $\frac{p}{\gamma}$:

$$h_s = z + \frac{\gamma (t - z)}{\gamma} = t \qquad (2)$$

The potential energy for all particles is therefore constant and equal to the energy of position of a particle on the surface.

If the velocity of a particle is v, then its kinetic energy is equal to the velocity head:

$$h_v = \frac{v^2}{2g} \qquad (3)$$

and the total energy of the particle with reference to the bottom of the channel is:

$$H = h_s + h_v = t + \frac{v^2}{2g} \qquad (4)$$

If the energy head for all particles in a cross-section is plotted vertically above the bottom and if the velocity is constant throughout this cross-section ($v = V$), then all these points will coincide. For these conditions, therefore, the energy line is obtained if for each cross-section the velocity head is plotted upward from the water surface and the points are connected. Applications of this expression to the determination of surface profiles and to related problems have shown excellent agreement with actual flow in numerous instances. It should not be overlooked, however, that Equations (1) and (4) are applicable only under the conditions stated.

---

[1] "Druck, Energie und Fliesszustand in Gerinnen mit grossem Gefälle," by Harald Lauffer, *Wasserkraft und Wasserwirtschaft* (Munich), Vol. 30, No. 7, 1935, pp. 78–82.

[2] Asst. at the Technical Univ., Graz, Austria.

[3] Associate Engr., U. S. Bureau of Reclamation, California Inst. of Technology, Pasadena, Calif.

Although the introduction of changes of direction in the stream line will entail certain difficulties, the influence of a uniform slope can be easily derived.

PRESSURE DISTRIBUTION

Again assuming straight-line parallel flow but with the stream lines inclined at an angle, ϕ, with the horizontal, if one considers a differential element with sides, dy and ds (the breadth being considered as unity throughout), the only forces acting in the normal direction are the pressure forces and a component of the weight (Fig. 2):

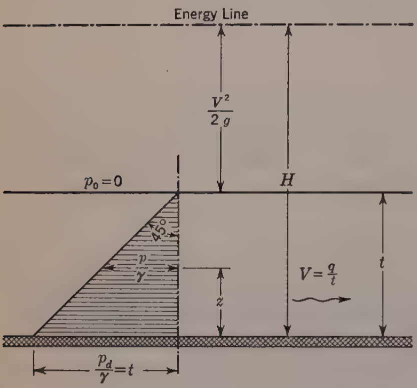

FIG. 1.—PRESSURE DISTRIBUTION AND ENERGY LINE FOR HORIZONTAL PARALLEL FLOW

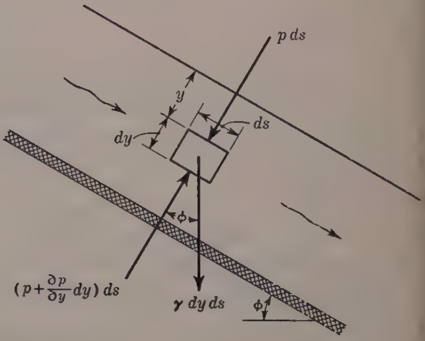

FIG. 2.—FORCES ON A NORMAL ELEMENT OF FLOW

$$p\,ds - \left(p + \frac{\partial p}{\partial y}dy\right)ds + \gamma\,dy\,ds\cos\phi = 0$$

or,

$$\frac{\partial p}{\partial y} = \gamma\cos\phi \qquad (5)$$

and, finally, for an external pressure, $p_0 = 0$ (Fig. 3),

$$p = y\gamma\cos\phi \qquad (6)$$

The pressure at the bottom is, then,

$$p_d = d\gamma\cos\phi = t\gamma\cos^2\phi \qquad (7)$$

The pressure diagram for a straight section, therefore, remains triangular, although the pressures are smaller for appreciable slopes than those given by Equation (1). The graphic determination of the pressure diagram is given in Fig. 3.

ENERGY LINE

If the general expression for the potential energy of a fluid particle in parallel flow is sought, there is obtained from Equations (6) and (7),

$$h_s = z + \frac{p}{\gamma} = t\cos^2\phi + z\sin^2\phi$$

For a vertical section (Fig. 3), the potential energy of the various particles is no longer constant, but is dependent upon z, and varies between $h_s = t$ for the upper surface and $h_s = t \cos^2 \phi$ for the bottom. On the other hand, in a normal section,

$$y = d - \frac{z}{\cos \phi}$$

and, therefore,

$$h_s = z + \frac{p}{\gamma} = z + \left(d - \frac{z}{\cos \phi}\right) \cos \phi = d \cos \phi \ldots\ldots\ldots\ldots (8)$$

The potential energy of all the particles in a normal section is thus seen to be the same.

The total energy of a particle in the normal section whose velocity is v is, therefore,

$$H = h_s + h_v = d \cos \phi + \frac{v^2}{2\,g} \ldots\ldots\ldots\ldots\ldots\ldots (9)$$

with reference to the bottom of the channel.

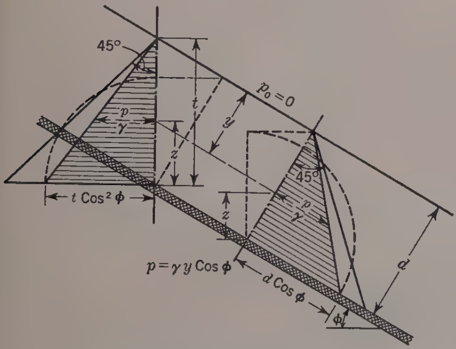

Fig. 3.—Pressure Distribution for General Case of Parallel Flow

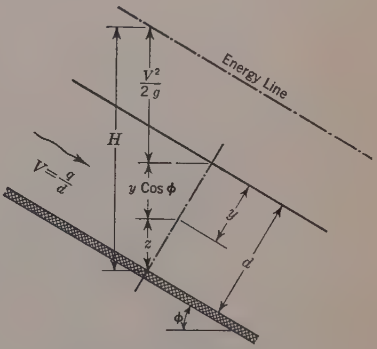

Fig. 4.—Energy Line for General Case of Parallel Flow

If the velocity across the normal section is constant, then the energy head for all the particles is the same and the energy line is again obtained by plotting H at each section (Fig. 4). It would be relatively simple to plot the velocity heads from points on the surface. When considering friction losses, however (that is, when the energy line is no longer parallel to the bed), this would lead to certain inconsistencies. One is led, therefore, to the following principle:

The energy line for parallel flow is obtained by plotting the energy head, $H = d \cos \phi + \frac{v^2}{2\,g}$, above the bottom point of each normal section.

Determination of the Position of the Surface and the Maximum Flow

For a given discharge, q, per unit width, the velocity is $V = \frac{q}{d}$. Introducing this value into Equation (9),

$$H = d \cos \phi + \frac{q^2}{2\,g\,d^2}$$

or,

$$d^3 \cos \phi - d^2 H + \frac{q^2}{2\,g} = 0 \quad \text{(10)}$$

For horizontal flow, this becomes the familiar equation for the q-curve,

$$t^3 - t^2\, H + \frac{q^2}{2\,g} = 0$$

For a given energy head the critical depth, t_c, is,

$$t_c = \tfrac{2}{3}\, H \quad \text{(11)}$$

and the maximum discharge is,

$$q_c = \sqrt{g}\, (\tfrac{2}{3}\, H)^{3/2} \quad \text{(12)}$$

If Equation (10) is divided by H^3, it becomes dimensionless, and, considering Equation (12), there results,

$$\left(\frac{d}{H}\right)^3 \cos \phi - \left(\frac{d}{H}\right)^2 + \frac{4}{27}\left(\frac{q}{q_c}\right)^2 = 0 \quad \text{(13)}$$

Equation (13) is independent of the units of measurement, and as plotted for various slopes in Fig. 5 is completely general. The solution for $\frac{d}{H}$ gives three real roots, of which one is negative and, therefore, may be disregarded.

Solutions of Equation (13) for $\frac{q}{q_c} = 0$ give the intersections of the curves with the $\frac{d}{H}$-axis, $\left(\frac{d}{H}\right)_1 = 0$, and $\left(\frac{d}{H}\right)_2 = \frac{1}{\cos \phi}$.

For a given stream thickness, d, and energy head, H, the discharge according to Equation (13) is,

$$\frac{q}{q_c} = \sqrt{\frac{27}{4}\left[\left(\frac{d}{H}\right)^2 - \left(\frac{d}{H}\right)^3 \cos \phi\right]} \quad \text{(14)}$$

The maximum value, q_{max}, of the discharge in a sloping channel is obtained by differentiating Equation (14) and equating to zero:

$$2\frac{d}{H} - 3\left(\frac{d}{H}\right)^2 \cos \phi = 0$$

and,

$$\frac{d}{H} = \frac{d_{q\,(max)}}{H} = \frac{2}{3}\,\frac{1}{\cos \phi} \quad \text{(15)}$$

Substituting the value of $\frac{d}{H}$ from Equation (15) in Equation (14),

$$\frac{q_{max}}{q_c} = \frac{1}{\cos \phi} \quad \text{(16)}$$

The maximum points of the q-curves for various slopes (Fig. 5) lie on a straight line through the origin whose slope is,

$$\frac{\;\frac{d_{q\,(\max)}}{H}\;}{\;\frac{q_{\max}}{q_c}\;} = \frac{2}{3} \qquad (17)$$

The calculation of the surface curve in a sloping rectangular channel for a particular discharge, q, and a particular energy head, H (measured from the bottom point of the section to be investigated), is carried out as follows: First, from Equation (12), calculate the critical discharge for the given energy head and for horizontal parallel flow. Next, obtain $\frac{q}{q_c}$ and determine the value of $\frac{d}{H}$ either by calculation from Equation (13), or graphically from Fig. 5. This gives the desired value of the thickness of the stream, d.

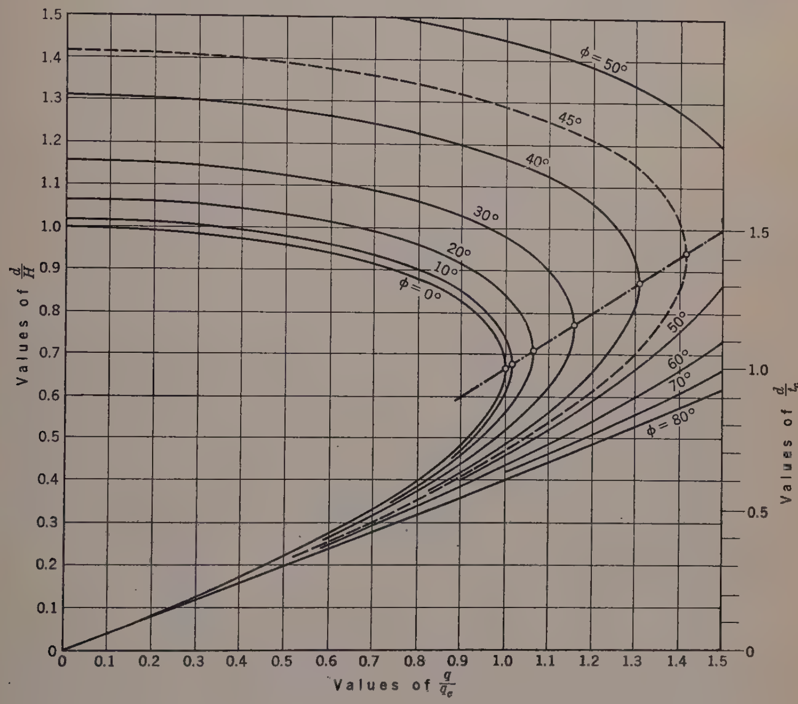

Fig. 5.—Curves of $\frac{q}{q_c}$ from Equation (13). Values of $\frac{d}{t_c}$ Are Valid Only for Horizontal Parallel Flow

In general there will be obtained two values, $d_1 > d_{q\,(\max)}$, and $d_2 < d_{q\,(\max)}$, which coincide for $q = q_{\max}$. The value obtained for $q > q_{\max}$ is useless since the energy head is then too small to support the necessary discharge.

DYNAMIC CAPACITY AND RESISTANCE TO FLOW

The dynamic capacity, K, is defined as the sum of the hydrostatic pressure and the transfer of momentum through a cross-section in unit time.[4] For parallel flow on a slope, ϕ, and for a normal section (Fig. 3),

$$K = \gamma\left(\frac{d^2}{2}\cos\phi + \frac{1}{g}V^2 d\right) \quad \text{.................. (18)}$$

The limiting value, K_c, of the dynamic capacity in horizontal parallel flow is obtained when,

$$d = t_c = \sqrt[3]{\frac{q^2}{g}} \quad \text{........................ (19)}$$

and is,

$$K_c = \gamma\frac{3}{2}\left(\frac{q^2}{g}\right)^{2/3} \quad \text{........................ (20)}$$

When Equation (18) is divided by t_c^2, and considering Equation (20), there is obtained the dimensionless equation for the dynamic capacity,

$$\frac{K}{K_c} = \frac{1}{3}\cos\phi\left(\frac{d}{t_c}\right)^2 + \frac{2}{3}\left(\frac{d}{t_c}\right)^{-1} \quad \text{................ (21)}$$

Equation (21) again is independent of absolute magnitudes, and the dynamic capacity curves shown in Fig. 6 for various slopes are completely general. Solving Equation (21) for $\frac{d}{t_c}$ yields two useful roots which for a given dynamic capacity can best be determined graphically from Fig. 6.

For a particular slope, the minimum value of the dynamic capacity is obtained by differentiating Equation (21) and equating to zero. This yields, for the thickness of the stream,

$$\frac{d_{k\ (\min)}}{t_c} = (\cos\phi)^{-1/3} \quad \text{...................... (22)}$$

The minimum dynamic capacity for a given discharge is, therefore,

$$\frac{K_{\min}}{K_c} = (\cos\phi)^{1/3} \quad \text{........................ (23)}$$

The minimum points on the dynamic capacity curve lie on the unit hyperbola,

$$\frac{d_{k\ (\min)}}{t_c} \times \frac{K_{\min}}{K_c} = 1 \quad \text{...................... (24)}$$

For $d = d_{k\ (\min)} = \frac{t_c}{\sqrt[3]{\cos\phi}}$, the velocity,

$$V_{k\ (\min)} = \frac{q_{k\ (\min)}}{d_{k\ (\min)}} = \sqrt{g\, d_{k\ (\min)} \cos\phi} \quad \text{.............. (25)}$$

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[4] "Bewegung des Wassers und dabei auftretende Kräfte," by Koch-Carstanjen, J. Springer, Berlin, 1926.

is equal to the wave velocity in a rectangular channel whose slope is ϕ. If the same channel is imagined to be in a horizontal position and subjected to a gravity field whose acceleration is $g \cos \phi$, the hydrostatic pressure is exactly the same as for the sloping channel in which the acceleration of gravity is g. The wave velocities, therefore, must be the same, and there results, for horizontal channels,

$$V_c = \sqrt{g \cos \phi \, d}$$

In a sloping channel, therefore, there is shooting flow if $d < d_{k\,(\mathrm{min})}$, and streaming flow if $d > d_{k\,(\mathrm{min})}$. Hence the locus of the surface for a minimum dynamic capacity coincides with the boundary between the two types of flow.

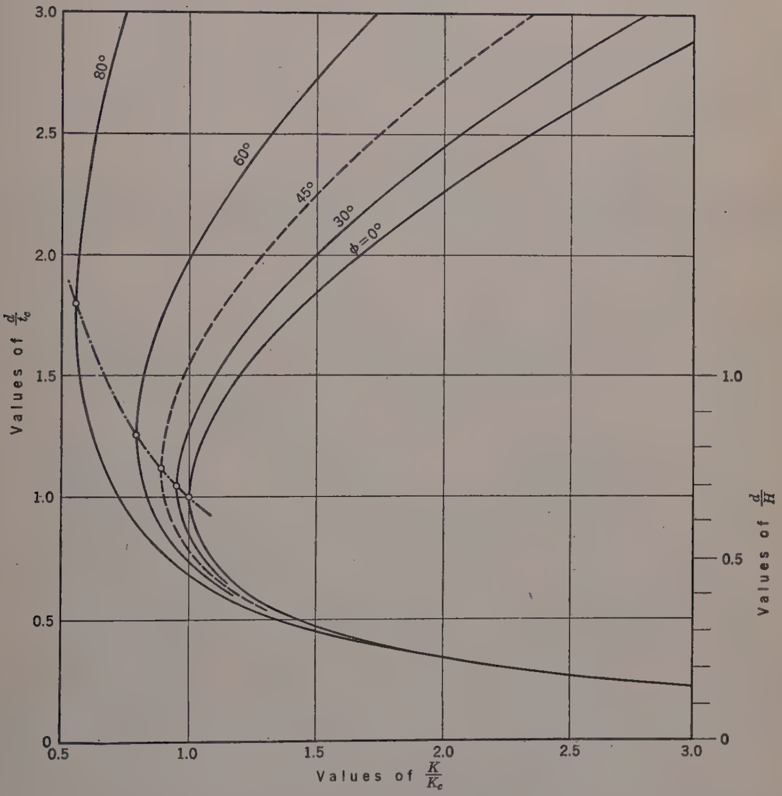

FIG. 6.—CURVES OF DYNAMIC CAPACITY FROM EQUATION (21). VALUES OF $\frac{d^2}{H}$ ARE VALID ONLY FOR HORIZONTAL PARALLEL FLOW

POSITION OF THE SURFACE FOR MAXIMUM DISCHARGE, AND THE CRITICAL DEPTH

The ratios, $\frac{d}{H}$ and $\frac{d}{t_c}$, can be related through the Froude number,

$$\mathsf{F} = \frac{V}{\sqrt{g\,d}} \qquad \text{(26)}$$

which was used by Safranez[5] as the "flow index" for the calculation of the hydraulic jump.

From Equation (9),

$$V = \sqrt{2\,g\,(H - d\cos\phi)}$$

Combining this with Equation (26),

$$\mathbf{F} = \sqrt{2\left(\frac{H}{d} - \cos\phi\right)}$$

Substituting the value of $\frac{d}{H}$ from Equation (15), the Froude number for maximum discharge is,

$$\mathbf{F}_{q\,(max)} = \sqrt{\cos\phi}$$

On the other hand, from Equation (19)

$$q = t_c^{3/2}\,g^{1/2}$$

and, therefore,

$$\mathbf{F} = \frac{q}{d\sqrt{g\,d}} = \left(\frac{t_c}{d}\right)^{3/2}$$

Whence, by use of Equation (22) the Froude number for the critical condition and minimum dynamic capacity is, again,

$$\mathbf{F}_{k\,(min)} = \sqrt{\cos\phi}$$

The position of the surface for maximum discharge for parallel flow, therefore, coincides with the boundary position between shooting and streaming flow (Fig. 7):

$$\mathbf{F}_{k\,(min)} = \mathbf{F}_{q\,(max)} = \mathbf{F}_c = \sqrt{\cos\phi}\,\ldots\ldots\ldots\ldots\ldots(27)$$

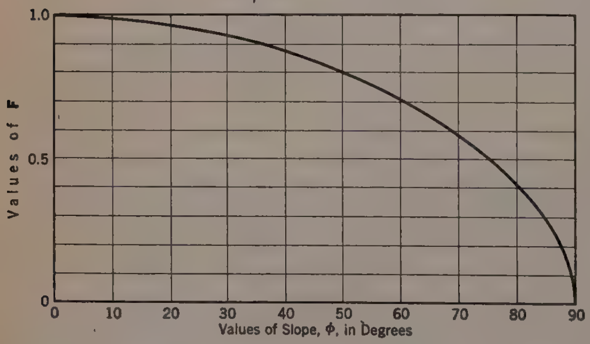

FIG. 7.—CRITICAL VALUE OF FROUDE'S NUMBER FOR MAXIMUM DISCHARGE AND MINIMUM DYNAMIC CAPACITY

The Froude number for this critical flow, therefore, will differ widely, for steep slopes, from its critical value for horizontal parallel flow ($\mathbf{F} = 1$).

Under the influence of curvature in the stream line this difference may be considerably magnified, without affecting the validity of the Froude model law (in which the only external forces are assumed to be gravity forces), and without necessitating replacement by any other arbitrary law as suggested by Engel for the interpretation of the results of experiments on Venturi canals.[6, 7]

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[5] "Wechselsprung und die Energievernichtung des Wassers," by K. Safranez, *Der Bauingenieur*, No. 8 (1927), p. 898.

[6] "Non-Uniform Flow of Water," by F. Engel, *The Engineer*, Vol. 155, 1932, p. 392.

[7] "Wassermessung mit öffenen, seitlich eingeschnürten Kanälen (Venturikanäle)," by F. Engel, *Zeitschrift* des Vereines Deutscher Ingenieure, Vol. 77, 1933, p. 1285.

INCREASES IN PRESSURE WITH CURVATURE OF STREAM LINE

Much effort has recently been expended toward introducing the influence of curvature of the stream lines into elementary methods of calculation. At overflow crests and in the so-called "bucket" the effect of centrifugal force is particularly important.[4, 5, 8]

In order to determine this increase in pressure experimentally, the total pressure is measured, and this is divided into static and dynamic parts. For horizontal parallel flow, this increment in pressure, Δh_p, is equal to the difference between the measured pressure head, h_p, and the vertical distance from the point in question to the surface of the water.

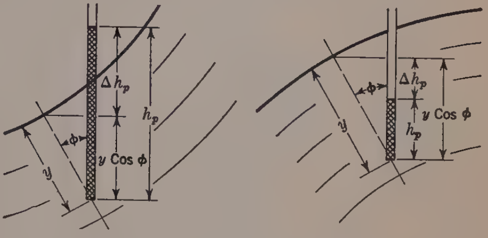

FIG. 8.—POSITIVE AND NEGATIVE INCREMENTS OF PRESSURE RESULTING FOR CURVATURE OF FLOW

For inclined stream lines the static pressure is no longer equal to the depth below the surface, and for concentric stream lines (considering Equation (6)), the increment in pressure head is,

$$\Delta h_p = h_p - y \cos \phi \ldots\ldots\ldots\ldots\ldots\ldots (28)$$

The determination of both positive and negative pressure increments due to curvature of the stream lines is indicated in Fig. 8.

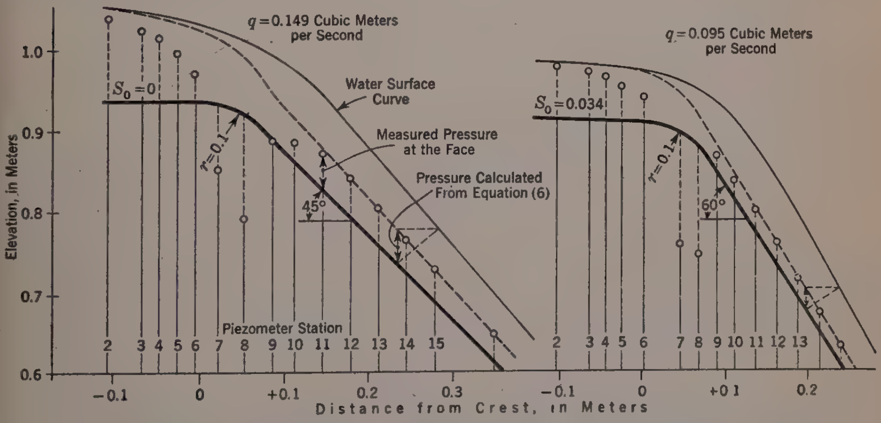

FIG. 9.—EXPERIMENTAL THEORETICAL PRESSURE INTENSITY ON DOWN-STREAM WEIR FACES

EXPERIMENTAL AGREEMENT

Few pressure measurements have been made in sections of channels having high gradients. Those that have been completed were for the most part carried out in regions under the influence of considerable curvature. Many

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[8] "Die Abflusserscheinungen und Druckverhältnisse an Klappenwehren," by H. Schwarzmann, R. Oldenbourg, Munich and Berlin, 1934.

valuable data are nevertheless available for checking the fundamental pressure equation (Equation (7)).

Particularly pertinent are the experiments of M. Hasumi at the Vienna Hydraulic Experiment Station. These experiments were concerned with the distribution of pressures along the faces of weirs.[9] The profile of the surface and the pressures along the face were determined for twenty-four different conditions of flow. Two of these experiments are plotted in Fig. 9, where the measured pressures are shown for comparison with those calculated from the position of the surface in accordance with Equation (7). The average of the bottom and surface slopes at each normal section was taken as the slope of the stream line in the calculation. Very good agreement between theory and fact was obtained where the curvatures were slight, the discrepancies being well within the limits of experimental accuracy.

CONCLUSION

It has been demonstrated, theoretically and experimentally, that for parallel flow with high gradients the pressure head in the interior of the liquid is not equal to the vertical distance from the surface, but is significantly smaller. It follows, therefore, that the q-line as well as the dynamic capacity is dependent upon the slope. The surface profile corresponding to the minimum dynamic capacity is both the boundary between shooting and streaming flow and the surface profile for maximum discharge. The Froude number for this condition of flow can vary between zero and 1, depending upon the slope.

NOMENCLATURE

The notation used in this translation is as follows:

d = thickness of stream perpendicular to direction of flow;
F = Froude's number;
F_c = critical value of Froude's number for parallel flow on any gradient;
g = acceleration of gravity;
h_p = pressure head $\left(\frac{p}{\gamma}\right)$;
h_s = potential energy head $(h_p + z)$;
h_v = velocity head;
H = total energy head above bed;
K = dynamic capacity;
p = intensity of hydrostatic pressure;
q = discharge per unit width of channel;
t = depth of stream, measured vertically;
v = velocity of a particle;
V = mean velocity $\left(\frac{q}{d}\right)$;

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[9] "Versuche über die Verteilung der Drucke an Wehrracken infolge des absturzenden Wassers," by R. Ehrenberger, *Die Wasserwirtschaft*, Wien, Vol. 22, 1929, H. 5.

y = distance of a particle below the water surface, measured perpendicularly to the direction of flow;

z = distance of a particle above the bed, measured vertically;

γ = unit weight of water;

ϕ = angle of inclination of bed with the horizontal;

t_c, v_c, q_c, and K_c = critical values of t, v, q, and K for horizontal parallel flow;

$d_{q\,(\max)}$, $q_{\max}$, and $\mathbf{F}_{q\,(\max)}$ = critical values of d, q, and $\mathbf{F}$ corresponding to the position of the surface for maximum discharge; and

$d_{k\,(\min)}$, $K_{\min}$, and $\mathbf{F}_{k\,(\min)}$ = critical values corresponding to the position of the water surface for minimum dynamic capacity.

EXPERIMENTAL INVESTIGATION OF THE ROUGHNESS PROBLEM[1]

BY H. SCHLICHTING,[2] ESQ.

TRANSLATED AND ABSTRACTED BY V. L. STREETER,[3]
JUN. AM. SOC. C. E.

SYNOPSIS

This paper presents a new experimental method for obtaining the resistance to flow along roughened surfaces, based on the universal velocity-distribution law. Systematic experiments on twenty-one geometrically simple, regular roughnesses are described.

In order to transfer the results conveniently to channels and pipes with other hydraulic radii, and to towed plates, the "equivalent sand roughness" was computed in each case. The resistance depends not only on the relative roughness, but also on the roughness density. The greatest resistance occurred not with the maximum roughness density, but with a considerably smaller value.

The resistance coefficient of the elementary roughness was obtained for each rough plate. In practically all cases this coefficient was independent of the roughness density for small values of the latter, and decreased sharply for higher values.

INTRODUCTION

As a result of numerous recent investigations ((1), (2), (3), (4), (5), (6))[4] the laws of turbulent flow in smooth pipes and channels and along smooth plates can be considered as satisfactorily understood from an experimental standpoint. The universal laws for velocity distribution at rough surfaces, however, have been only recently discovered.

In considering the universal velocity distribution at smooth and rough walls, the introduction of a dimensionless velocity, $\phi = \frac{v}{v_*}$, and a dimensionless distance from the wall, $\eta = \frac{y\,v}{\nu}$, has proved useful (2); v signifies the velocity at the distance; y, the distance from the wall; $v_* = \sqrt{\frac{\tau}{\rho}}$, the "shear-stress velocity" (a quantity with the dimensions of a velocity formed from the shear stress, τ, at the wall); ρ is the density; and ν, the kinematic viscosity. The extensive tests by J. Nikuradse (6) on velocity distributions in smooth pipes show that ϕ is a universal function of η, and that the plot of ϕ against log η is,

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[1] "Experimentelle Untersuchungen zum Rauhigskeitsproblem," by H. Schlichting, *Ingenieur-Archiv*, Vol. VII, No. 1 (February, 1936).

[2] Kaiser Wilhelm-Institut für Stromüngsforschung, Göttingen, Germany.

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[4] The numbers in parentheses refer to the Bibliography at the end of the paper.

$$\phi = A + B \log \eta = 5.5 + 5.75 \log \eta \quad \text{(1a)}$$

or,

$$\frac{v}{v_*} = 5.5 + 5.75 \log \frac{y\, v_*}{\nu} \quad \text{(1b)}$$

Theoretical considerations (3) show that this straight-line law may be expected to hold after the effect of viscosity on turbulence disappears. It is assumed, therefore, that Equations (1) will remain valid for the highest Reynolds numbers. The velocity-distribution law (Equations (1)) is a so-called "boundary-layer" law, since the velocity in the vicinity of the wall depends (except for ν and ρ) only upon the distance from, and the shear stress at, the wall. It is not influenced by occurrences at greater distances from the wall than the point considered; for example, the conditions at the other side of the pipe or channel have no effect. (Important use of this fact is made in transferring the results of pipe and channel experiments to towed plates.) The universal velocity-distribution law thus permits the computation of τ, when v, y, ρ, and ν are known. It is well known that the resistance law is closely related to the velocity-distribution law. The resistance law obtained in connection with Equations (1) is:

$$\frac{1}{\sqrt{\lambda}} = 2 \log (\mathbf{R} \sqrt{\lambda}) - 0.8 \quad \text{(2)}$$

in which $\lambda = \frac{dp}{dx} \frac{2\,D}{\rho\, V^2}$, the friction factor, and $\mathbf{R} = \frac{V\,D}{\nu}$, the Reynolds number; V is the mean velocity over the cross-section, and D, the diameter of the pipe. This resistance law is valid over as great a range as Equations (1)—that is, to arbitrarily high values of $\mathbf{R}$.

If the relation, $\phi = f(\log \eta)$, is plotted for rough pipes, using data from Nikuradse's experiments (7), a straight line likewise results for each relative roughness and each value of $\mathbf{R}$. (Nikuradse used pipes with sand glued to their interior surfaces to produce the roughness. The relative roughness is $\frac{k}{r}$ (k being the absolute roughness—that is, the height of the roughness—and r, the pipe radius). These lines are parallel, and their slope is the same ($B = 5.75$) as for smooth pipe. Since $\frac{y\, v_*}{\nu} \equiv \frac{y}{k} \frac{v_*\, k}{\nu}$, the velocity distribution law for rough pipe may also be written in the form:

$$\phi = A + 5.75 \log \frac{y}{k}; \quad A = \chi \left(\frac{v_*\, k}{\nu} \right) \quad \text{(3)}$$

According to the measurements of Nikuradse, the magnitude of A depends only on $\frac{v_*\, k}{\nu}$, which may be called the Reynolds number of the roughness. In connection with Equation (3), the universal resistance law for a rough wall is:

$$\lambda = \left(2 \log \frac{R}{k} + a \right)^{-2} \quad \text{(4)}$$

in which the magnitude of a likewise depends on $\frac{v_* k}{\nu}$. (R is the "equivalent" radius—that is, twice the hydraulic radius of the channel.) In the Nikuradse experiments with sand roughnesses, Equation (4) takes the form:

$$\lambda = \left(1.74 + 2 \log \frac{R}{k_s} \right)^{-2} \quad \text{(5)}$$

for completely developed turbulent flow.

The practical demands on roughness research will be completely fulfilled if the resistance of any roughness can be given. The resistance, however, must be given not only for pipes and channels, but also for towed plates in unconfined flow; and it must be possible to extend the results to other diameters, channel heights, or plate lengths, without additional experiments. Theoretically, it should be possible to compute the resistance of any given geometrical form of roughness; such computations, however, would seldom give the desired accuracy. It is all the more important, therefore, to develop a simple experimental set-up in which, with a minimum amount of testing, every roughness occurring in practice can easily be investigated. Such an arrangement has been developed in Göttingen, Germany, in the form of a rectangular channel with three smooth sides and one changeable rough side.

In the Nikuradse experiments roughness was characterized by one parameter—the roughness height, k (or k_s, the size of the sand grain). With the multiplicity of roughnesses occurring in practice, a single parameter will not suffice. It is necessary to introduce at least one more, say, the roughness density; that is, the number of roughness elements per unit of area. (In the Nikuradse experiments the roughness density was always the same, and nearly the maximum.) Hence, for a given geometrical form of roughness elements, the effect of roughness density on roughness resistance must be investigated.

THE TESTS

Description of Experimental Equipment.—The experimental channel is shown in Fig. 1. It consists of two sections with flanged connections, the down-stream section being used for the tests. The walls of the up-stream section are smooth. The down-stream channel was made from cast steel, and consists of two parts: (1) A rectangular open channel with smooth sides and bottom, with grooves in the sides into which the test plate is fitted; and (2) a cover which is laid over the test plate and bolted to the lower part. There are eight measuring sections, 40 cm apart, for obtaining static pressure. The velocity distribution at each station can be observed by introducing a Pitot tube into the flow through a hole in the side wall.

The Rough Plates.—The test plates were made from 5-mm sheet iron stiffened on the back by iron strips 15 mm by 15 mm in cross-section. After the face of the plates had been coated with a thin layer of tin, the roughness elements were soldered into place. Six groups of roughness elements were investigated, each at several roughness densities. The elements are shown in Fig. 2, and data on all the rough plates studied are presented in Table 1.

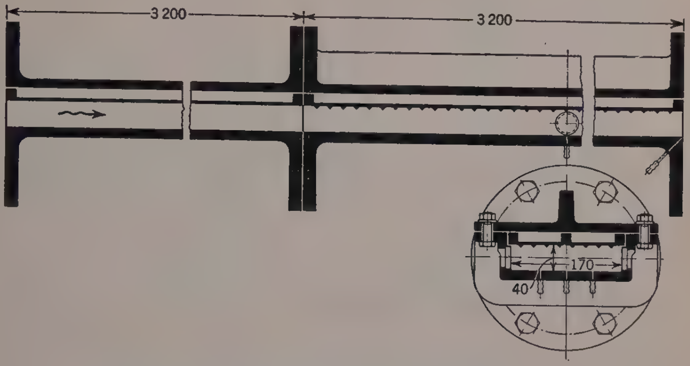


FIG. 1.—SECTIONS THROUGH TEST CHANNEL FOR ROUGHNESS MEASUREMENTS (DIMENSIONS IN CENTIMETERS)

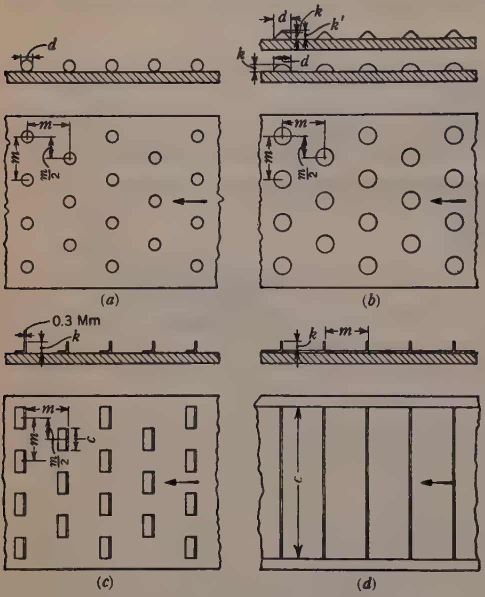
TABLE 1.—DATA ON TEST PLATES (SEE FIG. 2)

Type of roughness	Plate No.	d, in centimeters	m, in centimeters	$\frac{m}{d}$	k, in centimeters	c or k', in centimeters	b, in centimeters	$\frac{F_r}{F}$	$\frac{F_1}{F}$
Spheres:	XII	0.41	4	9.75	0.41	....	3.99	0.00785	0.992
	XII(*a*)*	1.0	10	10	1.0	....	*	0.00785	0.992
	III	0.41	2	4.88	0.41	....	3.99	0.0314	0.969
	I	0.41	1	2.44	0.41	....	3.96	0.126	0.874
	II	0.41	0.6	1.46	0.41	....	3.88	0.349	0.651
	V	0.41	†	†	0.41	....	3.68	0.907	0.093
	VI	0.21	1	4.86	0.21	....	3.99	0.0314	0.969
	IV	0.21	0.5	2.43	0.21	....	3.97	0.126	0.874
Spherical segments:	XIII	0.8	4	5	0.26	....	3.90	0.0087	0.969
	XIV	0.8	3	3.75	0.26	....	3.99	0.0155	0.944
	XV	0.8	2	2.5	0.26	....	3.98	0.0348	0.874
	XIX	0.8	†	†	0.26	....	3.85	0.251	0.093
Cones:	XXIII	0.8	4	5	0.375	0.425	3.99	0.0106	0.969
	XXIV	0.8	3	3.75	0.375	0.425	3.98	0.0189	0.944
	XXV	0.8	2	2.5	0.375	0.425	3.95	0.0425	0.874
"Short" angles:	XVI	....	4	....	0.30	0.8	4.0	0.0151	0.998
	XVIII	....	3	....	0.30	0.8	4.0	0.0269	0.996
	XVII	....	2	....	0.30	0.8	3.99	0.0605	0.994
"Long" angles:	XX	....	6	....	0.32	17	3.90	0.0538	0.995
	XXI	....	4	....	0.31	17	3.96	0.0776	0.992
	XXII	....	2	....	0.30	17	3.96	0.152	0.985

* Measured only in large wind tunnel. † Elements packed together as closely as possible.

Preliminary Experiments.—Preliminary experiments showed that the use of smooth plates in the up-stream, or "calming," section of the test channel had no effect on the measurement of pressure drop and velocity distribution in the down-stream section. Smooth plates, therefore, were used in the calming section.

Pressure-drop measurements were taken with a smooth top plate in the test section. The resulting values of λ, based on the equivalent diameter, D_h (that is, four times the cross-sectional area divided by the wetted perimeter), were in satisfactory agreement with those for smooth pipes.



(*a*) (*b*) (*c*) (*d*)

FIG. 2.—DETAILS OF TEST PLATES (SEE TABLE 1 FOR DIMENSIONS)
(*a*) SPHERES; (*b*) SPHERICAL SEGMENTS AND CONES; (*c*) "SHORT" ANGLES; (*d*) "LONG" ANGLES

With the aid of a small channel of rectangular cross-section (1.05 cm by 5 cm) with one smooth wall and one rough wall, studies were made to determine whether the velocity profiles near the rough surface and the smooth surface influence each other. This is important, as the method of evaluation described subsequently assumes that the friction layers on the smooth and rough walls are built up independently of each other, and are exactly the same as if all the walls were smooth (or rough). In Fig. 3 the velocity near each wall is plotted against the logarithm of the distance from that wall. The fact that the resulting velocity-distribution curve is made up of two intersecting straight lines proves that the assumption of independence is correct, since, according to the universal velocity-distribution law, with smooth as well as with rough surfaces, the velocity is proportional to the logarithm of the distance from the surface.

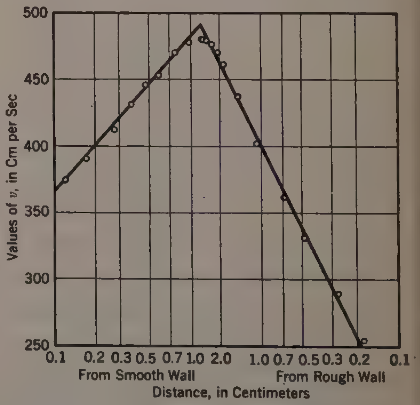


FIG. 3.—UNSYMMETRICAL VELOCITY DISTRIBUTION IN A CHANNEL WITH ONE SMOOTH AND ONE ROUGH WALL

The method of evaluating the test results also assumes two-dimensional flow in the middle part of the cross-section. Iso-velocity curves (Fig. 4) plotted for the exit cross-section of the test channel show that this condition was achieved.

Experimental Procedure.—Each rough plate was tested at at least five different mean velocities. In each case the following quantities were measured: (1) The velocity profile parallel to the short sides of the rectangle in the middle of the exit cross-section; (2) the pressure drop along the channel; (3) the temperature; and (4) the discharge.

EVALUATION OF EXPERIMENTAL DATA

General Considerations.—The object of this investigation is to determine a characteristic number for each of the roughnesses, such that the resistance of these roughnesses can be computed for other Reynolds numbers, $\mathbf{R} = \frac{V D_h}{\nu}$, and other relative roughnesses, $\frac{k}{R}$, than the ones measured. The symbol, k, signifies any suitable measure for the absolute roughness, and for uniform roughnesses may be best chosen as the greatest height of the roughness element; R is twice the hydraulic radius.

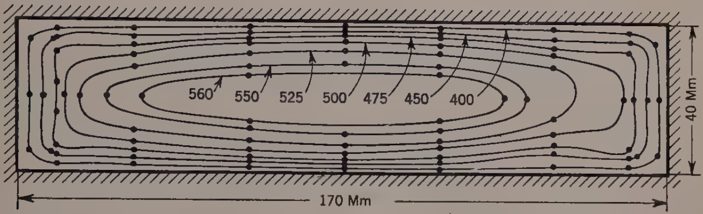


FIG. 4.—ISO-VELOCITY CURVES IN EXIT CROSS-SECTION OF TEST CHANNEL; BOTH WALLS SMOOTH. VELOCITIES IN CENTIMETERS PER SECOND

As the resistance of sand roughnesses (using the parameter, k_s) is known over a large range of Reynolds numbers and relative roughnesses, it proved expedient to evaluate the data in such a manner that they could be correlated with Nikuradse's experiments. This is not to say that sand roughness is a particularly favorable standard; actually, it is not, for it cannot be accurately reproduced. However, this fact is without importance in the present evaluation method, since consideration is given only to the equation expressing the resistance number as a function of $\mathbf{R}$ and $\frac{k_s}{R}$. No comparisons have been made with the roughness densities and absolute roughnesses. The function, $\lambda = f\left(\mathbf{R}, \frac{k_s}{R}\right)$, is particularly simple both for pure stream-line flow (small values of $\mathbf{R}$) and for completely developed turbulence. For small values of $\mathbf{R}$, λ ceases to be a function of $\frac{k}{R}$, and Equation (2) for smooth pipes applies. Nikuradse suggests the empirical equation, $\lambda = 0.0032 + \frac{0.221}{\mathbf{R}^{0.237}}$. For completely developed turbulent flow, λ, according to von Kármán (3), Prandtl (2), and Nikuradse (7) may be expressed by Equation (5) which is plotted in Fig. 5, with k_s replacing k. In the transi-

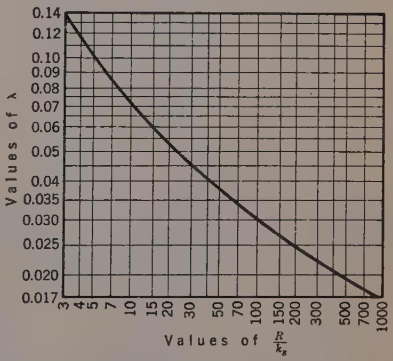


FIG. 5.—THE RESISTANCE COEFFICIENT, λ, AS A FUNCTION OF $\frac{R}{k_s}$.

tion zone between laminar flow and fully developed turbulent flow, where λ depends on both **R** and $\frac{R}{k}$, no analytical equation for λ has been found.

To the practicing engineer, the zone of fully developed turbulence is by far the most important, and, fortunately, the relationships are particularly simple in this zone. According to Nikuradse's measurements with sand roughnesses, completely developed turbulent flow exists for $\frac{v_* k_s}{\nu} > 70$, or $\frac{V D_h}{\nu} > \frac{198 D_h}{\sqrt{\lambda}\, k_s}$. This condition is met in all the tests of the present investigation.

Method of Evaluating the Results.—The problem of determining the resistance of surfaces geometrically similar to each of the roughnesses investigated, but for other relative roughnesses, is solved, if, for each roughness, the equivalent sand roughness, k_s, is specified. In other words, once k_s is determined for a given roughness, it is possible to compute the resistance for any size of pipe or channel having the same absolute roughness. (This assumes, of course, that fully developed turbulence exists.) The quantity, k_s, is the diameter of sand particle, obtained by the Nikuradse method, which would produce the same resistance as the roughness investigated. No particular physical significance need be attached to k_s; it is simply a characteristic number which can be substituted in Equation (5) to determine the value of λ for other channels with the same absolute roughness. Instead of k_s, the specification of a dimensionless number, $\alpha = \frac{k_s}{k}$, would be sufficient; k signifies the actual height (possibly the maximum height) of the roughness considered.

The computation of the equivalent sand roughness may be illustrated as follows, using the measurements of Hopf (8) and Fromm (9) for the three arbitrary roughnesses indicated in Table 2. These measurements were made for different channel depths, and, therefore, for different relative roughnesses. Plotting $\frac{1}{\sqrt{\lambda}}$ against $\log \frac{R}{k}$, the following relationship is determined, $\frac{1}{\sqrt{\lambda}} = a + 2 \log \frac{R}{k}$, which results in the values of a as given in Table 2. By comparison with Equation (5), the conversion coefficient, $\alpha = \frac{k_s}{k}$, is found to be expressed by $2 \log \alpha = 1.74 - a$. The values of α and k_s are likewise given in Table 2.

TABLE 2.—CONVERSION OF MEASUREMENTS BY HOPF AND FROMM TO EQUIVALENT SAND ROUGHNESSES

Type of roughness	k, in centimeters	a	α	k_s, in centimeters
Screen	0.0115	0.96	2.46	0.028
Waffle-iron	0.0427	1.36	1.52	0.065
Saw-profile	0.15	1.48	1.34	0.201

In this manner, any roughness may be converted into an equivalent sand roughness, provided λ is known as a function of $\frac{k}{R}$. However, to determine k_s it is not necessary to conduct experiments with different relative roughnesses; one measurement with a single relative roughness is sufficient, as will be shown later.

In the present investigation the determination of k_s is somewhat complicated by the fact that the resistance of the whole channel is the sum of the resistances of the rough and smooth walls together. More information, therefore, is needed concerning the resistance of the smooth wall in order to compute the resistance of the rough plate. The resistance of the smooth plate is obtained, with the help of Equation (1), from the measured velocity profile in its vicinity.

If τ_r and τ_g signify shear stress at the rough and smooth walls, respectively, one equation for them results from the equilibrium between the surface shear stress and the pressure drop:

$$\tau_r + \tau_g = b\frac{dp}{dx} \qquad (6)$$

in which b is the channel depth. The universal velocity-distribution laws for the friction layer along a smooth surface and along a rough surface are, respectively,

$$\frac{v}{v_{*g}} = 5.5 + 5.75 \log \frac{v_{*g}\, y}{\nu} \qquad (7)$$

and,

$$\frac{v}{v_{*r}} = A + 5.75 \log \frac{y}{k} \qquad (8)$$

in which v_{*g} and v_{*r} are the shear-stress velocities along the smooth and the rough surfaces, respectively. A is a characteristic function of the roughness concerned; it is constant for fully developed turbulence and, according to Nikuradse's experiments, its value is 8.48. Equation (8), therefore, may be written,

$$\frac{v}{v_{*r}} = 8.48 + 5.75 \log \frac{y}{k_s} \qquad (9)$$

Equation (7) will be used to determine the value of τ_g from the measured velocity profile. If the measured velocity is plotted against the logarithm of the distance from the smooth surface, a straight line results,

$$v = m_g + n_g \log y \qquad (10)$$

whose slope, n_g, determines τ_g at once. By comparison of Equations (7) and (10), $n_g = 5.75\, v_{*g}$, or,

$$v_{*g} = \sqrt{\frac{\tau_g}{\rho}} = \frac{n_g}{5.75} \qquad (11)$$

The slope of the profile at the smooth surface can be obtained graphically with good accuracy. (For this determination of v_{*g} it is assumed that the friction layers on the smooth surfaces and the rough surfaces do not affect each other. That this is true was shown by the preliminary experiments.)

It may be simpler to use the shear-stress velocities than the shear stresses themselves. Equation (6) then takes the form:

$$v_{*r}^2 + v_{*g}^2 = \frac{b}{\rho}\frac{dp}{dx} \qquad (12)$$

or,

$$v_{*r} = \sqrt{\frac{b}{\rho}\frac{dp}{dx} - v_{*g}^2} = \sqrt{\frac{b}{\rho}\frac{dp}{dx} - \left(\frac{n_g}{5.75}\right)^2} \qquad (13)$$

The problem of obtaining the equivalent Nikuradse sand roughnesses, k_s, will now be considered. The constant,[5] A, in Equation (8) must be determined for each roughness investigated:

$$A = \frac{v}{v_{*r}} - 5.75 \log \frac{y}{k} \qquad (14)$$

in which k may signify the height of the highest roughness element. The numerical values of k are given in Table 1. Substituting $k = \frac{k_s}{\alpha}$ in Equation (14), and comparing with Equation (9),

$$5.75 \log \alpha = 8.48 - A \qquad (15)$$

In evaluating experimental data, a mean value of A was computed from Equation (14) for each velocity profile, based on the value of v_{*r} as given by Equation (13).

A definition is required for "distance from the rough plate." It shall be defined as the distance from an imaginary smooth plate, replacing the rough plate, and placed so that the volume of liquid in the channel remains the same. The volume of the channel with both sides smooth is accurately known (cross-section, 4 cm by 17 cm). As the volume of the roughness elements can be computed the mean channel depth and, therefore, the mean depth with both smooth and rough walls can be obtained. These depths are given in Table 1.

Experimental Results.—The results of the tests are given in Table 3. For the engineer the most important result is the evaluation of the quantity, k_s, which, substituted in Equation (5), makes possible the computation of λ for other channels and pipes of the same absolute roughness. It is also applicable to towed plates, as explained hereinafter. In Nikuradse's tests on sand roughness, fully developed turbulence existed for $\frac{v_* k_s}{\nu} > 70$. The same limit must supposedly be assumed for the other roughnesses. In the conversion to other channel depths and pipe diameters, the limit, $\frac{v_* k_s}{\nu} > 70$, should be regarded. In pipes, $v_* = \frac{V\sqrt{\lambda}}{2.83}$, and fully developed turbulent flow occurs if, $\frac{V k_s}{\nu}\sqrt{\lambda} > 198$.

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[5] This is identical to the function, $\left(v_* \frac{k}{\nu}\right)$ used by Prandtl (2).

TABLE 3.—SUMMARY OF EXPERIMENTAL RESULTS

Plate No.	$\frac{v_{*r}}{V}$	A	k_s, in centimeters	$\frac{k_s}{k} = \alpha$	Plate No.	$\frac{v_{*r}}{V}$	A	k_s, in centimeters	$\frac{k_s}{k} = \alpha$
SPHERE ROUGHNESS: $k = 0.41$ CM					CONE ROUGHNESS: $k = 0.375$ CM				
XII	0.0689	12.2	0.093	0.227	XXIII	0.0652	13.1	0.059	0.159
III	0.0881	8.92	0.344	0.838	XXIV	0.0754	10.6	0.164	0.437
I	0.120	5.68	1.26	3.07	XXV	0.0894	8.49	0.374	0.996
II	0.131	5.15	1.56	3.81					
V	0.0854	9.65	0.257	0.626					
SPHERE ROUGHNESS: $k = 0.21$ CM					"SHORT ANGLE" ROUGHNESS: $k = 0.30$ CM				
VI	0.0779	8.98	0.172	0.819	XVI	0.0856	8.56	0.291	0.965
IV	0.106	5.27	0.759	3.61	XVIII	0.101	6.67	0.618	2.05
					XVII	0.124	4.53	1.47	4.86
SPHERICAL SEGMENT ROUGHNESS: $k = 0.26$ CM					"LONG ANGLE" ROUGHNESS: $k = 0.323$ CM				
XIII	0.0590	13.8	0.031	0.118	XX	0.137	4.17	1.81	5.61
XIV	0.0631	12.7	0.049	0.186	XXI	0.167	2.28	3.70	11.9
XV	0.0763	9.89	0.149	0.571	XXII	0.179	2.33	3.56	11.75
XIX	0.0909	7.64	0.365	1.40					

In Fig. 6 is shown the curve resulting from plotting the dimensionless velocity, $\frac{v}{v_{*r}}$, against log $\frac{y}{k_s}$ for the twenty-one rough plates investigated. All profiles agree well with Equation (9), which is taken as the basis for determining k_s. The plotted points (not shown) all fell close to the curve, indicating that for all plates the universal velocity-distribution law is well satisfied. Systematic variations occurred only with very small values of $\frac{y}{k_s}$.

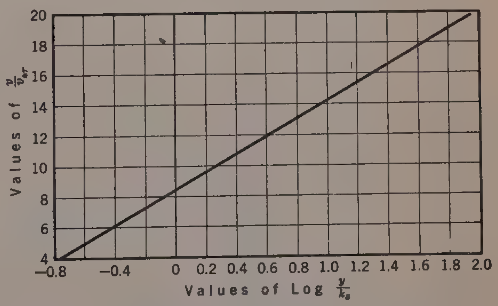

FIG. 6.—VELOCITY DISTRIBUTION FOR ALL 21 ROUGH PLATES, BASED ON EQUIVALENT SAND ROUGHNESS $\frac{v}{v_{*r}}$ AS A FUNCTION OF LOG $\frac{y}{k_s}$.

Of particular interest is the dependence of the resistance of rough plates, with the same roughness elements and the same distributional arrangement, on the roughness density. In Fig. 7, $\frac{v_{*r}}{V}$ is plotted against log $\frac{F_r}{F}$ for all roughnesses. (F_r denotes the total projection area of all the elements, on a plane perpendicular to the flow direction, and F, the area of the plate; $\frac{F_r}{F} = 0$ signifies a smooth plate.) For the sphere roughness, the greatest resistance occurred not with greatest density of roughness elements, but with $\frac{F_r}{F} = 0.4$. This is comprehensible, since with very high density of spheres, only their radius (or even less) can be considered as the roughness height. For the "long-angle" roughness the maximum value of the resistance can also be recognized; it occurs at about $\frac{F_r}{F} = 0.1$.

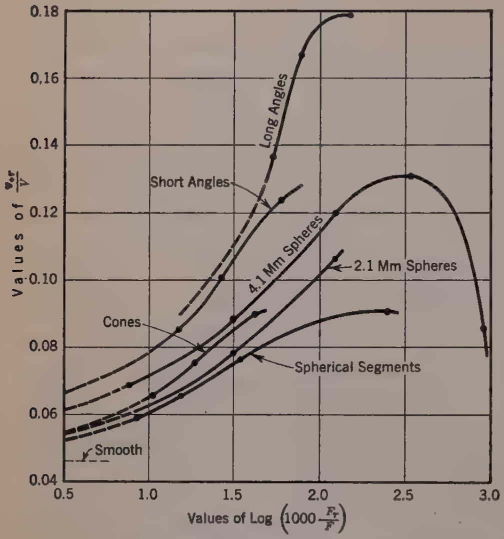

FIG. 7.—DEPENDENCE OF RESISTANCE ON ROUGHNESS DENSITY

In order to obtain a better insight into the dependence of resistance on the roughness density, a resistance coefficient, C_f, for a single roughness element will be determined. Let W_r signify the "roughness resistance" of the rough plate; that is, the difference between the total resistance, W, of the plate and the resistance, W_g, of the smooth area between the roughness elements:

$$W_r = W - W_g \quad \ldots (16)$$

Further, let v_k be the velocity at the distance, $y = k$, from the plate (k = the height of the roughness). The resistance coefficient is then defined as:

$$C_f = \frac{2\,W_r}{\rho\, v_k^2\, F_r} \quad \ldots\ldots (17)$$

v_k may be computed from Equation (8) when $\tau_r = \rho\, v_{*r}^2$ is known. If V_m is the maximum velocity and b_2 is the distance from the rough plate at which it occurs, it follows from Equation (8) that

$$\frac{V_m - v}{v_{*r}} = -\,5.75 \log \frac{y}{b_2} = -\,2.5 \log_e \frac{y}{b_2} \quad \ldots\ldots (18)$$

and by integrating between $y = 0$ and $y = b_2$ the mean velocity, $\bar{v}$, of the profile near the rough surface is found to be $V_m - 2.5\, v_{*r}$. Hence,

$$V_m - \bar{v} = 2.5\, v_{*r} \quad \ldots\ldots (19a)$$

Putting v_k for v and k for y in Equation (18),

$$V_m - v_k = -\,2.5\, v_{*r} \log_e \frac{k}{b_2} \quad \ldots\ldots (19b)$$

Subtracting Equation (19*b*) from Equation (19*a*) and dividing by $\bar{v}$,

$$\frac{v_k}{\bar{v}} = 1 - 2.5 \frac{v_{*r}}{\bar{v}} \left(\log_e \frac{b_2}{k} - 1 \right) \quad \ldots\ldots (20)$$

From Equation (16), $W_r = F\,\rho\, v_{*r}^2 - F_1\,\rho\, v_{*g}^2$. Dividing by $\rho\,\bar{v}^2$,

$$\frac{W_r}{\rho\,\bar{v}^2} = F\left(\frac{v_{*r}}{\bar{v}}\right)^2 - F_1\left(\frac{v_{*g}'}{\bar{v}}\right)^2$$

Whence,

$$C_f = \frac{2\,W_r}{\rho\, v_k^2\, F_r} = 2\,\frac{F}{F_r}\left(\frac{\bar{v}}{v_k}\right)^2\left[\left(\frac{v_{*r}}{\bar{v}}\right)^2 - \frac{F_1}{F}\left(\frac{v_{*g}'}{\bar{v}}\right)^2\right] \text{........(21)}$$

in which $\tau' = \rho(v_{*g}')^2$ is the shear stress on the smooth surface, F_1, between the roughness elements. Since $\frac{\bar{v}}{v_k}$ is known from Equation (20), C_f is expressed in terms of measurable quantities. The value of $\frac{v_{*g}'}{\bar{v}}$ depends somewhat on **R**, decreasing as **R** increases. As the second term in the bracket is usually small compared with the first term, $\frac{v_{*g}'}{\bar{v}}$ has been given a mean value, based on measurements on a smooth plate, of 0.0461. The values of $\frac{v_k}{\bar{v}}$, and the values of C_f computed therefrom by Equation (21), are given in Table 4. In Fig. 8, log $(100\, C_f)$ is plotted against log $\frac{1\,000\, F_r}{F}$. For the spherical segment, cone, and angle roughnesses, C_f is constant for small values of the roughness density. The resistance of a single roughness element is, therefore, dependent on $\frac{F_r}{F}$ only through the influence of v_k. For larger values of $\frac{F_r}{F}$, the C_f-curves drop as $\frac{F_r}{F}$ increases. In this zone the flow conditions around the individual elements

TABLE 4.—VALUES OF RESISTANCE COEFFICIENT, C_f, OF SINGLE ROUGHNESS ELEMENT

Type of roughness	Plate No.	$\frac{F_r}{F}$	$\frac{\tau_r}{\rho\,\bar{v}^2}$	C_f	$\frac{v_k}{\bar{v}}$
Spheres, d = 0.41 cm:	XII	0.00785	0.00474	0.908	0.862
	III	0.0314	0.00775	0.569	0.804
	I	0.126	0.0145	0.405	0.704
	II	0.349	0.0172	0.195	0.678
	V	0.907	0.00730	0.023	0.826
Spheres, d = 0.21 cm:	VI	0.0314	0.00606	0.520	0.702
	IV	0.126	0.0112	0.498	0.570
Spherical segments:	XIII	0.0087	0.00348	0.480	0.826
	XIV	0.0155	0.00398	0.469	0.799
	XV	0.0348	0.00582	0.388	0.767
	XIX	0.251	0.00825	0.102	0.702
Cones:	XXIII	0.0106	0.00425	0.552	0.865
	XXIV	0.0189	0.00569	0.561	0.832
	XXV	0.0425	0.00799	0.463	0.790
"Short" angles:	XVI	0.0151	0.00732	1.20	0.757
	XVIII	0.0269	0.0102	1.24	0.691
	XVII	0.0605	0.0154	1.19	0.607
"Long" angles:	XX	0.0538	0.0188	1.95	0.563
	XXI	0.0776	0.0279	3.62	0.428
	XXII	0.152	0.0321	2.54	0.378

influence each other by affecting v_k, so that the effective roughness height becomes smaller although the absolute roughness height remains the same.

The C_f-curve (Fig. 8) for the 4.1-mm spheres has a slight downward slope even at the smallest measured roughness densities. The two plates with 2.1-mm spheres yield values that fall, after a fashion, on the curve for the 4.1-mm spheres, which is to be expected from the geometrical similarity of the elements. The three plates with the long angles may be considered exceptions, as the C_f-curve first rises, then falls, with an increase in $\frac{F_r}{F}$. Treer (10) investigated identically the same roughness, although for values of $\frac{F_r}{F}$ of 0.5, 0.63, and 1.0, which are considerably greater than those in the present study. He found a decrease of resistance with increasing roughness density for these high roughness densities, which is in agreement with the present results.

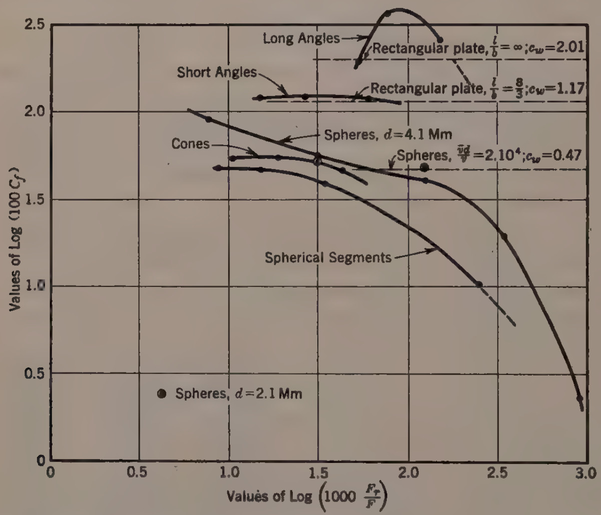

FIG. 8.—RESISTANCE COEFFICIENT OF A ROUGHNESS ELEMENT AS A FUNCTION OF ROUGHNESS DENSITY

Finally, C_f may be compared with the usual resistance coefficient, c_w, for unconfined flow. For rectangular plates placed perpendicular to the direction of flow, $c_w = 2.01$ and 1.17 for side ratios, $\frac{l}{b} = \infty$ and $\frac{8}{3}$, respectively (corresponding to the plates with long and short angles; $l = c$ and $b = k$), and is independent of **R**. For spheres with **R** $= 2 \times 10^4$ (which is about the Reynolds number for the sphere roughnesses), $c_w = 0.47$. These values are plotted in Fig. 8 and it can be seen that they agree very well with the C_f-values for small values of $\frac{F_r}{F}$. From this the following conclusion may be drawn: At low roughness densities, the resistance of a roughness element in the friction layer is practically the same as in confined flow with a velocity equal to the velocity at the distance, k, from the wall in the friction layer. For the roughnesses other than spheres, this comparison unfortunately cannot be made at this time, since c_w is not yet known.

Application to Towed Plates.—The total resistance of a ship is composed of skin-friction resistance, eddy resistance, and wave resistance, of which the first in many cases contributes by far the greatest part. The frictional resistance depends largely on the roughness of the sides of the ship. The increase in resistance resulting from roughness may amount to more than 50% of the total frictional resistance.

In pipes and channels the ratio of k to the thickness, δ, of the friction layer is constant throughout the entire length. With towed plates, δ increases with the distance from the front end of the plate, and hence, the ratio, $\frac{k}{\delta}$, which is a decisive factor in determining the resistance, decreases from front to rear. After passing through a short laminar zone, $\frac{k}{\delta}$ is comparatively large at the front of the plate, and completely developed turbulent flow occurs in that region. If the plate is long enough and the roughness small enough, the transition zone follows, and possibly even a region of laminar flow.

Prandtl, in 1927 and 1932 (11), (12), and von Kármán, in 1921 and 1930 (13), (4), showed how, from the equations of flow in a smooth pipe, the resistance equation for a smooth plate may be deduced. The derived equation agrees well with the results of tests. The same train of thought was later used by Prandtl and the writer (14, 15) to obtain equations for a rough plate, based on Nikuradse's measurements with sand roughnesses in pipes. These conversions were made possible by the discovery of the universal laws (Equations (7) and (8)) for turbulent flow in smooth and rough pipes.

The simplest relationships for the local resistance coefficient, $c_f' = \frac{2\,\tau}{\rho\, v^2}$, and the total resistance coefficient, $c_f = \frac{2\,W}{\rho\, v^2\, b\, l}$, are in the zone of fully developed turbulence, where the resistance coefficients depend only upon the relative roughness. (l = length of plate; b = width of plate; and v = velocity of plate.) The exact resistance equation is, however, still very complicated. For practical use it is more convenient to have simple interpolation formulas; the following are sufficiently accurate:

For rough plates $\left(\text{valid for } 2 \times 10^2 \leqq \frac{l}{k_s} \leqq 10^6\right)$:

$$c_f' = \left(2.87 + 1.58 \log \frac{l}{k_s}\right)^{-2.5} \quad \text{(22a)}$$

and,

$$c_f = \left(1.89 + 1.62 \log \frac{l}{k_s}\right)^{-2.5} \quad \text{(22b)}$$

For smooth plates (valid for $10^6 \leqq \mathbf{R} \leqq 10^9$):

$$c_f' = (2 \log \mathbf{R} - 0.65)^{-2.3} \quad \text{(23a)}$$

and,

$$c_f = 0.455\,(\log \mathsf{R})^{-2.58} \ldots\ldots\ldots\ldots\ldots\ldots (23b)$$

in which $\mathsf{R} = \frac{v\,l}{\nu}$. Equations (22) for rough plates correspond to Equation (5) for pipes. Accordingly, for every roughness for which k_s is known, c_f' and c_f may be at once determined. An example will be given: Length of ship, l, is 150 m; speed of ship, v, is 12 knots (6.17 m per sec); kinematic viscosity of water at 15° C, ν, is 0.00114 cm per sec; and $\mathsf{R} = \frac{v\,l}{\nu} = 8.12 \times 10^8$. By Equation (19b), c_f for a smooth ship is 1.62×10^{-3}. Assume that the ship has a roughness similar to the spherical segments (Test Plate XIII), which correspond somewhat to rivet heads ($k = 2.6$ mm; $d = 8$ mm; and $m = 40$). k_s as determined from Table 4 is 0.031 cm. Hence, from Equation (18b), with $\frac{l}{k_s} = 4.84 \times 10^5$, c_f, for the rough ship, is 2.43×10^{-3}, a 50% increase in resistance over that of the smooth plate. This is very large, but the roughness assumed is also considerable.

In order to show how the different parts of the ship contribute to the total frictional resistance, the resistances, W_1 and W_{10}, of the first and last tenths of its length will be computed in terms of W, the resistance of the whole ship. From Equation (22b),

$$\frac{W_1}{W} = \frac{1}{10}\,\frac{c_f\,\frac{l}{10}}{c_f\,(l)} = 0.149$$

and,

$$\frac{W_{10}}{W} = \frac{c_f\,(l) - \frac{9}{10}\,c_f\left(\frac{9\,l}{10}\right)}{c_f\,(l)} = 0.081$$

The first tenth of the ship's length is thus seen to produce 14.9%, and the last tenth only 8.1%, of the total resistance. It follows that a given roughness on the bow is significantly more injurious than on the stern.

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FLOW AROUND CUBES FIXED TO THE BOTTOM OF A FLUME[1]

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TRANSLATED AND ABSTRACTED BY ANDREAS LUKSCH,[3] ESQ.

From previous investigations it appears that the behavior of the flow and the bed-load directly at the bottom of the flume is the most important question in the problem of detritus movement.

In the present study, for reasons of practicability, schematized bed-load particles of cube shape were used. The objects of the experiments were: (1) To investigate the physical phenomena connected with the flow around a cube fastened to the bottom of a flume; and (2) to determine the quantitative effect of forces exerted by flowing water on such a cube. The final aim was to determine a relation between the amount of bed-load moved, the mean velocity of flow, and the size and unit weight of the bed-load particles.

The apparatus consisted of a tiltable wooden flume 15 cm wide and 18 cm high, with a measuring reach 2 m in length; a stilling-basin and rectangular weir; and a pump with a capacity of 16 liters per sec, by which the mean velocity in the flume could be varied between 1.0 m per sec and 0.1 m per sec. Ninety lead cubes, having a side dimension of 25 mm, were prepared for use as bed-load particles. Three of them were equipped with piezometer connections, two having three piezometer connections on each face. The leads from these connections were assembled in a short pipe at the bottom face of each cube, and this pipe also served to fasten the cube to the bottom of the flume. It was found subsequently that the readings on the bottom face of the cube were not reliable because the short pipe connection disturbed the flow. Therefore, a third cube was equipped with piezometer connections only on its bottom face, the leads being taken out at the top face. The first two cubes were placed 300 mm apart in the direction of flow, the readings for the down-stream cube serving as a check on those for the up-stream cube. The cube with piezometer connections on its bottom face was generally placed directly up stream from the lower cube, although in a few tests it was directly down stream therefrom. The pressure differences were so small that an inclined manometer bank ("micromanometer") was used to magnify the readings.

THE TEST SERIES AND GENERAL PROCEDURE

(1) In the main test series, in order to eliminate the influence of flume roughness, depth of flow, and shape of the particles, the roughness of the flume was held constant (a smoothly dressed wooden flume bottom was used), a

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[1] "Flow Around Cubes Fixed to the Bottom of a Flume," by V. N. Goncharov, *Transactions*, Scientific Research Inst. of Hydrotechnics, Leningrad, Vol. 17 (1935), pp. 77–112.

[2] Scientific Research Inst. of Hydrotechnics, Leningrad, U. S. S. R.

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depth of 100 mm was maintained, and the lead cubes were fastened to the bottom, but separated from it by a distance of 0.3 mm. Varying only the mean velocity, the relation between the pressure distribution on the cube faces and the mean velocity could thus be investigated. In order to study the effect of arrangement and density of particle distribution, three patterns were used: (*a*) A square, or "frontal," pattern in which the cubes were arranged in uniform rows normal to and in the direction of the flume axis; (*b*) an axial pattern in which the cubes were arranged in a single line along the axis of the flume; and (*c*) a chess-board pattern in which the cubes were arranged in staggered rows. For each pattern various degrees of density of cube distribution were used. The coefficient of this density (that is, the ratio of the volume per unit area occupied by the cubes to the volume per unit area that would be occupied by a solid layer of cubes), varied from 1.0 to 0.0069. The mean velocities in the tests of this series were 0.40, 0.55, 0.70, 0.85, and 1.0 m per sec.

(2) Additional tests were made in which the flume roughness was changed by gluing small sand grains to the smooth surface. Grains of two sizes (2 to 3 mm and 5 to 7 mm) were used. The experiments were conducted in the same manner as those for the main series, at a constant depth of about 100 mm, but using the chess-board pattern. The transverse rows of cubes were spaced 1 cm apart; in other words, the ratio of the distance l, between cubes in the direction of flow, to the length, D, of the side of the cube, was 4.0 $\left(\frac{l}{D} = 4.0\right)$.

(3) Tests were made to determine the influence of depth of flow on the pressure distribution on the cube faces, using a chess-board pattern with $\frac{l}{D} = 4.0$, mean velocities of about 0.4 and 0.9 m per sec, and depths of 147 and 50 mm.

(4) A special series of tests was carried out to investigate how the projection of one cube above a solid layer of surrounding cubes would affect the pressure distribution on the faces of this cube itself and the adjacent cubes. The bottom of the flume was covered with a solid layer of cubes, and one transverse row of cubes was adjusted to various elevations (0.3, 1.5, 3.0, 6.0, 9.0, and 11.5 mm) above the surrounding layer, forming a sill. The pressure distribution on the faces of a cube directly up stream from the sill, on the sill, and down stream from the sill was measured. The tests were carried out for mean velocities of 0.4 and 0.8 m per sec and a constant depth of 100 mm.

(5) A few experiments were also made to determine the pressure distribution on a cube surrounded by natural bed-load particles. For this purpose a piezometer cube was placed on edge, with two faces parallel to the flume walls, between pebble gravel having a mean diameter of about 35 mm. The experiments were carried out for mean velocities of from 0.42 to 0.57 m per sec, and the depth was varied from 146 to 62 mm in seven steps. The results of this series were also used as additional material on the question of the influence of depth of flow.

CONCLUSIONS FROM THE EXPERIMENTS

(1) The pressure coefficient, α, in the expression, $p = \alpha \gamma \frac{v^2}{2g}$, does not depend upon the mean velocity of flow, but it increases with the relative dis-

tance between the bed-load particles. Likewise it increases as the coefficient of density of cube distribution decreases, and as the depth of the flow decreases.

(2) For values of $\frac{l}{D}$ between 1.0 and 1.25 an up-stream cube protects the following down-stream cube from any flow reaction. With increasing values of $\frac{l}{D}$ this protecting influence decreases, and finally disappears completely at a value of $\frac{l}{D} = 12$; that is, for the latter value of $\frac{l}{D}$ the down-stream cube is no longer within the eddy wake of the up-stream cube.

(3) The pressure on the frontal face of the cube is always positive, whereas the pressure on the down-stream face is negative. The pressure on the top face of the cube is negative. On the bottom face the pressure decreases from the up-stream edge to the down-stream edge, and becomes negative only when the gap between the bottom face of the cube and the flume bottom is sufficiently wide.

(4) A change in the roughness of the flume bottom has practically no influence on the pressure distribution on the cube faces. Such an influence can be expected only if the dimensions of the roughness elements are of the same order as those of the bed-load particles themselves.

(5) The experiments with protruding cubes have shown that the pressure distribution on surrounding cubes is greatly influenced by even slight differences in elevation. This explains why, in the main test series, α had a small positive value for $\frac{l}{D} = 1.0$ (a solid layer of cubes), instead of being zero. The discrepancy is due to slight inaccuracies in the elevation of the cubes placed in a solid layer.

(6) From the experiments with pebble gravel surrounding one piezometer cube it is concluded that the relations existing between the pressure distribution and the density of cube distribution are valid also for natural bed-load. The preceding up-stream particle again affords a varying degree of protection to the down-stream particle, and frontal forces as well as lifting forces are observed also for the case of maximum flow protection.

(7) From the test data an empirical formula can be derived expressing the coefficient of completeness of bed-load motion, μ, as a function of the translatory and lift forces on the individual cube. In analyzing the process of bed-load motion, two different manners in which motion may start, can be distinguished—the particle may be pushed along the bottom, or it may be upset. The test data indicate that the two cases are nearly similar with respect to the forces required to start the motion, and the expression of equilibrium for the second case, the upsetting of the particle, can be used as the basis of the analysis. The expression for μ is,

$$\mu = \frac{0.14^2}{(\alpha + \alpha_1)^{3/2}} = \frac{0.0069\,\gamma^{3/2}\,V^3}{g^{3/2}\,(\gamma_1 - \gamma)^{3/2}\,D^{3/2}}$$

in which μ is the ratio of the volume of bed-load moved per unit area, to the volume per unit area that would be occupied by a solid layer of particles of the same height as those moved; α and α_1 are the pressure coefficients of the translatory and lift forces, respectively; V is the mean velocity of flow; D, the diameter of the bed-load particles; and γ and γ_1 are the unit weights of the water and the bed-load particles, respectively.

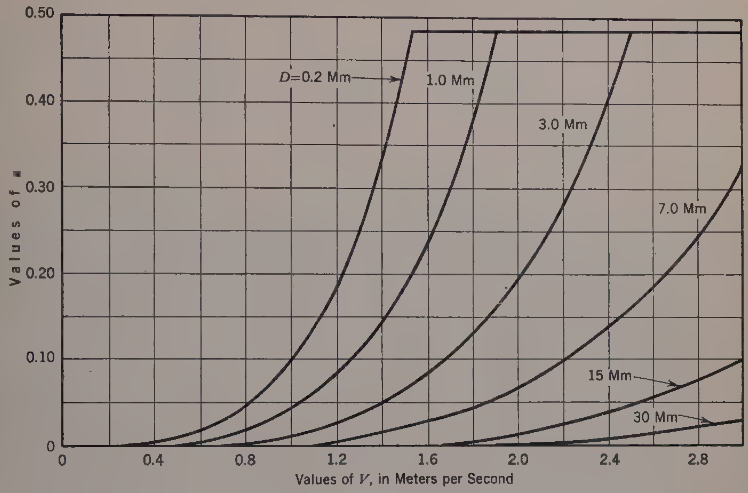

FIG. 1.—VALUES OF μ AS A FUNCTION OF V FOR VARIOUS DIAMETERS OF BED-LOAD PARTICLES

(8) Other considerations indicate the possibility of an approximate application of these results to the problem of natural bed-load movement. Fig. 1 shows the approximate relation between the coefficient, μ, and the mean velocity of flow for various sizes of bed-load particles.

FLOW IN EARTHEN CANALS OF COMPOUND CROSS-SECTION[1]

BY V. M. HEGLY,[2] ESQ.

TRANSLATED[3] AND ABSTRACTED BY CHILTON A. WRIGHT,[4] M. AM. SOC. C. E.

The research on flow in earthen canals of compound trapezoidal cross-section was undertaken to furnish data for the design of a barge canal 420 ft wide, running from Strasbourg, France, to Basel, Switzerland, and by-passing the rapids of the River Rhine. Low velocities in the shallow section would not hinder up-stream traffic, while the large discharge in the deep section would supply hydro-electric plants at the eight locks proposed.

EXPERIMENTAL LAYOUT

The three experimental canals used were constructed in the moats of an old powder plant on the Moselle River, at Metz (Fig. 1). All canals were geometrically similar in cross-section, as indicated in Figs. 2, 3, and 4. The dimensions of the first canal were one-twentieth of the proposed full-sized canal, which was 13 ft deep on one side and 33 ft on the other. The dimensions of both the second and third canals were one-fiftieth of the prototype, but the third canal was curved in plan on a 230-ft radius. The lengths of the models were 604, 706, and 230 ft, respectively. All were made of clay soil, carefully tamped and rolled on a bed slope of 0.0001, except as modified.

River water was led to each canal over sharp-crested measuring weirs provided with baffles. The depth of water in each canal was regulated by a needle-weir at the lower end, and float-gages were provided for recording the water-surface elevation. Cars, equipped with a point-gage and a Pitot tube with inclined manometer, were also provided for each canal. Special weighted wooden floats were used to measure surface velocities.

EXPERIMENTATION

The experimental procedure was similar in all canals. The desired flow was set at the measuring weir and the needle-gate was adjusted until the water surface was parallel to the bottom. (For some experiments in the 1 : 20 canal, a state of uniformly accelerating or decelerating flow was used also.) Longitudinal profiles of the water surface in each canal were taken above the top of the slope connecting the two parts of the cross-section, and on the center line of each part.

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[1] "Note sur l'Écoulement de l'Eau dans un canal à Profil Complexe," V. M. Hégly; *Annales des Ponts et Chaussées*, Mémoires et Documents, Vol. 106, No. 5, May 1936, pp. 445–528.

[2] Chf. Engr. (*Retired*), Dept. of Bridges and Roads, Metz, France.

[3] A more complete translation is on file in the Engineering Societies Library, New York, N. Y. (25 pages, 7 figures, 10 tables.)

[4] Hydr. Engr., National Hydraulic Laboratory, National Bureau of Standards, Washington, D. C.

At three stations in the 1 : 20 canal, Pitot tube traverses were made at ten verticals across the canal, but on the other canals only one station was used. The magnitude and direction of surface velocities were observed by tracing the paths of floats, released at ten points on a given cross-section. A study was made of the effect on the flow of various degrees of roughness on the canal bed. Several experiments were made in the 1 : 20 canal on a bed of gravelly soil, and in another group of tests the deep part of the canal was smooth clay soil while the shallow part was covered with large pebbles. The slope of the bed for these tests was approximately 0.001.

The curved canal was investigated with the deep part of the cross-section against first the concave side and then the convex side. After measurements with uniform flow had been made, the discharge was reduced progressively, causing decelerating flow, and the measurements were repeated. Five velocity verticals across the canal were measured with the Pitot tube at the upper end, the center, and the lower end of the canal. Comparative experiments were then made in an equivalent length of the straight 1 : 50 canal.

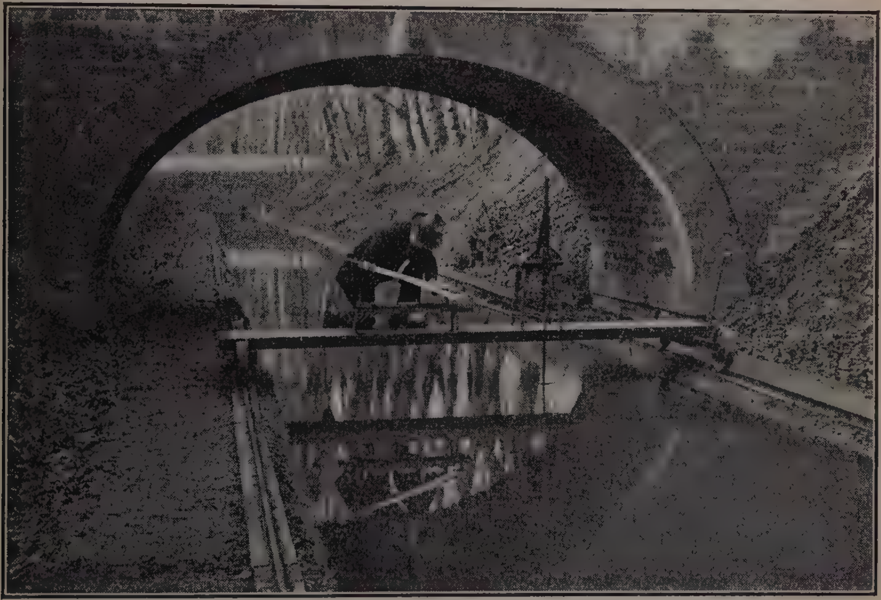


FIG. 1.—THE 1 : 50 MODEL CANAL, SHOWING CAR AND PITOT TUBE

RESULTS—STRAIGHT CANALS

Velocity Distribution.—Typical velocity-distribution diagrams obtained from the Pitot tube, are shown in Fig. 2 for two experiments in the 1 : 20 canal. Velocity ratios, deep to shallow part, were determined.

The velocity distribution in the deep part of the canal was typical of any trapezoidal canal. Over the shallow part, the curves of equal velocities are seen to remain flat nearly to the left bank, where they curved upward. At the

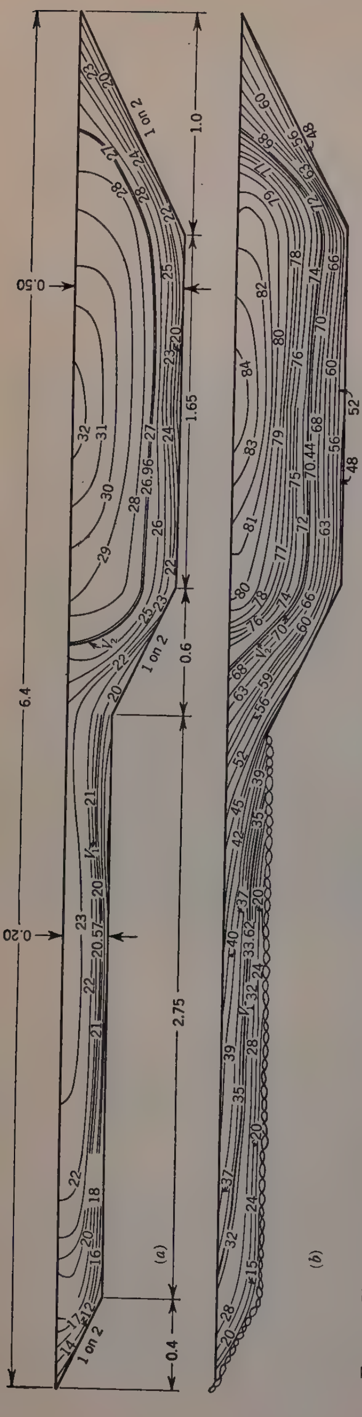
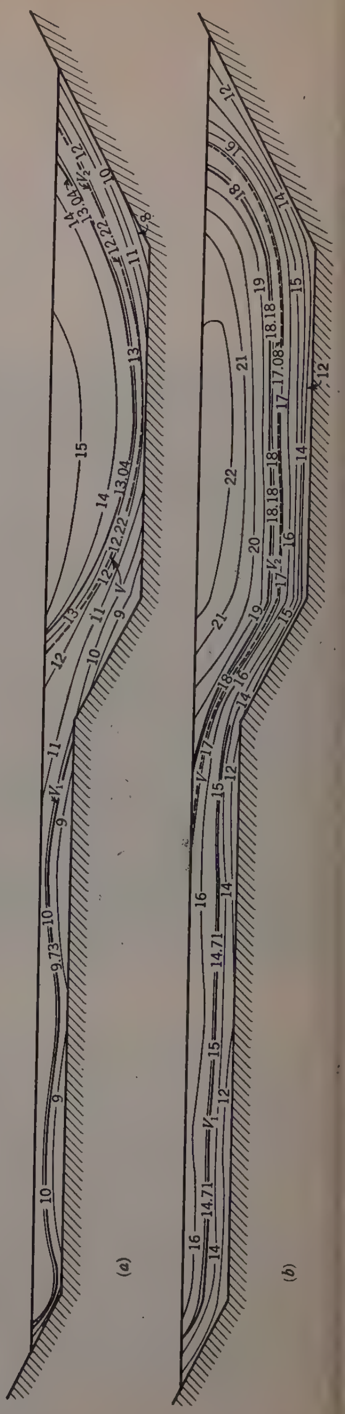


Fig. 2.—Velocity Distribution in 1 : 20 Canal (Dimensions, in Meters; Velocities, in Centimeters per Second). (a) Q = 460 Liters per Second; (b) Q = 1 100 Liters per Second



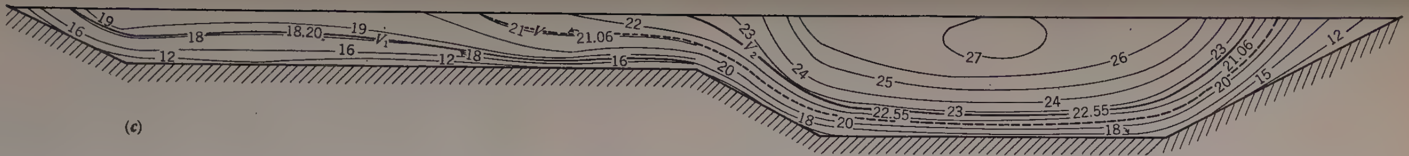


FIG. 3.—VELOCITY DISTRIBUTION IN 1 : 50 CANAL. (*a*) Q = 31.6 LITERS PER SECOND; (*b*) Q = 58.5 LITERS PER SECOND; AND (*c*) Q = 80.6 LITERS PER SECOND

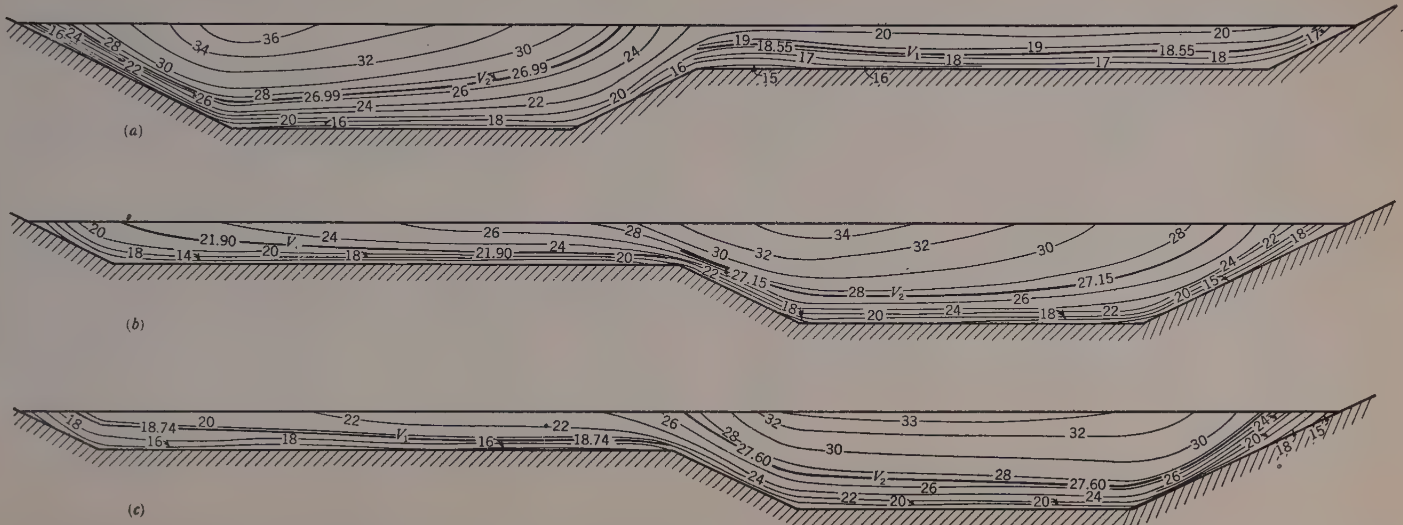


FIG. 4.—VELOCITY DISTRIBUTION IN CURVED CANAL, AND COMPARISON WITH STRAIGHT CANAL. (*a*) DEEP PART ON CONCAVE SIDE, Q = 72.4 LITERS PER SECOND; (*b*) DEEP PART ON CONVEX SIDE, Q = 76.0 LITERS PER SECOND; (*c*) STRAIGHT CANAL, Q = 75.2 LITERS PER SECOND

intermediate slope, they were deflected downward toward the higher velocities in the deep part. The rough bottom on the shallow part in Fig. 2(*b*) had the effect of causing this deflection to extend considerably over the cross-section.

Comparative diagrams for the 1 : 50 canal are shown in Fig. 3. The shapes of the velocity contours for the two model canals are identical and the relative positions of their mean velocity curves are the same.

When both parts of the cross-section were composed of the same material, the ratio of the mean velocities increased in proportion to the increase in the depth of water. The smallest ratio, 0.355, occurred for the minimum depth of 16.6 cm in the shallow part. Reducing the roughness of the walls has the same effect on this ratio as increasing the depth. Thus, for a normal depth of 20 cm in the shallow part and 50 cm in the deep part, the average ratio of mean velocities was 0.50 for the roughest walls, 0.62 for those of intermediate roughness, and 0.72 for the smoothest walls tested.

Oblique Currents.—The float observations for the 1 : 20 canal showed that, as long as the flow was uniform, no oblique currents were developed. For gradually accelerated flow, oblique currents toward the deep part were observed on the surface. The opposite was true for gradually decelerated flow. However, although these general tendencies were found, the floats were also very sensitive to accidental variations, turbulence, and wind effects.

For uniform flow, the water flowed practically independently in each of the two parallel parts of the section; practically no turbulence was observed above the connecting slope and, consequently, systematic oblique currents were not produced.

The flow in the 1 : 50 canal was very nearly uniform for most of the experiments, and no oblique currents were observed.

In the curved canal, the radius of curvature was too large to develop oblique currents at the low velocities prevailing for the condition of uniform flow. In addition the transverse water surface was practically horizontal at each station —no oblique currents.

Longitudinal Profiles.—The elevation of the water surface along the two axial longitudinal profiles was compared with that along the central axis. Both positive and negative deviations were found, but the general average was less than 1 mm. These observations emphasized that the transverse profile of the water surface is level for uniform flow and only slightly altered for nonuniform flows.

When the average surface slope differed slightly from that of uniform flow, super-elevation due to the generation of oblique currents was so small as to be hardly observable. Furthermore, these determinations were always extremely delicate, due to oscillations of the water surface, so that the exact rigorous determinations of the mean positions of the surface of the water were quite difficult.

Longitudinal profiles made in the 1 : 50 canal confirmed the previous results. The longitudinal slope was only slightly disturbed by small irregularities on the bottom, and these disturbances were found to be only local.

The elevations of the water surface across the curved canal for a given station were found to be practically the same at the five points measured. The slope of the water surface was substantially straight along the canal.

Relation of Mean Velocity to Surface Velocity—1 : 20 Canal.—In the 1 : 20 canal, the ratio of mean velocity to maximum surface velocity was nearly the same in all experiments, as well as for each part of the cross-section. The average ratio for the shallow part was 0.843 and for the deep part, 0.827. The ratios agree with those determined by Bazin and also provide a means of determining the mean velocity from measurement of the maximum surface velocity along the axis of the canal.

The proportional depth from the water surface to the line of mean velocity was also remarkably constant. Excluding the experiments with roughened bed, the average proportional depths were found to be 0.707 for the shallow part and 0.723 for the deep part, respectively, which agrees with figures given by other experimenters. For a large rectangular section the theoretical value should be 0.577.

Comparison of Computed and Measured Discharge.—The discharge computed from the Pitot tube observations averaged 0.3% higher than the measured discharge for the 1 : 20 canal and 6% lower for the 1 : 50 canal. In the 1 : 50 canal the mean velocity was computed from the maximum surface velocity obtained from the float measurements by utilizing the ratios (0.843 and 0.827) previously derived for the shallow and deep parts, respectively, and the resulting computed discharge was compared with that measured by the weir. The deviation in the twenty cases studied (selected from among the better experiments) ranged from +12% to −5% and averaged only −0.02 per cent. Thus, when this method of determining the mean velocity is applied with a little care, it yields results comparable with those given by direct Pitot-tube measurements.

Velocity-Depth Ratios.—In order to throw some light on the relation of the ratio of the mean velocity of the shallow and deep sections to the corresponding depth ratio, a study was made of experiments in which the depth ratio obviously influenced the velocity. In experiments in the 1 : 20 canal, the velocity was determined by the Pitot tube and was made comparable to the 1 : 50 canal by multiplying by the proper scale ratio, $\sqrt{2/5}$. The roughness of the walls (tamped clay soil) was practically the same in each case. Experiments from the 1 : 20 canal, in which the velocity was determined by means of floats, and the bed of the shallow part was smooth, but the deep part rough (gravelly soil), were also studied. Corresponding ratios of the depths and the hydraulic radii were obtained.

In all cases it was found that the ratio of the mean velocities tended to increase in proportion to the increase in the ratio of the depths (this is also true for ratio of the hydraulic radii). Furthermore, the ratio of the velocities increased with the value of the velocity itself. Such an increase is to be expected, since in the deeper stream the retarding effect of the shallow part is less, and the higher velocities usually occur at the greater depths. The velocity ratio should tend toward unity at greater depths; and the highest ratio, 0.91, occurred for the greatest depth ratio, 0.60. Similar results for the 1 : 20 canal showed that these relationships are independent of the scale. However, when there is a difference in roughness between the two parts of the canal section, the relationships do not hold, and $\frac{V_1}{V_2}$ is greater than for the 1 : 50 canal. This is due to the relatively greater increase of V_1, the velocity in the shallow part of the channel, which, in turn, is due to the greater smoothness of that part.

Roughness Categories—Coefficients of Velocity.—For classifying the test canals, the six roughness categories proposed by Bazin,[5] slightly modified to make them more complete, are quoted:

> Category 1: Very smooth walls (planed wood, smooth cement, brass, steel, asphalt-coated to a smooth surface).
>
> Category 2: Smooth walls (plank, ordinary concrete, stone masonry, well-jointed brick, riveted metal, etc.).
>
> Category 3: Comparatively smooth walls (concrete with a rough surface, ashlar and brick masonry with ordinary joints, tamped clay soil, etc.).
>
> Category 3(*a*): Mixed walls (walls of regular earth or covered by unjointed rubble masonry).
>
> Category 4: Canals in ordinary earth or rivers with a fine sand or gravel bottom.
>
> Category 5: Canals of exceptional roughness or rivers with a bed of large pebbles.

For determining the category to which the canals of the present experiments belong, the value of C was computed from the equation, $C = \frac{V}{\sqrt{R\,S}}$, in which V is the observed mean velocity. (Separate computations were made for the shallow and deep parts, using V_1 and V_2, respectively.) C is the factor, $\frac{87}{1 + \frac{\gamma}{\sqrt{R}}}$, which replaces the Chezy coefficient in Bazin's formula; γ being the roughness index.

The computed values of C were intermediate between those given in Bazin's table for Categories 3 and 3(*a*). This agreed well with the actual conditions at this time, since the canal was covered with uneven gravelly soil. Then, considering the canal as a matter of design, a value of C was selected by interpolation from the categories in the Bazin table corresponding to the actual conditions in the canal, and the mean velocities were computed.

After the canal had been covered by tamped clay soil, a somewhat smoother category (between Categories 2 and 3) was found more suitable. Finally, after the shallow part had been covered by pebbles, it was necessary to assume a somewhat rougher category for this than for the deep part, yet the difference was not as great as if the two parts had been considered as entirely independent canals. Values of C half-way between the limits of Categories 3 and 3(*a*) for the shallow part, and one-third nearer the smoother limit for the deep part, corresponded very well with those of the actual experiments. The computed velocities differed from those measured by from +27% to −28% with an average variation of 0.0.

The category indicated from the computed values of C for the 1 : 50 canal in experiments where the velocity was determined by floats was intermediate between Bazin's Categories 2 and 3. The coefficients showed a smaller degree of roughness than for the 1 : 20 canal, because of greater care taken in the construction of the smaller canal. To check the velocities, values of C half-way between those of Categories 2 and 3 were used. For the shallow part, the average difference of the computed velocities was found to be $+0.7\%$, with individual variations of from $+17$ to -9 per cent. However, for the deep part, the difference ranged from 0 to $+12\%$ and averaged $+3.5\%$, showing that a somewhat rougher category was necessary.

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[5] Given by M. René Koechlin in "Méchanisme de l'eau," 1924.

The results showed that the walls of the two canals conformed to Bazin's Categories 2, 3, and 3(*a*), and that the method used in computing the mean velocities for the two parts of the compound experimental canal, yields accurate results provided the proper category is selected.

Similitude.—Computations similar to those described were made for applying the various roughness categories of Bazin generally to canals of compound trapezoidal cross-section. It was desired to show that the law of similitude of Reech-Froude was applicable neither here nor, as commonly supposed, to open channels of other usual shapes. The computations were made on the basis of the designated ratio of the depth of water in the shallow part to that in the deep part, 2 to 5; that is, 4 m and 10 m for the prototype canal, 0.2 m and 0.5 m for the 1 : 20 canal, and 0.08 m and 0.20 m for the 1 : 50 canal. The slope chosen was 0.000082, that of the 1 : 20 canal (practically the same as the value of 0.0001 for the 1 : 50 canal).

The theoretical velocity for the proposed full-sized canal was compared with the corresponding velocities in the two model canals for each category. It was found that these ratios differed considerably from the expected ratios of similitude, $\sqrt{20}$ and $\sqrt{50}$. Furthermore, the error increased greatly as the walls became rougher.

According to the Reech-Froude law of similitude, to obtain a velocity in the prototype corresponding to that occurring in the model, a greater degree of roughness must be provided in the prototype. For example, if the shallow part of the 1 : 20 canal falls in Category 3, and the mean velocity therein is 15.6 cm per sec, the full-sized canal must fall in a category rougher than 5, if its mean velocity is to be $\sqrt{20}$ times that of the model, or 68.7 cm per sec. Similarly, it would be necessary to provide for the full-sized canal a category intermediate between Categories 3(*a*) and 4 in order that its mean velocity should be $\sqrt{50}$ times that of the 1 : 50 model. The same relation holds for the deep part.

The inapplicability of the law of similitude may be also shown directly from the experimental values, without taking any assumed values for C. This can be illustrated by comparing certain experiments for the 1 : 20 and the 1 : 50 canals. The slopes for these experiments were very nearly the same:

	1 : 20 Canal	1 : 50 Canal
V_1 (in centimeters per second)	22.9	12.9
V_2 (in centimeters per second)	31.2	19.4
Q (in liters per second)	54.8	54.8

By multiplying the values of V_1 and V_2 for the 1 : 50 canal by $\left(\frac{5}{2}\right)^{1/2}$, $V_1 = 20.4$

and $V_2 = 30.7$ cm per sec, which are slightly smaller than the actual values for the 1 : 20 canal. Multiplying the discharge for the 1 : 50 canal by $\left(\frac{5}{2}\right)^{3/2}$, $Q = 541$ liters per sec. This value is a little less than the actual discharge for the 1 : 20 canal. Thus, even for the model canals with practically the same degree of smoothness of the walls and bed, a small discrepancy still appears. If the slope of the two canals had been exactly the same, this discrepancy would have been greater. Furthermore, it would be much greater for the prototype canal, for which the dimensions would be increased twenty or thirty times.

For the full-sized canal it would be preferable to make computations directly, using actual dimensions and choosing a suitable roughness category for each part of the cross-section. Bazin's formula has been shown applicable equally to small canals and to large rivers, and should be used according to the actual conditions without depending upon relations of similarity.

RESULTS—CURVED CANAL

Velocity Distribution.—Typical cross-sections of the curved canal showing velocity curves derived from the Pitot tube observations are given in Figs. 4(*a*) and 4(*b*). The position of the maximum velocity for the curved canal was displaced toward the concave bank a distance ranging from 32 to 13 cm as compared with the similar straight canal. The displacement was found to be somewhat less for experiments in which the deep part was placed against the convex bank.

The Pitot-tube traverses were divided into three series for study: (I) Those made with the deep part of the canal against the concave bank; (II) those made with the shallow part against the concave bank; and (III) comparable traverses made in the straight 1 : 50 canal. The average value of the ratio, $\frac{V_1}{V_2}$, was found to be 0.716 for Series (I), 0.753 for Series (II), and 0.688 for Series (III). The last value differed little from that previously determined for this canal, and also agreed with the theoretical value, 0.67, determined from the designed depth ratio, $\frac{4}{10}$. The large value of the ratio for Series (I), 0.716, was due principally to the change in distribution of velocity in the deep part near the concave bank of the canal. The stronger velocities on this side were partly reduced by the friction against the bank, while the weaker velocities on the shallow part remained unchanged.

In Series (II) the increase in the ratio was more marked, because the higher velocity from the deep part invaded the shallow part and caused an increase of V_1. Furthermore, V_2, the velocity in the deep part near the concave bank, was reduced.

These experiments indicate that the effect of the curvature of the canal is to increase the ratio, $\frac{V_1}{V_2}$, and that the increase is more marked when the deep part is against the convex bank.

The total computed discharge averaged 1% lower than that measured, with a maximum variation from −10 to +6 per cent.

Surface Profiles.—The slopes of the water surface for these experiments, as determined from the longitudinal profiles, indicated that the slope for the curved canal was considerably greater than that for the corresponding straight canal, Series (III). As indicated previously, the frictional resistance was increased against the concave bank. The propelling force tends to be thus reduced and can be compensated for only by an increase in the slope of the water surface. The increase of slope was more marked for Series (I) than for Series (II), because the retarding frictional force was greater when the current was directed on to the bank of the canal than when it was directed on to the shallow part of the canal.

For experiments in which the discharge was successively reduced, the slope was definitely greater than that of uniform flow. For one experiment the depth of water on the shallow part was arbitrarily reduced from a normal of 8 cm to 5.7 cm to show the effect of excessive reduction in depth on the velocity ratio, $\frac{V_1}{V_2}$. The ratio became a minimum, lowering to 0.577.

Loss of Head Due to Curvature.—Flamant[6] refers to the studies of DuBuat on the subject of flow in curves and elbows and, after explaining how to calculate the loss of head due to the additional friction against the concave bank, gives the following formula for the slope of a watercourse on a smooth curve (metric units):

$$S = \frac{V^2}{d}\left(b + \tau\sqrt{\frac{l}{r}}\right) \qquad (1)$$

in which τ is the coefficient of fluid friction against the bank due to centrifugal force; d, the depth; l, the average width of bed at the curve; and r, the radius of curvature. Values of τ were not definitely determined, but apparently ranged between 0.0003 and 0.0005.

Equation (1) was developed for large rectangular canals. In applying it to the curved model canal, d was replaced by R, the mean hydraulic radius of the cross-section. The first term of the second member then became $S = \frac{V^2 b}{R}$. This is another form of the Chezy formula, with C replaced by $\frac{1}{\sqrt{b}}$. The value of C in the present experiments averaged close to 50, corresponding to a value for b of 0.0004, the average value suggested by Flamant. Consequently, the second term of Equation (1) must represent the loss of head caused by the effect of centrifugal force on the friction.

Equation (1) was applied to one experiment in Series (I) and a corresponding experiment in Series (II). In each case, $V = 19.3$ cm per sec; $R = 11.3$ cm; $l = 2.60$ m; and $r = 70$ m. Taking $b = 0.0004$, and $\tau = 0.0003$ (its minimum value), S was computed to be 0.000151. The actual average slope for the experiment in each series was 0.000150. Although this agreement was rather fortuitous, it indicates that Equation (1) is applicable to a typical model canal of curved plan. As Equation (1) does not take into account the developed length of the curve, it should be used with care for curves of small degree but of great length.

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[6] See the Third Edition of Flamant's "Hydraulique," specifically the parts entitled "Loss of Head Due to Elbows," p. 79, and "Theoretical Tests of Flow in Curves," p. 325.

Equation (1) was also applied to a geometrically similar full-sized canal by changing the dimensions according to scale. The coefficient, C, of Chezy's formula was changed slightly in accordance with the increased hydraulic radius, thus reducing b to 0.00033. Then, since λ, the scale ratio, is 50, $V = 0.193\sqrt{50} = 1.36$ m per sec; $R = 0.113 \times 50 = 5.65$ m; $l = 2.60 \times 50 = 130$ m; and $r = 70\sqrt{50} = 500$ m (in round numbers). Taking $b = 0.00033$, and $\tau = 0.0003$, S was computed to be 0.000154. The fact that the computed slope agrees with that of the 1 : 50 model justifies the assumption that the radius of curvature is proportional to $\sqrt{\lambda}$.

Roughness Coefficient.—Studies of the roughness coefficient were made in the same manner as for the first two canals. It was found that the effect of the radius of curvature upon C was small, since C depends mainly on the hydraulic radius and the degree of smoothness of the canal walls. Values of C for the curved canals agreed well with those of the previous straight canal. Consequently for Series (I) and (II) (curved canal), since the hydraulic radius was somewhat greater in the deep part than in the shallow part, C_2 was expected to be greater than C_1; however, the opposite was found to be the case. This was due to slight differences in the smoothness of the walls of the deep part, which occurred in spite of special care used in construction. It emphasizes the need for care in selecting a value of C for determining V. It was found necessary also to consider the curvature to some extent in the determination of C. Reference to Bazin's tables indicates that the curved canal should be placed in a category (2(*a*)), for approximately smooth walls, although the straight canal appears to be clearly in Category 2.

Similitude.—In order to compute the velocity in a full-sized canal, based upon the radius of curvature of the model canal, it would be necessary to use values of C from Category 5, or even from one a little rougher, this category being for earthen canals.

CONCLUSIONS

1.—For uniform flow in a canal of compound cross-section, when the slope of the bottom is the same in each part of the section, the slope of the water surface is also the same in each part of the section, and the transverse line of the water surface is horizontal.

2.—The discharge is divided between the two parts of the section (equal width) in proportion to the depth of the water and to the roughness of the walls in each part. Uniform flow is maintained as long as no change occurs in the direction of the canal, or in the slope, dimensions, or the roughness of the walls of either part.

3.—If the flow is steady, but non-uniform; that is, accelerating or decelerating (this type of flow differs but little from uniform flow for a navigation canal), oblique currents are produced from the shallow part of the canal to the deep part, or the reverse, depending upon whether the slope of the water surface is greater or smaller than that of the bed. These currents are somewhat irregular; they do not affect perceptibly either the mean velocity or the proportion of the total discharge flowing in each part. The oblique currents become clearly established only when the difference in slope between the water surface and the bed of the canal is considerable.

4.—In a uniform canal with a compound cross-section, and with conventionally dressed walls of earth or masonry, if the shallow part and the deep part are of the same width, and the ratio between the respective depths of water is 0.40, the ratio of the mean velocities (shallow to deep part) or the maximum velocities is about 0.67. If the ratio of depths becomes greater than 0.50, the velocity ratios increase to 0.75 or 0.80; thus the reduction of velocity in the shallow part is less accentuated. On the other hand, if the ratio of the depths drops to 0.30, the velocity ratio is reduced to 0.50. This is the most important conclusion drawn from the experiments. (The possibility of reducing the depth in an actual canal would be limited by the navigation requirements of the barges.)

5.—The equal-velocity contours derived from Pitot-tube traverses of the cross-section show identical distribution patterns for the two model canals. The patterns for each part considered separately are also similar to that of a simple trapezoidal canal, except in the region of the connecting slope. No special turbulence occurs on the water surface above the connecting slope. For the proposed full-sized canal, the pattern of velocity distribution between the two parts of the canal would be very similar to those of the model canals. Also, for any homologous vertical, the reduction of velocity from the surface to the bottom would follow the same law.

6.—When two model canals with only a small difference in scale are compared, the law of similitude of Reech-Froude does not apply exactly, but the divergence is very small. However, to extrapolate the results to the prototype canal demands too large a difference in scale. The walls of the prototype would have to be considerably rougher than those of the model in order to have the velocities at homologous points in the ratio of $\sqrt{\lambda}$.

7.—The flow in any canal with a compound cross-section, from 20 to 100 m wide, or in a river with a fairly regular deep and shallow cross-section, can be computed correctly by adding the discharges computed separately for each part of the section by the usual formulas. A fictitious vertical wall on the top of the connecting slope between the two parts is assumed in determining both the wetted perimeter and the area of each part. It is of the greatest importance to choose the correct coefficient for wall roughness, which is not necessarily the same for each part of the section.

8.—In the curved canal, the region of maximum velocity in the deep part of the section was displaced toward the concave bank. The amount of this displacement was practically the same regardless of whether the deep part is against the concave or the convex bank. When the deep part was against the concave bank, there was a noticeable increase in the ratio of the mean velocity of the shallow part to that of the deep part. When the deep part was against the convex bank, the increase was even more marked. Consequently, in both cases, the reduction of mean velocity in the shallow part was less than for the straight canal.

9.—For similarity of velocities, the radius of curvature of 70 m in the 1 : 50 canal corresponds to 500 m for the full-sized canal. For smaller radii the scale effect on the velocity will be more marked.[7] In Equation (1) the second member may be used for computing a close approximation to the additional loss of head due to curvature. However, from the data found for the radius of 70 m, it would be very difficult to compute the horizontal displacement of the position of maximum velocity, or the increase in the ratio, $\frac{V_1}{V_2}$, for any other radius.

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[7] The velocity will not be so nearly in proportion to the square root of the radius of curvature.

RESISTANCE TO FLOW IN CURVED OPEN CHANNELS[1]

BY SANJIVA PUTTU RAJU, ESQ.[2]

TRANSLATED AND ABSTRACTED BY CLARENCE E. BARDSLEY,[3]
M. AM. SOC. C. E.

INTRODUCTION

A number of investigations have been made to determine the loss of head in closed curved tubes or pipes, but researchers have given little thought to this loss in curved open channels. It is more difficult to make exact measurements on a free water surface because of the many uncertainties.

In minor investigations the head loss in open-channel bends in most cases has been neglected. In the cases of river regulation, water supply, irrigation, and high-head water-power installations, these losses have been determined approximately by empirical formulas.

A more accurate determination of these head losses in bends is desirable for better economic utilization of all canals, and, in particular, the feed canals leading to low-head power developments. The head loss, however small, tends to augment the total costs.

SELECTION OF THE BENDS

Open-channel bends can not be standardized like pipe bends. The bend length, deflection angle, radius of curvature, relation between radius of curvature and channel width, velocity of water, etc., are in each case dependent upon local conditions.

The present investigation was confined to 90° deflections. Two bends were investigated: One having a uniform width, B, of 300 mm (11.8 in.) and a radius of curvature, r, of 1 500 mm (59.0 in.); and the other having the same width but a radius of curvature of 300 mm (11.8 in.). All channel sections were rectangular in form and the height of the side walls was 200 mm (7.87 in.). The channel bed was horizontal throughout its entire length in all the test runs.

METHOD OF INVESTIGATION

The total head loss was first determined for the straight channel section. When either bend was inserted, a disturbing influence appeared in the flow that increased the head loss. Considering equal center line lengths of straight and curved channel sections, the difference between the total friction head loss and the bend resistance loss was readily ascertained by subtraction. The bend resistance loss, of course, included the losses caused by eddying, cross-currents, mixing, etc.

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[1] This research, "Versuch über den Strömungswiderstand gekrümmter offener Kanäle," by Mr. Sanjiva Puttu Raju, was performed under the direction of Prof. Dr. Ing. Dieter Thoma, Director, The Munich Hydr. Inst. of the Eng. Coll., Munich, Germany. The paper appeared in the *Transactions* of that Institute, *Mitteilungen* des Hydraulischen Instituts der Technischen Hochschule, München, Vol. 6, 1933, pp. 45 to 60. The publishers of the *Transactions* are R. Oldenbourg of Munich and Berlin.

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In analyzing the data, the bend resistance coefficient was expressed in its relation to the velocity, to the shape of the channel (ratio of depth of flow to breadth of channel), and to the Reynolds number. Also, a co-ordination of bend resistances in open channels and closed circular pipes was made.

The following symbols are applicable to the discussion (numerical subscripts refer to the stations at which measurements were made):

A = area of a given cross-section;
B = width of channel;
d = depth of channel;
$D = 4\,R_m$ = diameter of circular pipe (used in comparing losses in curved open channels to those in pipe bends), in meters;
g = acceleration of gravity;
h_b = head loss due to bend, in meters;
h_f = total friction head loss in a straight channel section (or pipe), in meters;
L = length of channel (or pipe) in which h_f occurs;
Q = discharge;
r = radius of curvature of center line of bend;
R_m = mean hydraulic radius throughout the entire significant length of the channel, in meters;
$\mathbf{R}$ = Reynolds number. For open channels, $\mathbf{R} = \frac{4\,V_m\,R_m}{\nu}$; for pipes, $\mathbf{R} = \frac{V\,D}{\nu}$;
V = average velocity at a given station, in meters per second;
V_b = mean velocity in bend, in meters per second;
V_m = mean velocity throughout the entire significant length of the channel, in meters per second;
Z = observed elevation of water surface, in meters;
ζ = bend loss coefficient (dimensionless);
λ = friction coefficient in Darcy-Weisbach equation for straight sections; and
ν = kinematic viscosity (in centimeter-gram-second units).

The bend loss coefficient, ζ, was calculated from the relationship,

$$\zeta = \frac{h_b}{\frac{V_b^2}{2\,g}}$$

The friction coefficient, λ, for straight-channel sections was evaluated from the Darcy-Weisbach equation,

$$h_f = \lambda \frac{V_m^2}{2\,g} \frac{L}{4\,R_m}$$

and the friction-head loss in pipes was expressed by the following form of the

Darcy-Weisbach equation, which was used for comparisons:

$$h_f = \lambda \frac{V^2}{2\,g} \frac{L}{D}$$

The total friction loss in the straight channel section was determined from the measurements, by equations similar to the following:

$$h_f = (Z_3 - Z_{12}) - \frac{V_{12}^2 - V_3^2}{2\,g}$$

in which $V_{12} = \frac{Q}{A_{12}}$ and $V_3 = \frac{Q}{A_3}$. The last term, of course, represents the surface lowering due to acceleration.

The temperature of the water was measured throughout the experiments and used to compute ν. The temperature varied from 12.8° to 13.6° C (55.0° to 56.5° F).

From measurements made with the bend in place, the bend loss was similarly determined by,

$$h_b = Z_3 - Z_{12} - \frac{V_{12}^2 - V_3^2}{2\,g} - h_f$$

With the apparatus precisely calibrated, by careful point-gage determinations of the elevation of the water surface, and by exact weighings of the discharge, the various desired relationships were ascertained by aid of the foregoing equations.

THE TEST FLUME AND ITS USE

The test flume (Fig. 1) was 7 m (22.97 ft) long and constructed of very smooth brass plate, 2 mm (0.079 in.) thick. Each plate was a meter in length, and joined together with the greatest exactness. They were also well braced at top and bottom, and upon filling with water, practically no deviation from true lines was discernible. Stations were selected along the straight flume at 500-mm (19.7-in.) intervals, fourteen stations in all. Additional stations were selected in the bends. In the bed, at each station, three holes were drilled for manometer or gage-well connections.

In making the test runs, every precaution was taken to prevent extraneous errors.[4]

All calculations were based on the losses between Stations 3 and 12 of the flume, thus eliminating the disturbing conditions in the approach and discharge channels, yet taking into account all stations where the effect of centrifugal force was measurable at the surface.

Taking into consideration the best performance of the apparatus, a velocity range of around 0.35 to 0.70 m per sec (1.15 to 2.30 ft per sec) was selected, corresponding to values of **R** of about 60 000 to 180 000.

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[4] The channel was even wiped out and polished after each run of the experiment. In his original paper Mr. Raju states in minutest detail how the channel was aligned and how carefully all measurements were taken. Possibly such an elaborate discussion of his technique is not warranted by the meagerness of his findings, but most certainly the effort was in the right direction. More extensive programs dealing with the resistance losses in open-channel bends should be prosecuted.—TRANSLATOR.

CONCLUSIONS

From the detailed tabulations[5] and graphs, the following conclusions were drawn:

1.—The highest computed value of λ was 0.02243 (corresponding to **R** = 64 400), and its lowest value was 0.01783 (for **R** = 174 000). A smoothed curve showing the observed variation of λ with **R** is given in Fig. 2.

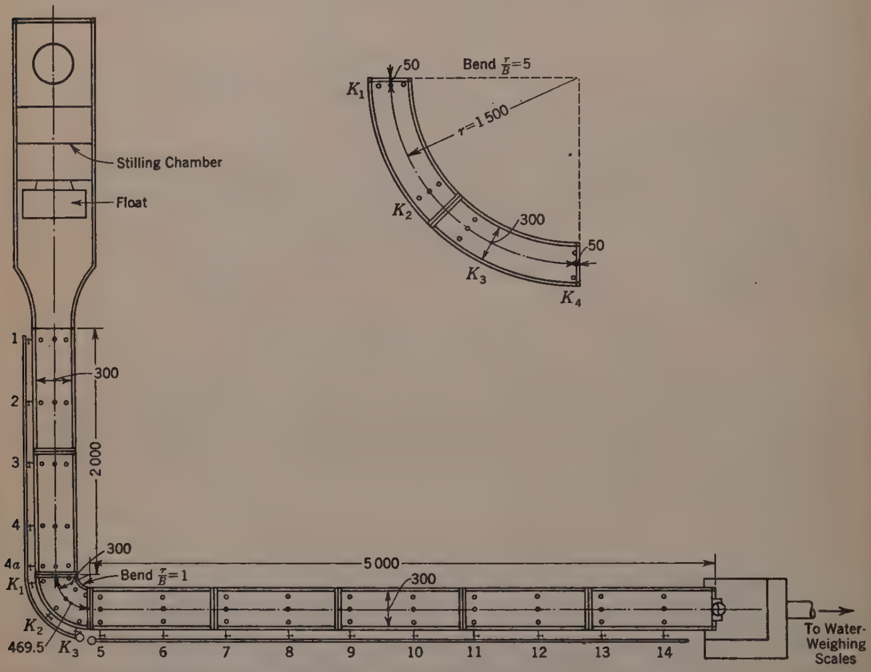

FIG. 1.—THE TEST FLUME. (DIAMETERS, IN MILLIMETERS)

2.—In contradistinction to a pipe in which the cross-section constantly remains the same, the size and shape of the cross-section of an open channel will change with the quantity of water flowing and the throttling at the outlet; therefore, the frictional resistance is not only dependent on **R** but on the shape of the channel (as defined by the ratio, d/B) as well.

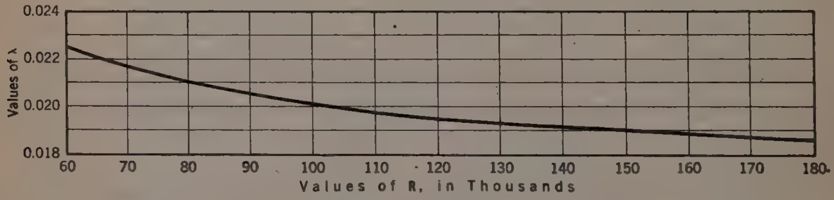

FIG. 2.—DARCY-WEISBACH FRICTION COEFFICIENT, λ, AS A FUNCTION OF **R**

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[5] Not published here.

3.—For the bend, $\frac{r}{B} = 5$, ζ did not deviate far from the value, 0.04, throughout the observed range of **R** (Fig. 3).

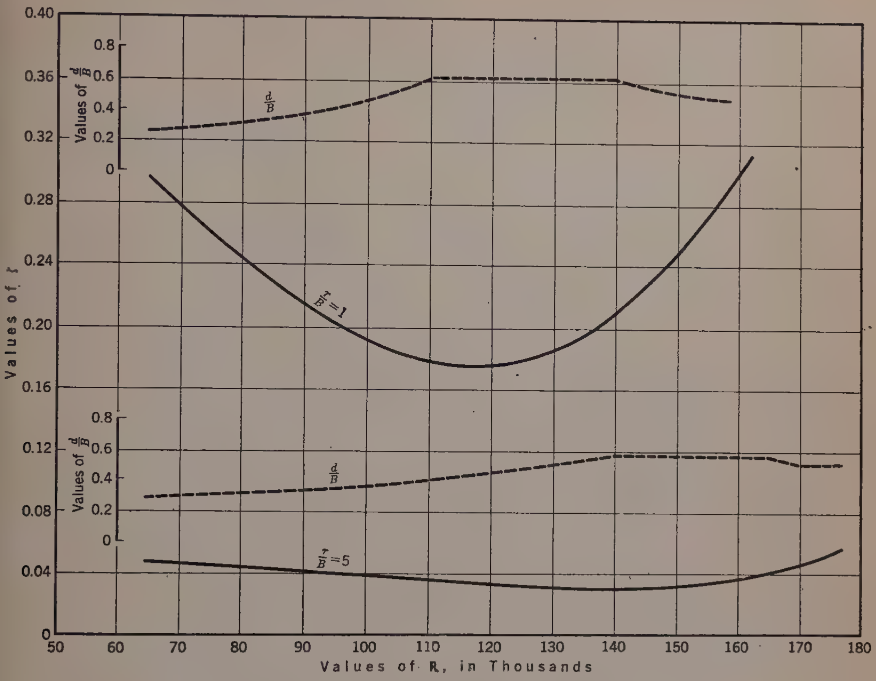

FIG. 3.—BEND RESISTANCE COEFFICIENT, ζ, AS A FUNCTION OF **R**

4.—For the bend, $\frac{r}{B} = 1$, ζ varied considerably, first decreasing and then increasing as **R** increased (Fig. 3). The values of ζ lie between 0.17 and 0.31, and are about five to nine times as large as they are for the bend, $\frac{r}{B} = 5$.

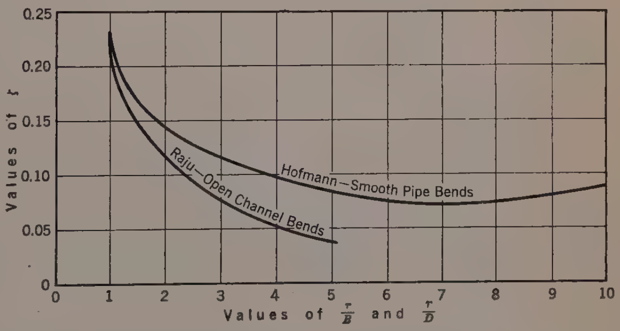

FIG. 4.—BEND RESISTANCE COEFFICIENT, ζ, AS A FUNCTION OF r/B (IN OPEN CHANNELS) AND r/D (IN SMOOTH PIPES)

5.—Values of ζ for bends whose $\frac{r}{B}$ lies between 1 and 5 would fall in the space between the two curves of Fig. 3.

6.—When the bend relation for open channels $\left(\frac{r}{B}\right)$ and the bend relation for smooth pipes $\left(\frac{r}{D}\right)$, as determined by Hofmann,[6] were put on the same basis and the ζ-value for pipes was computed for the same value of **R**, it was found $\left(\text{for } \frac{r}{B} = 1 \text{ and } \frac{r}{D} = 1\right)$ that ζ had the same value in pipes and open channels ($\zeta = 0.233$ for **R** $= 146\,000$). (See Fig. 4.)

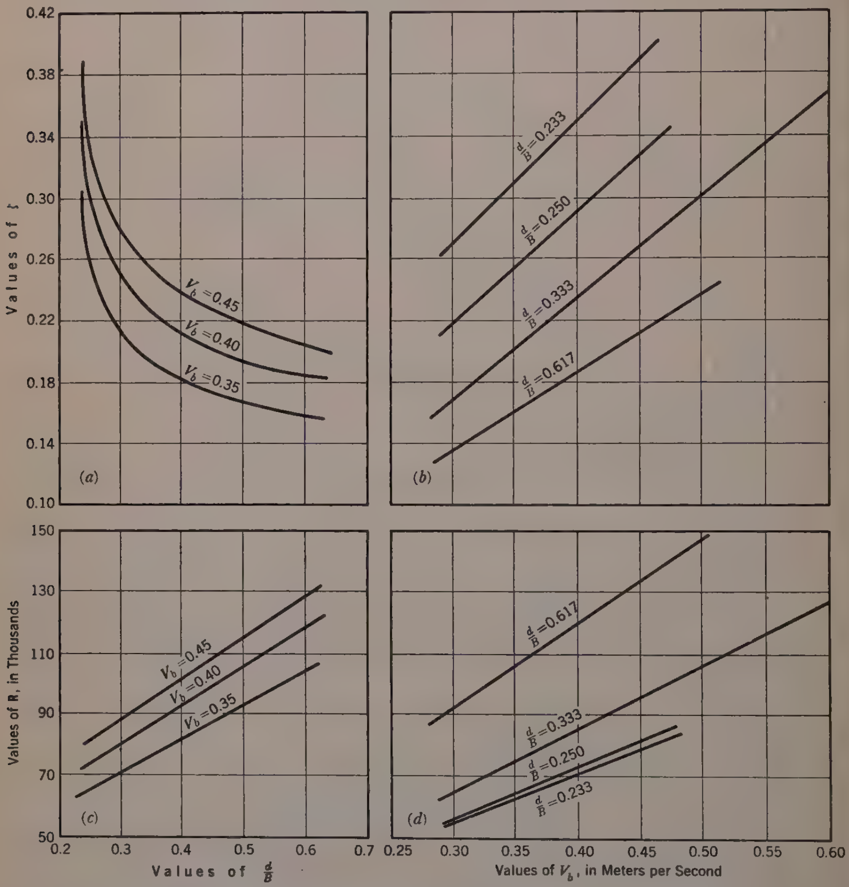

FIG. 5.—BEND RESISTANCE COEFFICIENT, ζ, FOR THE BEND $r/B = 1$, AS A FUNCTION (*a*) OF d/B, AND (*b*) OF V_b. CORRESPONDING VALUES OF **R** ARE SHOWN IN (*c*) AND (*d*)

7.—For open channels, when the bend relation, $\frac{r}{B}$, increases, ζ appears to decrease faster than in closed pipes for similar increases in $\frac{r}{D}$ (see Fig. 4). The bend resistance for the greater bend relations, $\frac{r}{B}$, for open channels seems to be smaller than for completely filled pipes.

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[6] "Der Verlust in 90° Rohrkrümmern mit gleichbleibenden Kreisquerschnitt," by A. Hofmann, *Mitteilungen* des Hydraulischen Instituts der Technischen Hochschule, München, Vol. 3, p. 45. This paper, "Loss in 90° Pipe Bends of Constant Circular Cross-Section," by the late Albert Hofmann, was translated from the German by Woodburn, Bardsley and Gutmann, and appears in *Transactions*, Hydr. Inst. of the Munich Technical Univ., p. 29; published in 1935 by the Am. Soc. of Mech. Engrs., New York, N. Y. The paper in German was printed in 1929.

8.—For the bend, $\frac{r}{B} = 1$, the influence of the form of the cross-section $\left(\text{as represented by the ratio, } \frac{d}{B}\right)$ on ζ is indicated in Fig. 5(*a*). (Observations on which these curves are based were made at depths of 70, 75, 100, and 185. B was constant at 300 mm.)

For equal water velocities, ζ decreases with increase in $\frac{d}{B}$. This decrease is rapid to a depth of about one-third the width, and then slower to a value of approximately one-half the width. It appears that ζ approaches nearly a constant value when $\frac{d}{B} = \frac{2}{3}$.

9.—The coefficient, ζ, increases with the velocity (Fig. 5(*b*)).

10.—In this experiment, the bends caused a difference in elevation of the water surface on the outer and inner sides of the channel for a distance of 1.3 B before and 2 B after the bend.

11.—The necessary conditions for small bend losses, for a given water velocity, are: (*a*) A large value of the ratio, $\frac{r}{B}$; and (*b*) a large value of the ratio $\frac{d}{B}$, wherever possible about $\frac{d}{B} \doteq \frac{2}{3}$.

RELATIONSHIP BETWEEN PRIMING HEADS OF MODELS AND OF PROTOTYPE OF SELF-PRIMING SIPHON[1]

BY ALESSANDRO VERONESE,[2] ESQ.

TRANSLATED AND ABSTRACTED BY F. L. PANUZIO,[3]
JUN. AM. SOC. C. E.

The priming head of a self-priming siphon is defined as the difference of elevation between the overflow sill and the water level in the forebay at which the siphon is completely primed, or the head that is created by complete vacuum in the siphon throat. The evaluation of this quantity is important: (1) Because it is an index of the sensitiveness of the structure; and (2) because in each case it is needed to understand and to establish the basic theory free of the effect of any works adjacent to the siphon.

The present experimental research was carried out to set up the relationship that exists between the priming head, *h*, of a model siphon and the priming head of its prototype. The experiments were performed on three siphons of different types (Fig. 1). For each type, three models of different scales, $\frac{1}{\lambda}$, were used: Model I (Camuzzoni Canal): $\lambda = 20$, 10, and 5; Model II (Carron): $\lambda = 20$, 12, and 8; and Model III (S. Caterina): $\lambda = 25$, 15, and 8.

The models were constructed of cement, with glass side-walls through which the priming phenomena could be observed. They were installed at the end of a canal, supplied through a calibrated orifice. Two piezometers, inclined in order to amplify the movement of the liquid, indicated, respectively, the elevation, *h*, of the water level in the forebay, and the elevation, *h'*, of the pressure or vacuum in the upper throat of the siphon. These elevations are referred to the overflow sill.

The elevation, *h'*, is measured, first, to obtain the changes in the internal pressure of the siphon during the period of priming; and, second, to determine the exact point of complete priming (which is indicated by an instantaneous increase of *h'*). The values of *h'* at different times during the priming period indicate the different priming characteristics of the test siphons. Thus, throughout the priming period:

(1) For models of the Camuzzoni siphon, $h' < h$. This indicates that the upper throat pressure was greater than atmospheric pressure.

(2) For models of the S. Caterina siphon, $h' > h$, indicating a pressure less than atmospheric pressure.

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[1] "Ricerche sulla relazione che intercede tra l'altezza di adescamento dei sifoni autolivellatori sperimentati in modello e quella dell'originale," by Alessandro Veronese, *L'Energia Elettrica* (Milan), Book VII, Vol. XI, July, 1934.

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(3) For models of the Carron siphon, $h' \cong h$; that is, the internal pressure was about equal to atmospheric pressure.

MINIMUM PRIMING HEAD

To establish a steady flow condition which could be sustained for some time without priming the siphon, small discharges were used to start the experiments. After the forebay level was allowed to fall below the elevation of the overflow sill, the experiment was continued by increasing the discharge very slowly until the level, h_m, which caused priming was reached. This elevation was considered as the minimum for priming.

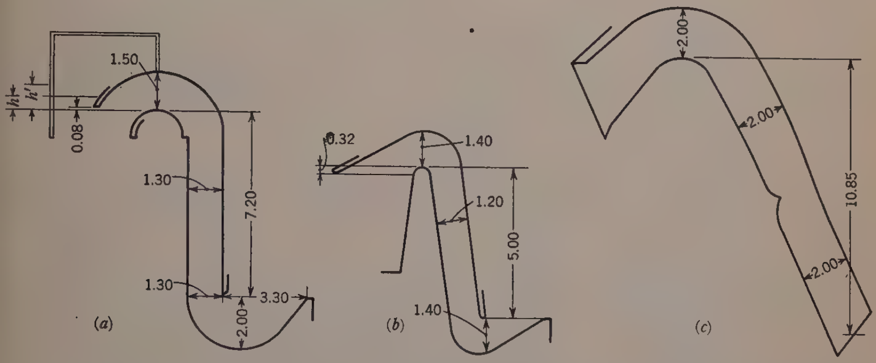

FIG. 1.—INTERNAL OUTLINES OF PROTOTYPE SIPHONS (DIMENSIONS, IN METERS). (a) CAMUZZONI; (b) CARRON; (c) S. CATERINA. THROAT WIDTHS, RESPECTIVELY, 2.0, 1.5, AND 3.0 METERS

Subsequently, experiments with larger discharges were performed to determine the priming time for increased mean velocities of rise in the forebay. The observations showed that the priming head in these experiments was always larger than the minimum.

The model results, as compared to observations made on the prototype ($\lambda = 1$), are shown in Table 1. According to the laws of geometric similitude, the relationship between the priming heads of model and prototype should be, $h_0 = \lambda h_m$, in which h_0 is the priming head in the prototype. However, this is not verified by Table 2, which gives the values shown in Table 1, multiplied by λ. The product, λh_m, instead of being constant and equal to h_0, has values differing by a function of λ. This function can be represented graphically when the values of h_m are obtained for at least three different scale models of the same type of siphon (Fig. 2).

TABLE 1.—MINIMUM PRIMING HEAD, h_m, IN MODELS AND PROTOTYPES

Type of siphon	h_m, in Centimeters, for λ =							
	25	20	15	12	10	8	5	1
Camuzzoni	....	2.65	....	....	2.81	....	3.37	13.0
Carron	....	7.37	....	7.33	....	7.45	....	18.3
S. Caterina	4.82	....	4.95	....	....	6.90	....	40.0

The value of h_0 can then be determined by extrapolation. In the present case, of course, field measurements of h_0 were available to confirm this step.

TABLE 2.—VALUES OF $\lambda\ h_m$

Type of siphon	h_m, IN CENTIMETERS, FOR λ =							
	25	20	15	12	10	8	5	1
Camuzzoni	....	53.0	....	....	28.10	....	16.85	13.0
Carron	....	147.4	....	87.96	....	59.60	....	18.3
S. Caterina	120.50	....	74.25	....	....	55.20	....	40.0

Study of Table 2 and Fig. 2 shows that the equation, $\lambda\ h_m = f(\lambda)$, has a different form for each type of siphon tested; that is, a general law that gives h_0 as a function of λ and of h_m does not exist. Hence, the only way to determine the value of h_0 for a proposed siphon is to test at least three models, of different scales, of the same type of siphon.

On the other hand, cases arise in which only an approximate indication of the value of h_0 is desired. This point is illustrated, for example, by some projects that have the design fixed by proposed designs and by contracts. In such cases, given the proposed internal outline of the siphon, one can determine from tests on a single model a value of h_0 that should at least indicate the advisability of continuing the studies or of immediately changing the proposed design. This approximate value of h_0 can be determined from the value of h_m for a single model, when h_0 is made to depend on h_m, on λ, and on the priming time, T_m, in the model for the minimum priming head, h_m.

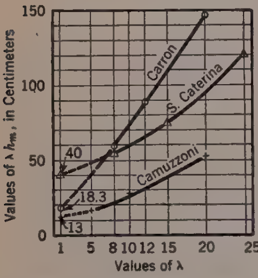

FIG. 2.—EXPERIMENTAL AND EXTRAPOLATED VALUES OF $\lambda\ h_m$ AS A FUNCTION OF λ

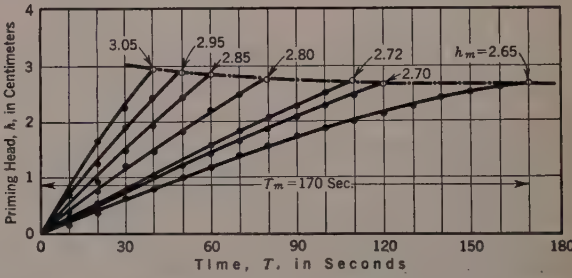

FIG. 3.—TYPICAL CHART OF h AS A FUNCTION OF T, TOTAL PRIMING TIME (CAMUZZONI-TYPE MODEL, $\lambda = 20$)

PRIMING TIME

The priming time, T, in the present investigation is defined as the interval between the start of the overflow and the instant at which the siphon is completely primed. Such a period is divided into two parts: (1) The interval between the start of the overflow and the moment at which the water elevation reaches the priming head; and (2) the interval between that moment and the completion of priming. (Some writers define the priming time as comprising only the second phase, which is very short—only a few seconds—and which is approximately constant regardless of the rate of rise of the water in the forebay.)

The first phase is preparatory and its duration influences the priming head. If it is sufficiently short—that is, if the rate of rise of water surface in the forebay is sufficiently great—complete priming no longer takes place at the minimum priming head, h_m. Hence, T_m may be defined as the minimum value of T for which the siphon primes itself at the head, h_m (see Fig. 3).

Experimental values of T_m, and other pertinent data, are given in Table 3. It appears that $\frac{T_m}{\sqrt{\lambda}}$ is approximately constant. Therefore, T_0, the priming time for the minimum priming head for the prototypes ($\lambda = 1$), should be equal to approximately 37, 60, and 32 sec for the Camuzzoni, Carron, and S. Caterina siphons, respectively. As a check on this hypothesis, it was possible to perform an experiment only on the Carron prototype, for which T_0 was accurately found to average 60 sec.

TABLE 3.—VALUES OF T_m AND $\frac{T_m}{\sqrt{\lambda}}$

CAMUZZONI TYPE			CARRON TYPE			S. CATERINA TYPE		
λ	T_m, in seconds	$\frac{T_m}{\sqrt{\lambda}}$	λ	T_m, in seconds	$\frac{T_m}{\sqrt{\lambda}}$	λ	T_m, in seconds	$\frac{T_m}{\sqrt{\lambda}}$
20	170	38.03	20	270	60.40	25	165	33.00
10	120	37.97	12	210	60.69	15	120	31.00
5	80	35.71	8	170	60.07	8	90	31.80
Approximate average		37	Approximate average		60	Approximate average		32

An approximate formula that gives h_0 in terms of h_m, λ, and T_m of a single model, can be set up as follows: Assume the equation,

$$h_0 = f(\lambda, h_m)$$

to be of the form:

$$h_0 = \lambda^n h_m \qquad (1)$$

Then,

$$n = \frac{\log h_0 - \log h_m}{\log \lambda} \qquad (2)$$

The values of n for each model investigated, computed from Equation (2), are shown in Column (4) of Table 4. Inspection shows that n varies not only from siphon to siphon but from model to model of the same siphon. However, its average value, n_m, can be expressed as a function of T_m.

From the experimental data, it appears that $\frac{T_m\, n_m}{\sqrt{\lambda}}$ is approximately constant. The values of the constant for the three siphons are 25, 22, and 25, respectively, and the average of the three is 24. Hence, it follows that, approximately, in Equation (1),

$$n = \frac{24\sqrt{\lambda}}{\nu\, T_m} \qquad (3)$$

TABLE 4.—APPROXIMATE DETERMINATION OF h_0

Siphon type	λ	h_m, in centimeters	n		h_0	
			From Equation (2)	From Equation (3)	From Equation (1)†	Actual
(1)	(2)	(3)	(4)	(5)	(6)	(7)
Camuzzoni	20	2.65	0.53	0.63	17.7	....
	10	2.81	0.67	0.63	12.0	....
	5	3.37	0.83	0.67	9.9	....
	..	....	0.68*	....	....	....
	1	....	....	....	....	13.0
Carron	20	7.37	0.30	0.40	24.4	....
	12	7.33	0.36	0.40	20.4	....
	8	7.45	0.43	0.40	17.1	....
	..	....	0.36*	....	....	....
	1	....	....	....	....	18.3
S. Caterina	25	4.82	0.66	0.73	50.5	....
	15	4.95	0.77	0.77	39.9	....
	8	6.90	0.84	0.75	32.8	....
	..	....	0.76*	....	....	....
	1	....	....	....	....	40.0

* Average value, designated n_m. † Using values of n from Column (5).

In Table 4, Column (6), Equation (1) is applied to the data from each model to predict the approximate value of h_0 for the prototype.

Comparison of Columns (6) and (7) indicates that the closest values of h_0 are obtained from models in which λ lies between 10 and 15, inclusive.

RATE OF RISE OF WATER IN THE FOREBAY

An inspection of Fig. 3 shows, as previously observed, that the priming head increases with a reduction in the priming time, or, in other words, with an increase of the mean rate of rise, v, of the water in the forebay. Furthermore, Fig. 4 shows that the equation, $h = f(v)$, is linear.[4]

It was previously stated that the values of T_0 (minimum) for the three prototypes should be equal to 37, 60, and 32 sec, respectively. The minimum priming heads are, respectively, 13, 18.3, and 40 cm. Therefore, the maximum rates of rise in the forebay at which the prototypes prime at the minimum elevation are, respectively, 3.5, 3.05, and 12.5 mm per sec. These rates of rise seldom occur in Nature; hence the prototype siphons generally prime at the minimum elevations.

From Fig. 4 the slope, $\tan\gamma$, of the equation, $h = f(v)$, is easily obtained. Similar curves for the other models of the same prototype yield other values of $\tan\gamma$. The value of $\tan\gamma_0$ in the prototype can be deduced through extrapolation from diagrams like Fig. 5, that give $\log\tan\gamma$ as a function of λ. Having determined the minimum priming elevation, the priming time corresponding to that of the prototype, and the value of $\tan\gamma_0$, it is possible to trace a diagram of the type shown in Fig. 4, for the prototype itself.

Intensive studies were made of the possibility of determining $\tan\gamma_0$ from a single value of $\tan\gamma$, but these experiments have shown that a practical formula cannot be found that will give data even of the accuracy of the value of h_0 as determined from a single value of h_m. Therefore, the value of tan γ_0 ought to be determined from experiments on homologous models to at least three different scales.

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[4] See, also, "Siphon-Spillway Models Tested Against Prototypes," by Herbert H. Wheaton, Assoc. M. Am. Soc. C. E., *Engineering News-Record*, August 18, 1932.

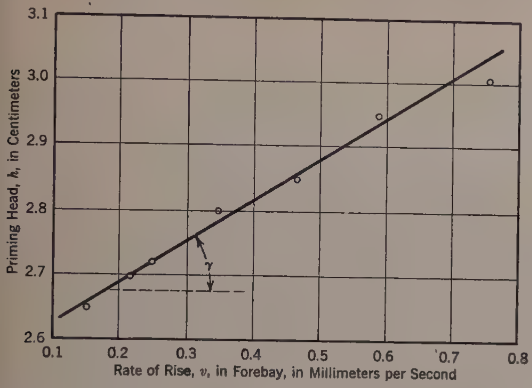

FIG. 4.—TYPICAL CHART OF h AS A FUNCTION OF v (CAMUZZONI-TYPE MODEL, $\lambda = 20$)

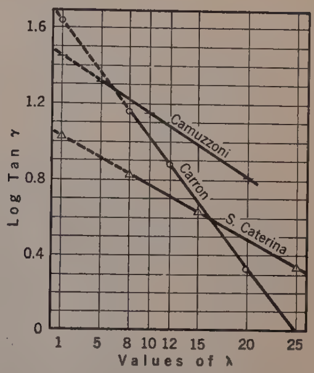

FIG. 5.—EXPERIMENTAL AND EXTRAPOLATED VALUES OF LOG TAN γ AS A FUNCTION OF λ

CONCLUSIONS

The results of the present investigation confirm the known geometric, kinematic, and dynamic relations of similitude between elements of the model of the prototype for quantities that are a function only of the elements that enter into the expression of the same relations. If the quantities involve other elements, the relations are not applicable, as in the case under investigation.

For this condition, without doubt, there exists a special law of similitude, like that expressed by Equations (1) and (3). In the majority of cases, however, Equations (1) and (3) give questionable practical results. Therefore, the system of experimenting on models to as many different scales as may be necessary to arrive at satisfactory and persuasive practical results is recommended.

LAWS GOVERNING PERCOLATION THROUGH EARTH DAMS AND THEIR FOUNDATIONS[1]

BY BURGHARD KOERNER,[2] ESQ.

TRANSLATED AND ABSTRACTED BY DONALD P. BARNES,[3]
ASSOC. M. AM. SOC. C. E.

The laws describing the percolation through rolled or hydraulic-fill dams are fundamentally the same as those describing the movement of water in natural, undisturbed soils. Whenever the structure of the material is such as to form an intricate network of fine channels which exhibit statistical regularity, the movement of water through that material can be expressed mathematically.

The forms in which water occurs in the soil have been classified by F. Zunker,[4] as illustrated in Fig. 1. "Ground-water" is characterized as completely filling the voids and moving under positive pressures which increase, in general, with depth. Two zones of "capillary" water are distinguished, that in which the voids are completely filled with water ("closed"), and that in which the voids are partly occupied with air and water vapor ("open"). Included under the term "tensile" water are the "corner" water under negative pressure, and the "water envelope" which at atmospheric pressure surrounds a pocket of air. Unlike the foregoing forms, the "hygroscopic" water has an abnormally high density and is bound closely to the particles by molecular forces.

The movement of water effected by gravity and friction forces alone is termed "pure" seepage; if, in addition, the movement is influenced by capillary forces, it is called "capillary" seepage. Movement anywhere in the completely saturated zone, however, will be "pure" seepage in the sense intended as long as the boundaries of the zone remain fixed.

For pure seepage in a completely saturated zone, the validity of the Darcy law must now be generally conceded. Although some writers still dispute its applicability, the burden of reliable experimental evidence (particularly the work of Ehrenberger[5]) is conclusive. The Darcy law is, $v = k\,J$, or $v = f\,(J)$, in which v does not represent the actual velocity of the water, but is, rather, an effective velocity defined as $v = \frac{Q}{A}$ in which Q is the quantity passing through the cross-section, A, of the soil in unit time. J represents the difference in piezometric heights between the two ends of a small soil prism through which the water is flowing, divided by the length of that prism, or, $J = \frac{d\left(z + \frac{p}{\gamma}\right)}{ds}$ in which z is the height of the water above datum; p is the hydrostatic pressure; γ is the specific weight of water; and s is the dimension in the direction of flow.

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[1] "Erforschung der physikalischen Gesetzen nach welchen die Durchsickerung des Wassers durch eine Talsperre oder durch den Untergrund stattfindet," by Burghard Koerner, 1er Congres des Grands Barrages, Vol. IV, Stockholm, 1933.

[2] Director, River Construction Div., Prussian Experiment Station for River and Marine Construction and Soil Mechanics.

[3] Associate Engr., U. S. Bureau of Reclamation, Denver, Colo.

[4] Handbuch der Bodenlehre, F. Zunker, Vol. 6, 1930.

[5] "Versuche über die Ergiebigkeit von Brunnen und die Bestimmung der Durchlässigkeit des Sandes," by R. Ehrenberger, *Zeitschrift* d. Österr. Ing. und Arch. Vereines, Nos. 9 to 14 inc., 1928.

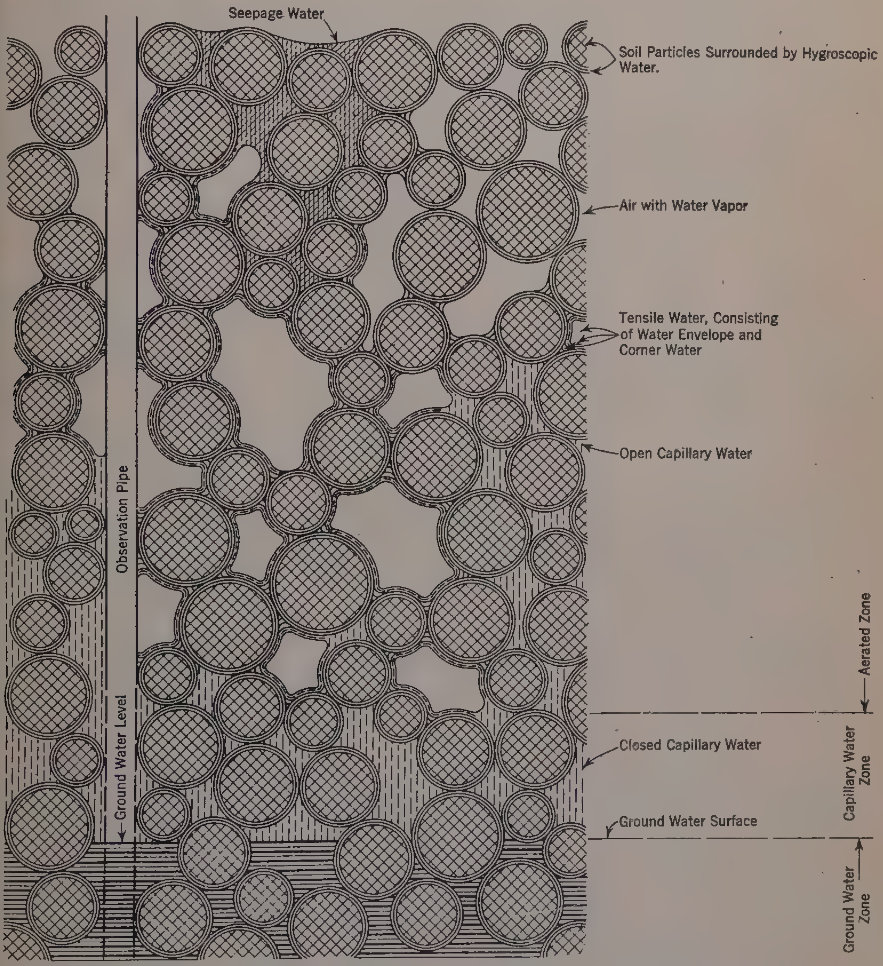

FIG. 1.—FORMS IN WHICH WATER OCCURS IN THE SOIL (AFTER ZUNKER)

The permeability, k, with dimensions of length divided by time, is a function of the viscosity of the water, the porosity of the soil, and the shape of the soil particles—hence, also, of temperature, hygroscopicity, mean effective particle size, and void volume. Numerous expressions have been developed to define k in terms of these properties. Whether such formulas are of any practical significance, or whether direct experimental determinations are quicker and more convenient, is debatable.

The Darcy law is valid for that range of velocities within which the Hagen-Poiseuille law describing the proportionality between the average velocity, V, and the gradient is satisfied: $V = \frac{g\,\rho\,a\,J}{8\,\pi\,\mu}$. Here, g is the acceleration of gravity; ρ is density; a is the cross-sectional area of the capillary channel; and μ is absolute viscosity. As the velocity increases, a point is reached at which turbulent flow begins and the simple proportionality ceases. Ehrenberger found this critical velocity to be 0.29 cm per sec (about 300 000 ft per yr) at 15° C. Other experimenters find from 0.3 to 0.4 cm per sec for capillary channels. It is reasonable, therefore, to assume that the Darcy law is applicable within any of the velocity ranges found in ground-water movement.

If, based on the equation,

$$v = k\,\frac{d\left(z + \frac{p}{\gamma}\right)}{dy}$$

the expression, $k\left(z + \frac{p}{\gamma}\right)$, is described as the potential whose derivative is the velocity, v = grad. ϕ; then, from the continuity of the velocity field, ϕ must satisfy the Laplace equation,

$$\Delta\,\phi = \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2} = 0$$

This concept of percolation as potential flow is founded on the premise that the forces produced by changes in velocity are negligible. It permits the application of the potential theory to either the mathematical or graphical analysis of percolation whenever the boundary conditions are known.

In the partly saturated or "open" capillary zone, the air offers strong resistance to displacement by the water. The water tends to pre-empt the smaller voids, and the air trapped in the larger voids materially reduces the void volume available for seepage. Schönwälder[6] found the permeability of sand in the aerated zone to be one-half that in the saturated zone, and Zunker likewise advocates the applicability of the Darcy law to flow in the aerated zone if a modified coefficient of permeability is used. More experimental evidence is needed on this point.

Flow in the capillary zones is divided into two classes, styled "attributive capillary flow" and "capillary seepage." The attributive flow is conceived as that in the fringe adjacent to the zone of positive pressure, whereas the capillary seepage may have any direction. It is doubted whether any insight into the laws governing attributive capillary flow will ever be gained.

In conjunction with the movement of water over the sealing layer or core of an earth dam, capillary seepage is important. Although Zunker supports the applicability of the Darcy law to flow of this character, the effects of the entrapped air, the tensile water, etc., would seem to cast doubt upon the validity of such application.

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[6] "Der Kulturtechniker," 1928.

Various equations have been derived for the capillary height, H_c. From considerations based on the Laplace equation there is obtained, $H_c = \frac{2\,\alpha}{\gamma\, r}$ cm, in which α is the available surface tension; γ is the specific weight of water in grams per cubic centimeter; and r is the radius of the capillary tube. Kozeny[7] and Zunker,[4] respectively, offer the formulas,

$$H = \frac{6\,\alpha\,(1 - p_a)}{\gamma\, p_a\, D_e}$$

and,

$$H = \frac{60\,\alpha\,(1 - p_a)\,U}{\gamma\, p_e}$$

Here, p_a is the porosity or the apparent void volume; p_e is the effective void volume; D_e is the effective grain size; and U is the specific surface. In practice, however, experimental determinations will probably be more satisfactory than calculations from such formulas, although the difficulty in reproducing natural modes of deposition should be emphasized.

Only in so far as the Darcy law is applicable can practical problems be treated mathematically. In determining the permeability, k, the difficulty either of extracting satisfactory "undisturbed" samples, or of testing the soil in place, must be faced. Cohesive soils have been successfully extracted and tested in the undisturbed state, but granular soils have generally failed to yield results. Furthermore, because of the non-uniformities in material which necessarily result from the effects of changing weather during construction and the method of deposition, the cost of the large number of tests that would be needed to obtain an adequate knowledge of the variation in k throughout a dam would be almost prohibitive.

The difficulty of applying the Darcy law if any variation exists in the value of k is best seen after an inspection of the equations involved. The equation,

$$v = \frac{Q}{A} = k\,J = k\,\frac{dz}{dy}$$

is applicable if two points on the boundary of the seepage zone are known; for example, given a uniform material, the up-stream and down-stream water levels, and a horizontal or inclined impervious foundation (see Figs. 2 and 3), the seepage quantity for unit width, Q_0, is,

$$Q_0 = \frac{k}{2\,y}\,(z^2 - h_0^2)$$

and the equation of the surface line is,

$$\beta\, y = H \log_e \frac{H - h_0}{H - z} - (z - h_0)$$

in which β is the tangent of the angle with the horizontal made by the foundation plane, and H is the "entrance height." Both these calculations, however, and the potential net normally determined from the same assumptions, break down if either the entrance or the outlet surface is other than vertical.

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[7] Über kapillare Leitung des Wassers im Boden . . . , by J. Kozeny, Sitzungsberichte der Akademie der Wissenschaft, Vienna, 1927.

To add to the uncertainties, Schaffernak has shown experimentally that the free surface always breaks out at some point higher than the tail-water. The existence of this phenomenon, termed the "hanging outlet," is likewise confirmed by Ehrenberger's tests.[5] The hanging outlet causes the stream lines to bend so sharply toward the face that the premises of the Darcy law are no longer fulfilled. Even without this complication the outlet elevation of the free surface could not be determined uniquely because of the disturbing influence of the attributive capillary flow. The entire process of percolation through earth dams (except between impervious boundaries, as might be the case under a masonry structure) is thus seen to be indeterminate.

The use of models for predicting the positions of saturation lines and estimating seepage quantities in earth dams is equally subject to grave limitations. Not the least of these limitations is the virtual impossibility of reproducing in a model the numerous irregularities present in the prototype as a result of variations in material, moisture content, tamping, rolling, etc.

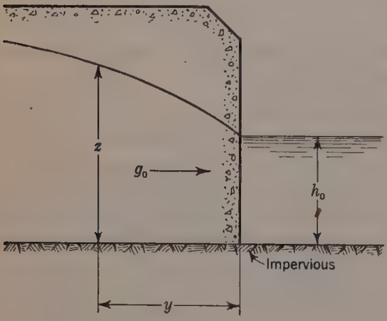

FIG. 2

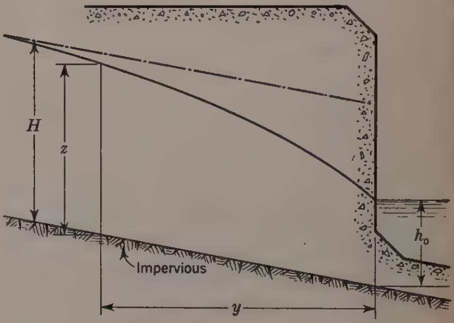

FIG. 3

If, however, such considerations are deemed of minor significance, the equations expressing the similitude relations become significant. For identical cohesionless materials in geometrically similar structures, it can easily be shown that seepage quantities are related to each other as the first power of the scale ratio, n, or, $Q_0 = n\, q_0$ for unit length in model and prototype. (Capital letters refer to the prototype and lower case letters to the model.) The velocities are equal: $V = v$. When the permeabilities of the materials in model and prototype differ, the seepage quantities and velocities are related by the following equations: $\frac{Q_0}{q_0} = n\frac{k'}{k}$, and $\frac{V}{v} = \frac{k'}{k}$, in which k' is the permeability of the model.

Unfortunately, since cohesionless soils devoid of capillary effects do not exist, these simple relationships have little practical significance. The effect of attributive capillary flow is illustrated in Fig. 4. Since the capacity for flow in the capillary zone is roughly the same in model and prototype, whereas for a material unaffected by capillary action the seepage quantity diminishes in proportion to the scale, it is clear that the model would indicate too high a discharge. The capillary forces, also, tend to act like suction forces on the percolation surface, raising it and causing a redistribution of the directions of flow so that the free surface lines and flow nets are no longer geometrically similar.

It is remotely possible that the difficulties introduced by capillary effects may be overcome by the choice of suitable liquids. If a liquid can be found such that its capillary height is $\frac{1}{n}$ times that of the liquid used in the prototype, the permeability remaining unchanged, it is only necessary to use this value as the model scale in order to maintain similitude. When, however, the model material exhibits a different permeability for the new liquid, a different model material, or a different combination of model material and model liquid must be sought such that the required capillary height and permeability are obtained. The development of such a combination is a laborious process which thus far has met with little success.

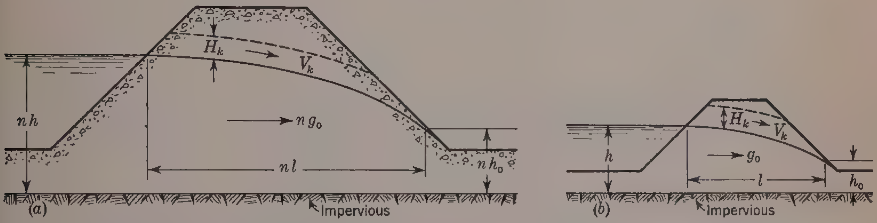

FIG. 4.—EFFECT OF ATTRIBUTIVE CAPILLARY FLOW ON MODEL RESULTS. (*a*) PROTOTYPE; (*b*) MODEL

To obtain similitude in models of cohesive soils is practically impossible because of the direct dependence of permeability upon pressure and hence its variation with depth. The changes in permeability with respect to time would also be difficult to reproduce. The effects of capillarity, however, might be negligible, since the capillary height in cohesive soils would generally be sufficient to saturate the dam to its crest in the prototype as well as in the model.

It is thought that there is little promise that either analytical treatment or model tests will ever yield significant results. Even careful observations of seepage in completed structures will seldom be applicable to new structures, because of the wide variety of conditions always encountered. Nevertheless, such measurements would add greatly to the existing knowledge and should be given every encouragement. Systematic study of foundation permeabilities is of particular importance. The construction of the "permeability profile" from tests on numerous samples, as proposed by Charles Terzaghi, M. Am. Soc. C. E., should be carried out wherever possible and the results made available to the profession.

ENERGY LOSSES IN OPEN CHANNEL EXPANSIONS[1]

BY SAMUEL FINLAY, ESQ., AND JORGE ALTAMIRANO, ESQ.

TRANSLATED AND ABSTRACTED BY J. C. STEVENS,[2]
M. AM. SOC. C. E.

This paper presents the results[3] of 104 experiments on the loss of energy occasioned by expanding the sides of a rectangular conduit at various angles.

The experiments were conducted in a canal (Fig. 1) which was lined with lumber for a length of 6 m to produce rectangular cross-sections. The entrance of the expansion was formed by two arcs framed from lumber to form a rectangular throat. The expansion consisted of straight vertical sides, meeting the sides of the flume at distances from the throat varying with the angle of dilation. Water was controlled by a head-gate and wasteway. The canal

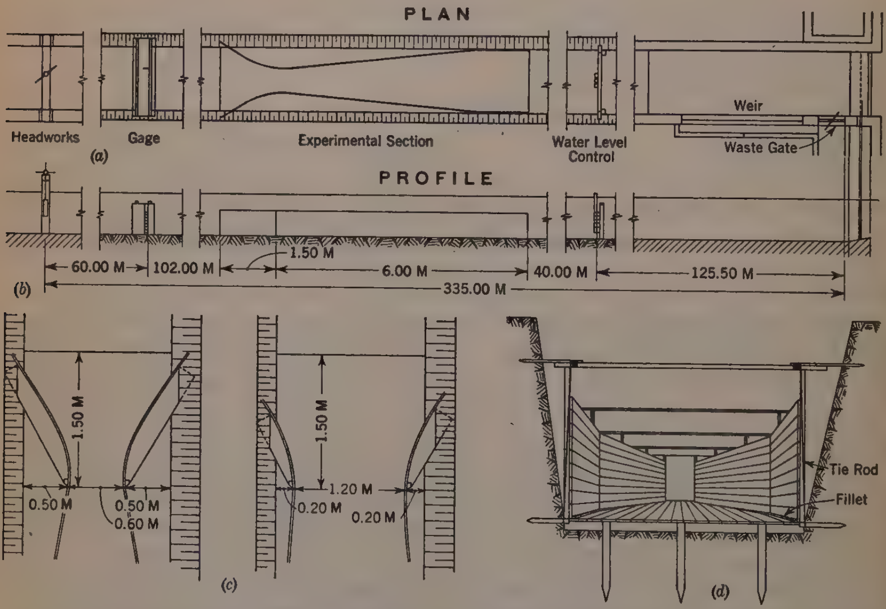

FIG. 1.—THE EXPERIMENTAL CANAL. (*a*) PLAN; (*b*) ELEVATION; (*c*) DETAILS OF THROAT; (*d*) VIEW LOOKING UP STREAM INTO THROAT

was rated by a current meter and the flow for each experiment was taken from the rating table. Discharges of 200, 300, and 400 liters per sec were used. Once the desired flow was established, the water depth through the expansion was regulated by flash-boards. This could be done between depths of 0.4 m and 0.9 m without affecting the flow.

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[1] "Perdida de Carga por Ensanches"; a Thesis presented to the Universidad Católica of Santiago, Chile, 1906.

[2] Cons. Hydr. Engr. (Stevens & Koon), Portland, Ore.

[3] TRANSLATOR'S NOTE.—The original data have been recomputed on a different basis from that used by Messrs. Finlay and Altamirano, in order to express the energy losses in terms of the kinetic energy at the smallest cross-section of the expansion.

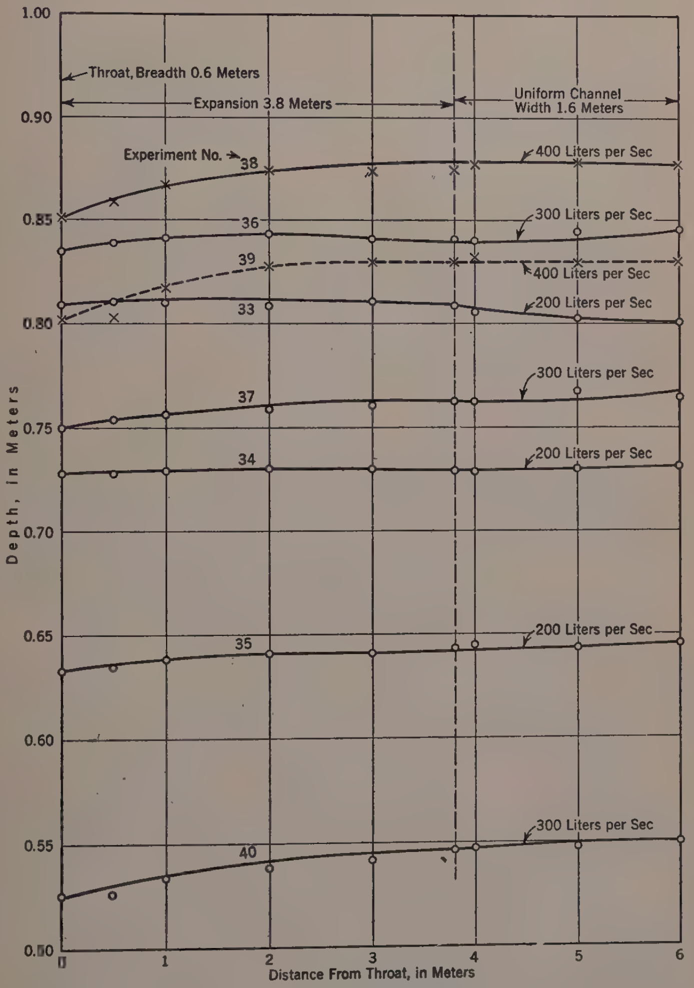

Fig. 2.—Surface Curves for Angle of Dilation of 15 Degrees

Elevations of water surface were obtained by means of an engineer's level reading on a level rod having a sharp point in the end. This point was lowered until it just touched the water surface, admitting of quite delicate adjustment. Elevations were secured in the center line of the expansion at distances of 0, 0.5, 1, 2, 3, 4, 5, and 6 m from the throat, and also at the end of the expansion. The flume had a drop of 2 cm in its 6-m length.

Care was taken in the construction to avoid "pockets" in which eddies could occur. The throat entrance was nicely curved and the sides of the expansion left the throat at a tangent to the curve.

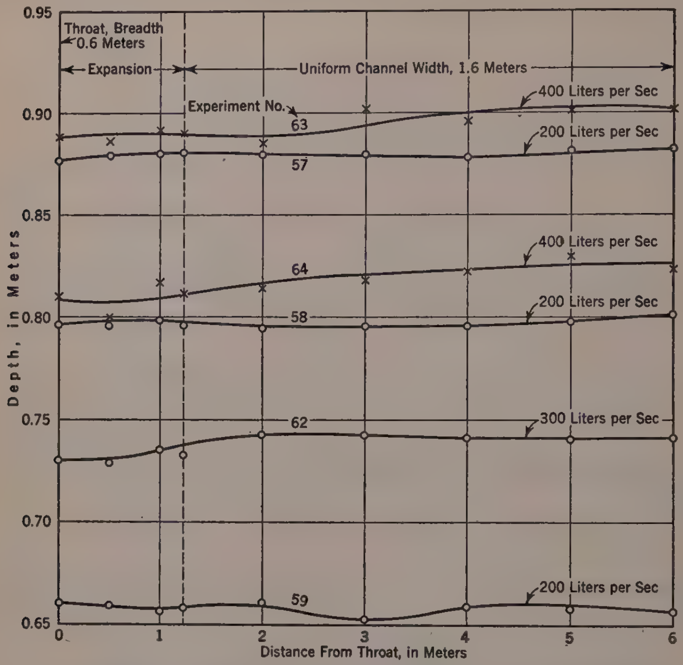

FIG. 3.—SURFACE CURVES FOR ANGLE OF DILATION OF 45 DEGREES

Areas were computed as the product of depth and width, and velocities were obtained by dividing flow by areas.

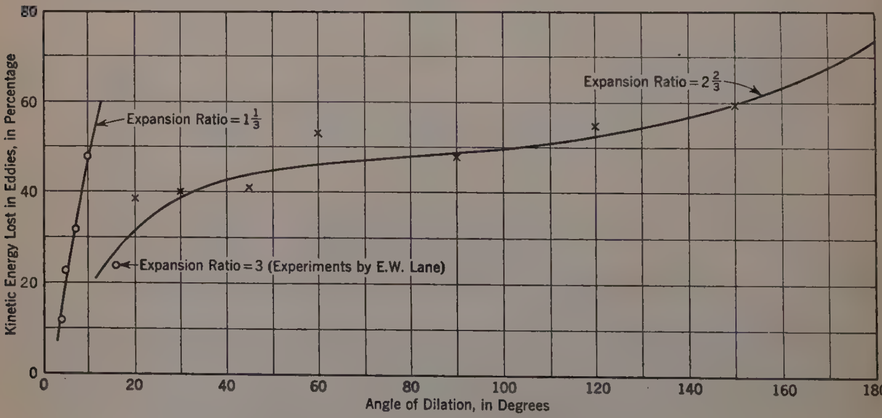

FIG. 4.—LOSS OF KINETIC ENERGY IN EXPANSION

In analyzing the data, it is evident that the total energy loss and velocity head recovery are not limited to the length of the expansion itself, but that the disturbance and adjustment continue for some distance down stream. Fig. 2 shows the surface profiles for the series of experiments with an angle of dilation of 15° and Fig. 3 shows similar curves for the series with a dilation angle of 45 degrees. For the 15° angle most of the velocity head recovery occurs within the expansion, and with the 45° angle most of the recovery occurs below the expansion.

In order to isolate the loss from impact and eddies, it is necessary to deduct the friction losses. These losses are generally small, ranging from 1% to 10% of the velocity head at the entrance. They were computed for certain controlling experiments and a set of curves was prepared from which the friction losses for the remainder of the experiments were read.

The eddy loss was taken as a percentage of the velocity head at the entrance of the expansion.

Experiments were made with throat widths of 0.6 m and 1.2 m. The width at the end of all expansions was 1.6 m. For dilation angles of 10°, and less, the "expansion ratio" (that is, the ratio of the width at the end of the transition to that at the throat) was $1\frac{1}{3}$; for greater angles it was $2\frac{2}{3}$.

The results of the tests are summarized graphically in Fig. 4, which shows the relation between the percentage of kinetic energy lost in eddies, and the angle of dilation. Distinct curves were found for each of the two expansion ratios.[4]

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[4] TRANSLATOR'S NOTE.—In connection with the studies preliminary to design of the works of the Miami Conservancy District a few experiments were made on an open-channel parabolic expansion. ("Experiments on the Flow of Water through Contractions in an Open Channel," by E. W. Lane, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. LXXXIII (1919–20), p. 1149.) The ratio of expansion was 3. If the sides had been straight the angle of dilation would have been 16 degrees. The average eddy loss (exclusive of channel friction) of five runs with discharges between 3.0 and 3.2 cu ft per sec was 24% of the velocity head at the entrance of the expansion. This result is plotted in Fig. 4; it appears to be entirely consistent with the results of Messrs. Finlay and Altamirano.

DIAGRAM FOR DETERMINING DIAMETER OF FRANCIS- OR PROPELLER-TYPE TURBINE[1]

BY K. AXEL AHLFORS,[2] ESQ.

TRANSLATED BY JAMES G. WOODBURN,[3] ASSOC. M. AM. SOC. C. E.

In the following discussion, let:

n_s = specific speed;
n = speed, in revolutions per minute;
P_B = horse-power output of wheel;
H = net head on wheel, in feet;
Q = discharge, in cubic feet per second;
w = 62.4 lb per cu ft;
e = wheel efficiency;
D = wheel diameter, in inches (measured on discharge side);
$n_1 = \frac{n}{\sqrt{h}}$ = speed of a given wheel under a 1-ft head;
$Q_1 = \frac{Q}{\sqrt{h}}$ = discharge of a given wheel under a 1-ft head;
$n_u = \frac{Dn}{\sqrt{h}} = Dn_1$ = speed of a 1-in. wheel under a 1-ft head;
$Q_u = \frac{Q}{D^2\sqrt{h}} = \frac{Q_1}{D^2}$ = discharge of a 1-in. wheel under a 1-ft head; and,
k = a numerical constant = $\frac{\sqrt[3]{n_u}}{Q_u}$.

The usual equation for the specific speed of a turbine is,

$$n_s = \frac{n\sqrt{P}}{H^{5/4}} = \frac{n\sqrt{\frac{Q\,w\,H\,e}{550}}}{H^{5/4}} \qquad \text{(1)}$$

If $H = 1$ ft,

$$n_s = n_1\sqrt{\frac{Q_1\,w\,e}{550}} \qquad \text{(2)}$$

The efficiency depends on the specific speed and the load, and can be assumed to average 0.80 at full load. With this value,

$$n_s = 0.302\,n_1\sqrt{Q_1} \qquad (3a)$$

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[1] "Diagramm zur Bestimmung des Laufraddurchmessers einer Francis- oder Propellerturbine," by K. Axel Ahlfors, *Wasserkraft und Wasserwirtschaft*, June 1, 1935.

[2] Helsingfors, Finland.

[3] Prof. of Hydr. Eng., Univ. of Wisconsin, Madison, Wis.

or,

$$n_1 = 3.31 \frac{n_s}{\sqrt{Q_1}} \ldots\ldots\ldots\ldots\ldots\ldots (3b)$$

which is a simple relation between discharge, speed under 1-ft head, and specific speed.

Assigning to n_s a constant value (for example, 75), values of n_1 are computed from assumed values of Q_1, and the curve for $n_s = 75$ is drawn with values of Q_1 as ordinates and of n_1 as abscissas. Similar curves are drawn for other values of n_s and a diagram, Fig. 1, is thus obtained which gives the discharge, Q_1, for a given turbine speed, n_1, when the specific speed, n_s, is known.

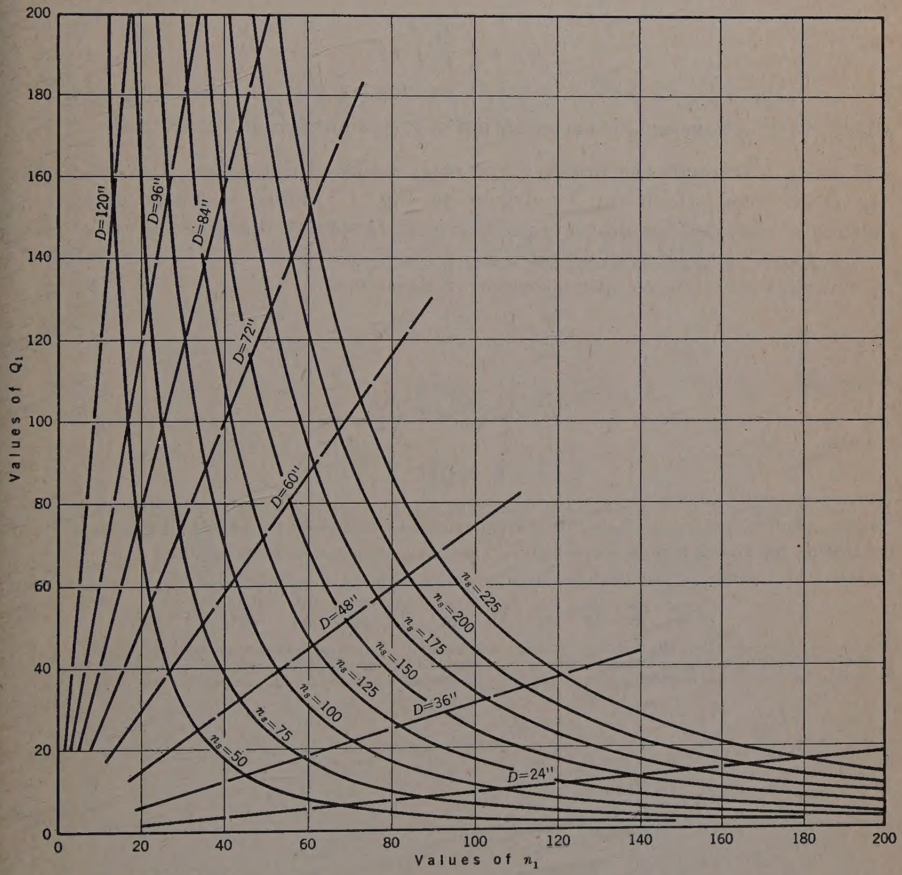

FIG. 1.—TURBINE SELECTION DIAGRAM FOR FRANCIS AND PROPELLER TURBINES

If the discharge and speed remain constant while the diameter increases, the friction losses in the wheel increase; simultaneously, however, the exit losses become smaller. Thus, for a given discharge and a fixed speed, there is a certain diameter corresponding to the least losses and the greatest efficiency, and if all conditions which affect the efficiency are considered, it is possible to set up a simple equation for the most favorable wheel diameter. Without following through the derivation,[4,5] it may be noted here that the maximum diameter is given by the equation,

$$D = K\sqrt[3]{\frac{Q}{n}} = K\sqrt[3]{\frac{Q_1}{n_1}} \qquad (4)$$

n which, for turbines with $n_s \geqq 45$, K has an average value of 53.

Equation (4) can also be written,

$$Q_1 = \left(\frac{D}{K}\right)^3 n_1$$

which, with $\frac{D}{K}$ constant, is the equation of a straight line in terms of Q_1 and n_1 and passing through the origin. For each assumed diameter, D, a different line is obtained which can be drawn on Fig. 1. Thus, a simple graphical method is provided for determining the most favorable diameter of a turbine.

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[4] *Wasserkraft und Wasserwirtschaft*, 1929, No. 12, p. 7.

[5] Translator's Note: The derivation of Equation (4) appears to be:

$$\frac{Q}{n} = \frac{Q_u D^2 \sqrt{h}}{\frac{n_u \sqrt{h}}{D}} = \frac{Q_u}{n_u} D^3$$

from which,

$$D = K\sqrt[3]{\frac{Q}{n}} = K\sqrt[3]{\frac{Q_1}{n_1}}$$

in which,

$$K = \sqrt[3]{\frac{Q_u}{n_u}}$$

Computation of K for 7 propeller-type and 22 Francis-type turbines listed on page 215 in Barrow's "Waterpower Engineering," shows that Mr. Ahlfors' average value of 53 fits the propeller type fairly closely, but that for the Francis type, K varies with n_s, approximately according to the relation, $K = \frac{4\,300}{n_s + 44}$. By this relation, n_s and K have the following corresponding values:

n_s:	30	40	50	60	70	80	90	100	110
K:	58	51	46	41	38	35	32	30	28

It appears, therefore, that the diameter lines based on $K = 53$ are not accurate enough for higher values of n_s for Francis turbines, and that additional diagrams based on values of K of, for instance, 45, 37, and 29, would be needed for determination of proper diameter of wheels over the complete range of n_s.

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R. E. Bakenhus
16 Dec. 1937

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NO. 10

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[*] Mr. Poole died October 14, 1937.